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DIFFERENTIAL EQUATIONS OF THE DEVICE SHELL WITH ARBITRARY GEOMETRY OF THE MERIDIAN LINE

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Abstract. *The theory describes the shell part of the apparatus as a surface with an arbitrary geometric outline and general acting factors. A mathematical model is constructed, and boundary conditions are formulated to determine the coordinate deformation functions of the shell part under any external disturbances. The methodology for calculating the elastic deformations of its surface with an arbitrary outline of the meridian line is also described. When analyzing the nature of a phenomenon and determining how to combat negative impacts on inertial navigation devices caused by certain factors, it is crucial to calculate the coordinate functions of the deformation of the vehicle's shell under the influence of spatial disturbances. It has been proved that inaccuracies or excessive simplifications lead to errors in the integration of the shell equations, and thus to errors in the calculation of the coordinate functions of the surface deformation and distortion of the meaning of the phenomenon. The equations for determining partial frequencies have been developed, revealing that oscillatory processes on the float's surface affect each other in all directions. Therefore, it is possible to determine the degree of influence for specific mass and dimensional modifications of the RMS. The scientific foundations have been laid for a deep analysis of the dynamics of the vehicle's shell under full-scale conditions. Additionally, a reasoned comparative analysis with the classical cylindrical modification of the float has been revealed. It is now possible to optimize the weight and size characteristics of the device. Theoretical foundations for improving the accuracy and reliability of float devices (and inertial navigation systems in general)*

are being developed based on passive methods of sound insulation and their combination with other methods, such as active and auto-compensation.
Keywords: *shell part, boundary conditions, mathematical modeling, arbitrary geometry of the shell, coordinate functions*

Introduction. Penetrating acoustic radiation generates elastic deformations on the surface of the float. Moreover, the radial components are most susceptible to this influence, which is explained by the lower stiffness of the shell in the plane of the frame compared to the other two - longitudinal and circumferential. Let us analyze the possibility of reducing this effect by abandoning the classical geometry of a circular cylinder in favor of a shell of revolution with a non-zero Gaussian curvature of the side surface - convex or concave (Fig. 1). As a partial example, the case of a circular cylinder is derived from the obtained regularities of elastic motion of the float surface in three directions. To do this, we should assume the Lamé constants $A_1 = 1$; $A_2 = R = const$.

The available scientific data make it possible to use ready-made differential equations of state for shells. At the same time, the authors simplified each specific problem, taking into account the problem being solved. Thus, in their final form, they may differ quite significantly. Therefore, there is a need to construct the differential equations of the shell dynamics in the most general form - with an arbitrary geometry of the meridian line delineation but to be able to obtain other variants of the shell of rotation - convex, concave, etc[1,2]. This creates opportunities for scientifically based conclusions on comparative analysis and the selection of ways to optimize the design.

Literature review and problem statement. To build a plane of a given angular orientation on moving objects, three-axis gyrostabilized platforms are used. They are known to have coordinate system creation errors due to the influence of a number of factors, of which cross-tie can be considered the main one [3-5]. But, basically, its drift is still caused by the errors of two-stage gyroscopes (for example, of the DUSU type), which serve as the platform's sensing elements.

An in-depth analysis of the errors of a two-stage float gyroscope under kinematic, power disturbance, unbalance of the moving part, gravity of current leads, due to the action of cross-angle velocity, and other external factors is given in the scientific literature [2]. At the same time, as it turned out, other negative factors that occur in the field but have not yet been identified require serious study. We are talking about penetrating acoustic radiation, which leads to wave processes in the suspension and additional measurement errors. And, importantly, it is necessary to create computational models that would take into account not only, and not so much, the actual effect of sound fields, but their simultaneous influence with the angular movement of the base. The computational model analyzes the effect of the joint action of acoustic radiation and low-frequency angular motion of an object on the error of a two-stage float gyroscope. The kinematic impact acts on the device through the supports, while the acoustic impact acts through the environment. The elastic movements of the float surface resulting from a sound wave, subject to the angular motion of the base, cause Coriolis moments of inertia forces to appear[6]. These forces act as a false' angular velocity on the input axis for the gyroscope and result in measurement errors (or output signal drift in the case of an integrating gyroscope).

The **purpose** of the research is to formulate a reasonable formulation of the boundary conditions of the state of a cylindrical float of an industrial-strength two-stage gyroscope. The use of a generalized approach allows for the expansion of the problem's boundaries and a better understanding of the phenomenon. The coordinate functions of the deformed surface of the float can be calculated to clarify the regularity of wave processes in the gyroscope suspension and their contribution to the appearance of additional device errors, under certain boundary conditions.

The **object** of research is a commercially manufactured two-stage float gyroscope of the DUSU2-6AS series, which is used on long-range aircraft. The device consists of two coaxial cylinders separated by a heavy liquid. The inner one contains the gyro unit itself[7].

Shell with arbitrary geometry of the meridian line.

A shell is formed by rotating any curve around the chosen axis of symmetry. Let's assume that this curve does not cross the axis of rotation (Fig. 1). The reference (inertial) and bound coordinate systems are shown in the diagram, where the following notations are used: $r = f(z)$ – rotation curve; r – is the distance from the axis of rotation to point M.

Let's take the length of the shell equal to l. Then $z \in [0, l]$.

Let φ be the coordinate that defines the position of the point M on the parallel. Then, for an infinitesimal distance between two points on the median surface π of the shell, we can write the following relation

$$dS^2 = dS_1^2 + dS_2^2,$$

where dS_1 is the arc differential along the meridian; dS_2 is the arc differential along the parallel.

Since $dS_1^2 = \left[1 + f'^2(z)\right] dz^2$;

$dS_2^2 = r^2 d\varphi^2 = \left[f(z)\right]^2 d\varphi^2$,

then

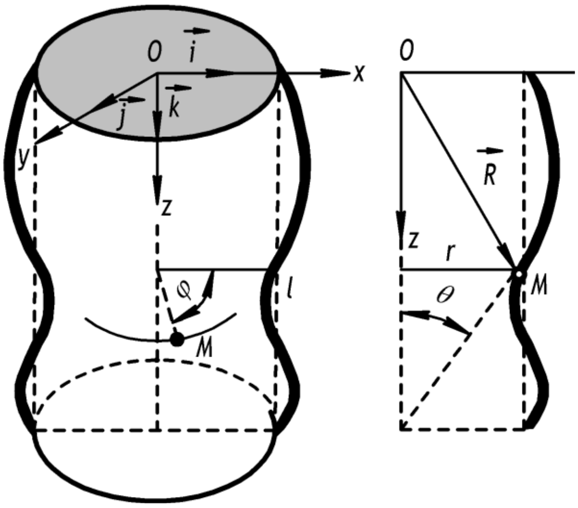

Fig. 1. Moving vehicle in the form of a shell with an arbitrary geometry of the meridian line

$$dS^2 = \left[1 + f'^2(z)\right] dz^2 + \left[f(z)\right]^2 d\varphi^2 . \quad (1)$$

Hence, it follows that the coordinates α_1 and α_2 the appropriate choice is [8,9].

$$\alpha_1 = z; \quad \alpha_2 = \varphi .$$

Then

$$A_1 = \sqrt{1 + f'^2(z)}; \quad A_2 = f(z). \quad (2)$$

This enables us to draw an important conclusion: the Lamé parameters A_1 and A_2 are functions of a single coordinate, *z*.

Other curved coordinates can also be selected as a coordinate α_i, for example, $\alpha_1 = \theta,\ \alpha_2 = \varphi$.

Let us write the equations of the shell's motion in the coordinates $\alpha_1 = z$, and $\alpha_2 = \varphi$. In this case, the expressions should be replaced with the appropriate notation:

$$U_1 \Rightarrow U_z; \quad U_2 \Rightarrow U_\varphi; \quad W \Rightarrow W; \quad \alpha_1 \Rightarrow z; \quad \alpha_2 \Rightarrow \varphi.$$

With this in mind, these equations take on a different form. The equation in the *z* coordinate is written in the form

$$\frac{Eh}{1-\nu^2}\frac{\partial}{\partial z}\left[\frac{A_2}{A_1}\frac{\partial U_z}{\partial z} + \nu\frac{\partial U_\varphi}{\partial \varphi} + \frac{\nu}{A_1}\frac{\partial A_2}{\partial z}U_z + W\left(\frac{A_2}{R_1} + \nu\frac{A_2}{R_2}\right)\right] +$$

$$+\frac{Eh}{2(1+\nu)}\frac{1}{A_1}\left(A_1\frac{\partial^2 U_\varphi}{\partial z \partial \varphi} + \frac{A_1^2}{A_2}\frac{\partial^2 U_z}{\partial \varphi^2} - \frac{A_1}{A_2}\frac{\partial A_2}{\partial z}\frac{\partial U_\varphi}{\partial \varphi}\right) -$$

$$-\frac{Eh}{1-\nu^2}\frac{\partial A_2}{\partial z}\left[\nu\frac{1}{A_1}\frac{\partial U_z}{\partial z}+\frac{1}{A_2}\frac{\partial U_\varphi}{\partial \varphi}+\frac{1}{A_1A_2}\frac{\partial A_2}{\partial z}U_z+W\left(\nu\frac{1}{R_1}+\frac{1}{R_2}\right)\right]+$$

$$+\frac{Eh^3}{12\left(1-\nu^2\right)}\frac{1}{R_1}\frac{\partial}{\partial z}\left(\frac{A_2}{A_1^3}\frac{\partial A_1}{\partial z}\frac{\partial W}{\partial z}-\frac{A_2}{A_1^2}\frac{\partial^2 W}{\partial z^2}-\frac{\nu}{A_2}\frac{\partial^2 W}{\partial \varphi^2}-\frac{A_2}{A_1R_1^2}\frac{\partial R_1}{\partial z}U_z+\right.$$

$$\left.+\frac{A_2}{A_1R_1}\frac{\partial U_z}{\partial z}+\frac{\nu}{R_2}\frac{\partial U_\varphi}{\partial \varphi}-\frac{\nu}{A_1^2}\frac{\partial A_2}{\partial z}\frac{\partial W}{\partial z}+\frac{\nu}{A_1R_1}\frac{\partial A_2}{\partial z}U_z\right)+ \tag{3}$$

$$+\frac{Eh^3}{12(1+\nu)}\frac{1}{A_1}\frac{1}{R_1}\left[-\frac{\partial}{\partial \varphi}\left(\frac{A_1}{A_2}\frac{\partial^2 W}{\partial z\partial \varphi}\right)+\frac{\partial}{\partial \varphi}\left(\frac{A_1}{A_2^2}\frac{\partial A_2}{\partial z}\frac{\partial W}{\partial \varphi}\right)+\right.$$

$$\left.+\frac{\partial}{\partial \varphi}\left(\frac{A_1^2}{A_2R_1}\frac{\partial U_z}{\partial \varphi}\right)-\frac{\partial}{\partial \varphi}\left(\frac{A_1}{A_2R_1}\frac{\partial A_2}{\partial z}U_z\right)+\frac{\partial}{\partial \varphi}\left(\frac{A_1}{R_2}\frac{\partial U_\varphi}{\partial z}\right)\right]+$$

$$+\frac{Eh^3}{12(1+\nu)}\frac{\partial}{\partial \varphi}\left(-\frac{1}{A_2R_1}\frac{\partial^2 W}{\partial z\partial \varphi}+\frac{1}{A_2^2R_1}\frac{\partial A_2}{\partial z}\frac{\partial W}{\partial \varphi}+\frac{A_1}{A_2R_1^2}\frac{\partial U_z}{\partial \varphi}-\right.$$

$$\left.-\frac{1}{A_2R_1^2}\frac{\partial A_2}{\partial z}U_z+\frac{1}{R_1R_2}\frac{\partial U_\varphi}{\partial z}\right)-\frac{Eh^3}{12\left(1-\nu^2\right)}\frac{1}{R_1}\frac{\partial A_2}{\partial z}\left(\frac{\nu}{A_1^3}\frac{\partial A_1}{\partial z}\frac{\partial W}{\partial z}-\right.$$

$$+\frac{\nu}{A_1R_1}\frac{\partial U_z}{\partial z}+\frac{1}{A_2R_2}\frac{\partial U_z}{\partial \varphi}-$$

$$\left.-\frac{1}{A_1^2A_2}\frac{\partial A_2}{\partial z}\frac{\partial w}{\partial z}+\frac{1}{A_1A_2R_1}\frac{\partial A_2}{\partial z}U_z\right.=A_1A_2\left(-q_1+\rho h\frac{\partial^2 U_z}{dt^2}\right).$$

The φ coordinate equation takes the form

$$\frac{Eh}{1-\nu^2}\frac{\partial}{\partial \varphi}\left[\nu\frac{\partial U_z}{\partial z}+\frac{A_1}{A_2}\frac{\partial U_\varphi}{\partial \varphi}+\frac{1}{A_2}\frac{\partial A_2}{\partial z}U_z+W\left(\frac{\nu A_1}{R_1}+\frac{A_1}{R_2}\right)\right]+$$

$$+\frac{Eh}{2(1+\nu)}\frac{1}{A_2}\frac{\partial}{\partial z}\left(\frac{A_2^2}{A_1}\frac{\partial U_\varphi}{\partial z}+A_2\frac{\partial U_z}{\partial \varphi}-\frac{A_2}{A_1}\frac{\partial A_2}{\partial z}U_\varphi\right)+$$

$$+\frac{Eh^3}{12\left(1-\nu^2\right)}\frac{1}{R_2}\frac{\partial}{\partial \varphi}\left(\frac{\nu}{A_1^2}\frac{\partial A_1}{\partial z}\frac{\partial W}{\partial z}-\frac{\nu}{A_1}\frac{\partial^2 W}{\partial z^2}-\frac{A_1}{A_2^2}\frac{\partial^2 W}{\partial \varphi^2}-\frac{\nu}{R_1^2}\frac{\partial R_1}{\partial z}U_z+\right.$$

$$\left.+\frac{\nu}{R_1}\frac{\partial U_z}{\partial z}+\frac{A_1}{A_2R_2}\frac{\partial U_\varphi}{\partial \varphi}-\frac{1}{A_1A_2}\frac{\partial A_2}{\partial z}\frac{\partial W}{\partial z}+\frac{1}{A_2R_1}\frac{\partial A_2}{\partial z}U_z\right)+$$

$$+\frac{Eh^3}{12(1+\nu)}\frac{1}{A_2}\frac{1}{R_1}\frac{\partial}{\partial z}\left(-\frac{A_2}{A_1}\frac{\partial^2 W}{\partial z\partial \varphi}+\frac{1}{A_1}\frac{\partial A_2}{\partial z}\frac{\partial W}{\partial \varphi}+\frac{A_2}{R_1}\frac{\partial U_z}{\partial \varphi}-\frac{A_2}{A_1R_1}\frac{\partial A_2}{\partial z}U_z+\right.$$

$$\left.+\frac{A_2^2}{A_1R_2}\frac{\partial U_\varphi}{\partial \varphi}\right)+\frac{Eh^3}{12(1+\nu)}\frac{\partial}{\partial z}\left(-\frac{1}{A_1R_2}\frac{\partial^2 W}{\partial z\partial \varphi}+\frac{1}{A_1A_2R_2}\frac{\partial A_2}{\partial z}\frac{\partial W}{\partial \varphi}+\frac{1}{R_1R_2}\frac{\partial U_z}{\partial \varphi}-\right.$$

$$\left.-\frac{1}{A_1R_1R_2}\frac{\partial A_2}{\partial z}U_z+\frac{A_2}{A_1R_2^2}\frac{\partial U_\varphi}{\partial z}\right)+ \tag{4}$$

$$+\frac{Eh^3}{12(1+\nu)}\frac{1}{R_1}\frac{\partial A_2}{\partial z}\left(-\frac{1}{A_1A_2}\frac{\partial^2 W}{\partial z\partial\varphi}+\frac{1}{A_1A_2^2}\frac{\partial A_2}{\partial z}\frac{\partial W}{\partial\varphi}+\frac{1}{A_2R_1}\frac{\partial U_z}{\partial\varphi}-\right.$$

$$\left.-\frac{1}{A_1A_2R_1}U_z+\frac{1}{A_1R_2}\frac{\partial U_\varphi}{\partial z}\right)=A_1A_2\left(-q_2+\rho h\frac{\partial^2 U_\varphi}{\partial t^2}\right).$$

Finally, we get:

$$\frac{Eh^3}{12(1-\nu^2)}\frac{1}{A_1}\frac{\partial}{\partial z}\left[\frac{\partial}{\partial z}\left(\frac{A_2}{A_1^3}\frac{\partial A_1}{\partial z}\frac{\partial W}{\partial z}-\frac{A_2}{A_1^2}\frac{\partial^2 W}{\partial z^2}-\frac{\nu}{A_2}\frac{\partial^2 W}{\partial\varphi^2}-\frac{A_2}{A_1R_1^2}\frac{\partial R_1}{\partial z}U_z+\right.\right.$$

$$\left.\left.+\frac{A_2}{A_1R_1}\frac{\partial U_z}{\partial z}+\frac{\nu}{R_2}\frac{\partial U_\varphi}{\partial\varphi}-\frac{\nu}{A_1^2}\frac{\partial A_2}{\partial z}\frac{\partial W}{\partial z}+\frac{\nu}{A_1R_1}\frac{\partial A_2}{\partial z}U_z\right)\right]+\frac{Eh^3}{12(1+\nu)}\frac{1}{A_1}\frac{\partial}{\partial z}\times$$

$$\times\left[\frac{1}{A_1}\frac{\partial}{\partial\varphi}\left(-\frac{A_1}{A_2}\frac{\partial^2 W}{\partial z\partial\varphi}+\frac{A_1}{A_2^2}\frac{\partial A_2}{\partial z}\frac{\partial W}{\partial\varphi}+\frac{A_1^2}{A_2R_1}\frac{\partial U_z}{\partial\varphi}-\frac{A_1}{A_2R_1}\frac{\partial A_2}{\partial z}U_z+\frac{A_1}{R_2}\frac{\partial U_\varphi}{\partial z}\right)\right]-$$

$$-\frac{Eh^3}{12(1-\nu^2)}\left\{\frac{1}{A_1}\frac{\partial}{\partial z}\left[\frac{\partial A_2}{\partial z}\left(\frac{\nu}{A_1^3}\frac{\partial A_1}{\partial z}\frac{\partial W}{\partial z}-\frac{\nu}{A_1^2}\frac{\partial^2 W}{\partial z^2}-\frac{1}{A_2^2}\frac{\partial^2 W}{\partial\varphi^2}-\frac{\nu}{A_1R_1^2}\frac{\partial R_1}{\partial z}U_z+\right.\right.\right.$$

$$\left.\left.\left.+\frac{\nu}{A_1R_1}\frac{\partial U_z}{\partial z}+\frac{1}{A_2R_2}\frac{\partial U_\varphi}{\partial\varphi}-\frac{1}{A_1^2A_2}\frac{\partial A_2}{\partial z}\frac{\partial W}{\partial z}+\frac{1}{A_1A_2R_1}\frac{\partial A_2}{\partial z}U_z\right)\right]\right\}- \qquad (5)$$

$$-\frac{Eh^3}{12(1-\nu^2)}\frac{1}{A_1^2}\frac{\partial A_1}{\partial z}\frac{\partial}{\partial z}\left(\frac{A_2}{A_1^3}\frac{\partial A_1}{\partial z}\frac{\partial W}{\partial z}-\frac{A_2}{A_1^2}\frac{\partial^2 W}{\partial z^2}-\frac{\nu}{A_2}\frac{\partial^2 W}{\varphi^2}-\frac{A_2}{A_1R_1^2}\frac{\partial R_1}{\partial z}U_z+\right.$$

$$\left.+\frac{A_2}{A_1R_1}\frac{\partial U_z}{\partial z}+\frac{\nu}{R_2}\frac{\partial U_\varphi}{\partial\varphi}-\frac{\nu}{A_1^2}\frac{\partial A_2}{\partial z}\frac{\partial W}{\partial z}+\frac{\nu}{A_1R_1}\frac{\partial A_2}{\partial z}U_z\right)-$$

$$-\frac{Eh^3}{12(1+\nu)}\frac{1}{A_1^3}\frac{\partial A_1}{\partial z}\frac{\partial}{\partial\varphi}\left(-\frac{A_1}{A_2}\frac{\partial^2 W}{\partial z\partial\varphi}+\frac{A_1}{A_2^2}\frac{\partial A_2}{\partial z}\frac{\partial W}{\partial\varphi}+\frac{A_1^2}{A_2R_1}\frac{\partial U_z}{\partial\varphi}-\frac{A_1}{A_2R_1}\frac{\partial A_2}{\partial z}U_z+\right.$$

$$\left.+\frac{A_1}{R_2}\frac{\partial U_\varphi}{\partial z}\right)+\frac{Eh^3}{12(1-\nu^2)}\frac{1}{A_1^2}\frac{\partial A_1}{\partial z}\frac{\partial A_2}{\partial z}\left(\frac{\nu}{A_1^3}\frac{\partial A_1}{\partial z}\frac{\partial W}{\partial z}-\frac{\nu}{A_1^2}\frac{\partial^2 W}{\partial z^2}-\frac{1}{A_2^2}\frac{\partial^2 W}{\partial\varphi^2}-\right.$$

$$\left.-\frac{\nu}{A_1R_1^2}\frac{\partial R_1}{\partial z}U_z+\frac{\nu}{A_1R_1}\frac{\partial U_z}{\partial z}+\frac{1}{A_2R_2}\frac{\partial U_\varphi}{\partial\varphi}-\frac{1}{A_1^2A_2}\frac{\partial A_2}{\partial z}\frac{\partial W}{\partial z}+\frac{1}{A_1A_2R_1}\frac{\partial A_2}{\partial z}U_z\right)+$$

$$+\frac{Eh^3}{12(1-\nu^2)}\frac{1}{A_2}\frac{\partial}{\partial\varphi}\left[\frac{\partial}{\partial\varphi}\left(\frac{\nu}{A_1^2}\frac{\partial A_1}{\partial z}\frac{\partial W}{\partial z}-\frac{A_1}{A_2^2}\frac{\partial^2 W}{\partial\varphi^2}-\frac{\nu}{A_1}\frac{\partial^2 W}{\partial z^2}-\frac{\nu}{R_1^2}\frac{\partial R_1}{\partial z}U_z+\right.\right.$$

$$\left.\left.+\frac{A_1}{A_2R_2}\frac{\partial U_\varphi}{\partial\varphi}+\frac{\nu}{R_1}\frac{\partial U_z}{\partial z}-\frac{1}{A_1A_2}\frac{\partial A_2}{\partial z}\frac{\partial W}{\partial z}+\frac{1}{A_2R_1}\frac{\partial A_2}{\partial z}U_z\right)\right]+$$

$$+\frac{Eh^3}{12(1+\nu)}\frac{1}{A_2}\frac{\partial}{\partial\varphi}\left[\frac{1}{A_2}\frac{\partial}{\partial z}\left(-\frac{A_2}{A_1}\frac{\partial^2 W}{\partial z\partial\varphi}+\frac{1}{A_1}\frac{\partial A_2}{\partial z}\frac{\partial W}{\partial\varphi}+\frac{A_2^2}{A_1 R_2}\frac{\partial U_\varphi}{\partial z}+\frac{A_2}{R_1}\frac{\partial U_z}{\partial\varphi}-\frac{A_2}{A_1 R_1}\frac{\partial A_2}{\partial z}U_z\right)\right]-$$

$$-\frac{Eh}{1-\nu^2}A_1A_2\left\{\frac{1}{R_1}\left[\frac{1}{A_1}\frac{\partial U_z}{\partial z}+\nu\frac{1}{A_2}\frac{\partial U_\varphi}{\partial\varphi}+\frac{\nu}{A_1A_2}\frac{\partial A_2}{\partial z}U_z+W\left(\frac{1}{R_1}+\frac{\nu}{R_2}\right)\right]+\right.$$

$$\left.+\frac{1}{R_2}\left[\frac{\nu}{A_1}\frac{\partial u_z}{\partial z}+\frac{1}{A_2}\frac{\partial U_\varphi}{\partial\varphi}+\frac{1}{A_1A_2}\frac{\partial A_2}{\partial z}U_z+W\left(\frac{\nu}{R_1}+\frac{1}{R_2}\right)\right]\right\}=-A_1A_2\left(q_n+\rho h\frac{\partial^2 W}{\partial t^2}\right).$$

Thus, the equations of the investigated shell have been greatly simplified compared to the original ones.

As a special case, the equations of a circular cylindrical shell are derived from expressions (3) – (5). To do this, it is enough to take the following values of the Lamé constants:

$$A_1=1; \quad A_2=R=\text{const}$$

Differential equations of a special shape shell

Consider the convex (Fig. 2, a) and concave (Fig. 2, b) shells of rotation relative to the axis of rotation. In both cases, it is assumed that $OD=BA=R,$ and the curve $f(z),$ which forms the shell of rotation, is symmetric with respect to the line CM, which intersects the axis of rotation in the middle ($OB = l$; $OC = CB = \frac{l}{2}$).

We also assume that

$$f(0)=f(l)=R=\text{const}. \tag{6}$$

Consider the coordinate system $C_1z_1r_1$ (Fig. 2). The relationship between this system and the reference Ozr is defined by the relations

$$r=r_1+R; \qquad z = z_1+\frac{l}{2}$$

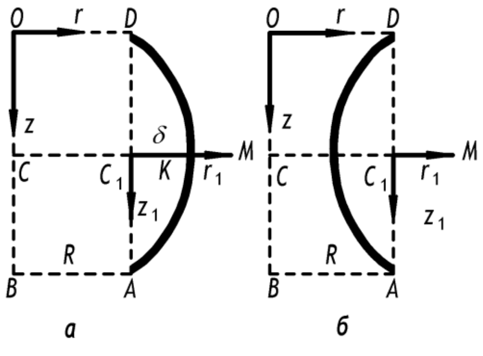

Fig. 2. A special shape of the rotation shell: a) convex; b) concave

In the frame of reference $C_1\, z_1\, r_1$ the shape of the shell (meridian line) is given by the expression

$$r_1 = \pm f_1(z_1), \tag{7}$$

where the «+» sign corresponds to Fig. 10.2, *a,* and the «–» – sign corresponds to Fig. 2, *b*.

Let us find out for which class of curves $f_1(z_1)$ the following conditions are fulfilled:

$$f_1(-z_1) = f_1(z_1); \qquad f_1\left(\pm\tfrac{l}{2}\right) = 0;$$

functions $\left[+f_1(z_1)\right]$ are strictly convex, and functions $\left[-f_1(z_1)\right]$ are strictly concave; the point with coordinate $z_1 = 0$ is the point of extremum for the functions $\pm f_1(z_1)$;

function $f_1(z_1)$ is decreasing if $\forall z_1 \in \left(0; \frac{l}{2}\right)$ (Fig. 2, *a*) and increasing if $\forall z_1 \in \left(0; \frac{l}{2}\right)$ (Fig. 2, *b)*.

Now let's consider an example. Let's say that

$$F_1(z_1) = a_2 - a_0 z_1^2, \; a_2 > 0; \; a_0 > 0.$$

Obviously, $f_1(-z_1) = f_1(z_1)$. Then, according to Fig. 2, we have:

$$f_1\left(\pm\frac{l}{2}\right) = 0 \quad \Rightarrow \quad a_2 - a_0\frac{l^2}{4} = 0.$$

Hence $a_2 = a_0\frac{l^2}{4}$. With that in mind, we can write it down:

$$f_1(z_1) = a_0\left(\frac{l^2}{4} - z_1^2\right), \quad a_0 > 0. \tag{8}$$

We denote the rise of the parabola C_1K at the point $z_1 = 0$ by δ (Fig. 2, a). Then

$$\delta = a_0\frac{l^2}{4} \quad \Rightarrow \quad a_0 = \frac{4\delta}{l^2}.$$

It can be written down that

$$f_1(z_1) = \delta - \frac{4\delta}{l^2} z_1^2 = \delta\left(1 - 4\frac{z_1^2}{l^2}\right). \tag{9}$$

The equation of the shell meridian line in the *Ozr* reference coordinate system is:

$$r = f(z) = R + \delta\left[1 - \frac{4}{l^2}\left(z - \frac{l}{2}\right)^2\right]. \tag{10}$$

Or

$$f(z)=R\left[1+\frac{\delta}{R}-\frac{4\delta}{Rl^2}\left(z-\frac{l}{2}\right)^2\right]; \tag{11}$$

$$f'(z)=-\frac{8\delta}{l^2}\left(z-\frac{l}{2}\right); \tag{12}$$

$$A_1=\sqrt{1+64\frac{\delta^2}{l^4}\left(z-\frac{l}{2}\right)^2}\ ; \tag{13}$$

$$A_2=R\left[1+\frac{\delta}{R}-\frac{4\delta}{Rl^2}\left(z-\frac{l}{2}\right)^2\right]. \tag{14}$$

In lines of curvature, the rules for differentiating unit vectors $\vec{e}_i$ are as follows:

$$\frac{1}{A_1}\frac{\partial\vec{e}_1}{\partial\alpha_1}=-\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha}\vec{e}_2-\frac{1}{R_1}\vec{e}_3;\ \frac{1}{A_1}\frac{\partial\vec{e}_2}{\partial\alpha_1}=\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_1}\vec{e}_1;\ \frac{1}{A_1}\frac{\partial\vec{e}_3}{\partial\alpha_1}=\frac{1}{R_1}\vec{e}_1, \tag{15}$$

where $\vec{e}_3$ is the normal vector to the surface.

These expressions will be useful for further calculations.

Let's move on to deriving the formulas for calculating the curvature of coordinate lines $\left(\frac{1}{R_1}\right)$ and $\left(\frac{1}{R_2}\right)$ in the undeformed state of the shell surface.

From Fig. 1 we obtain the obvious vector equation for the points of the median surface:

$$\vec{R}=\vec{R}(z,\varphi)=\vec{i}r\cos\varphi+\vec{j}r\sin\varphi+\vec{k}z=\vec{i}f(z)\cos\varphi+\vec{j}f(z)\sin\varphi+\vec{k}z. \tag{16}$$

Then

$$\left.\begin{aligned}&\vec{R}_z=\frac{\partial\vec{R}}{\partial z}=\vec{i}f'(z)\cos\varphi+\vec{j}f'(z)\sin\varphi+\vec{k};\quad \vec{e}_1=\vec{e}_z=\frac{1}{\left|\vec{R}_z\right|};\\&\vec{R}_z=\frac{1}{A_1}\left[\vec{i}f'(z)\cos\varphi+jf'(z)\sin\varphi+\vec{k}\right],\quad A_1=\sqrt{1+f'^2(z)};\end{aligned}\right\} \tag{17}$$

$$\begin{gathered}\vec{R}_\varphi=\left[-\vec{i}r\sin\varphi+\vec{j}r\cos\varphi\right]\Rightarrow\\ \Rightarrow\vec{e}_2=\vec{e}_\varphi=\frac{1}{\left|R_\varphi\right|}\left(-\vec{i}r\sin\varphi+\vec{j}r\cos\varphi\right)=\frac{1}{A_2}\left(-\vec{i}r\sin\varphi+\vec{j}r\cos\varphi\right).\end{gathered} \tag{18}$$

$$\vec{e}_3=\vec{e}_1\times\vec{e}_2=\frac{1}{A_1}\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\ f'(z)\cos\varphi&f'(z)\sin\varphi&1\\ -\sin\varphi&\cos\varphi&0\end{vmatrix}=\frac{1}{A_1}\left[-\vec{i}\cos\varphi+\vec{j}\sin\varphi+\vec{k}f'(z)\right]. \tag{19}$$

From formulas (15) it follows:

$$\frac{1}{R_1}=\frac{1}{A_1}\vec{e}_1\frac{\partial\vec{e}_3}{\partial z};$$

$$\frac{\partial\vec{e}_3}{\partial z}=-\frac{1}{A_1^2}\frac{\partial A_1}{\partial z}\left[-\vec{i}\cos\varphi-\vec{j}\sin\varphi+\vec{k}f'(z)+\frac{\vec{k}}{A_1}f''(z)\right]=$$

$$=\vec{i}\frac{1}{A_1^2}\frac{\partial A_1}{\partial z}\cos\varphi+\vec{j}\frac{1}{A_1^2}\frac{\partial A_1}{\partial z}\sin\varphi+\vec{k}\left[-\frac{1}{A_1^2}\frac{\partial A_1}{\partial z}f'(z)+\frac{1}{A_1}f''(z)\right]. \qquad (20)$$

Then

$$\vec{e}_1\frac{\partial\vec{e}_3}{\partial z}=\frac{1}{A_1^3}f'(z)\cos^2\varphi+\frac{1}{A_1^3}f'(z)\sin^2\varphi-\frac{1}{A_1^3}\frac{\partial A_1}{\partial z}f'(z)+\frac{1}{A_1^2}f''(z)=\frac{1}{A_1^2}f''(z).$$

Taking into account the above, we can write down the known ratio –

$$\frac{1}{R_1}=\frac{1}{A_1^3}f''(z)=\frac{f''(z)}{\left[1+f'^2(z)\right]^{\frac{3}{2}}}. \qquad (21)$$

Consequently,

$$\frac{1}{R_2}=-\frac{1}{A_1A_2}, \qquad (22)$$

or

$$\frac{1}{R_2}=\frac{1}{R_2(z)}=-\frac{1}{f(z)\sqrt{1+f'^2(z)}}. \qquad (23)$$

If the shell of revolution is a cylindrical surface, then $f(z)=R=\text{const}$, where R is the radius of the shell.

The Codazzi identity for the considered shell of revolution will change as follows:

$$\frac{\partial}{\partial\alpha_1}\left(\frac{A_2}{R_2}\right)=\frac{\partial}{\partial z}\left[A_2\left(-\frac{1}{A_1A_2}\right)\right]=-\frac{\partial}{\partial z}\left(\frac{1}{A_1}\right)=\frac{1}{A_1^2}\frac{\partial A_1}{\partial z}; \qquad A_1=\sqrt{1+f'^2}(z);$$

$$\frac{\partial A_1}{\partial z}=\frac{1}{A_1}f''(z)f'(z).$$

And so, $\frac{\partial}{\partial\alpha_1}\left(\frac{A_2}{R_2}\right)\Rightarrow\frac{f''(z)f'(z)}{\left[1+f'^2(z)\right]^{\frac{3}{2}}}$.

The right side will look like this $\frac{1}{R_1}\frac{\partial A_2}{\partial\alpha_1} \Rightarrow \frac{f''(z)\,f'(z)}{\left[1+f'^2(z)\right]^{\frac{3}{2}}}$.

The second ratio turns to zero.

Let's clarify the relations for calculating the Lamé parameters A_1 and

A_2, as well as the curvature $\left(\frac{1}{R_1}\right)$ and $\left(\frac{1}{R_2}\right)$. In the course of further research, we will assume that the following conditions must be met $\zeta = \frac{\delta}{R} < 1$.

Parameter A_2 should be given in the form

$$A_2 = R + \delta - \delta\left(\frac{2z}{l} - 1\right)^2 = (R+\delta)\left[1 - \frac{\delta}{R+\delta}\left(\frac{2z}{l} - 1\right)^2\right] = (R+\delta)\left[1 - \xi(z)\right], \quad (24)$$

where $\xi(z) = \frac{\delta}{R+\delta}\left(\frac{2z}{l} - 1\right)^2$.

It is easy to establish that the function $\xi(z) \le 1$ at $\forall\, z \in [0, l]$, i.e. $0 \le \xi(z) << 1$.

Parameter A_1 s represented as

$$A_1 = \left[1 + f'^2(z)\right]^{\frac{1}{2}} = \left[1 + 16\frac{\delta^2}{l^2}\left(\frac{2z}{l} - 1\right)^2 \xi(z)\right]^{\frac{1}{2}} = \left[1 + 16\zeta(1+\zeta)\eta^2\,\xi(z)\right]^{\frac{1}{2}},$$

where $\eta = \frac{R}{l}$.

The final parameter value

$$A_1 = \left[1 + 2\mu\,\xi(z)\right]^{\frac{1}{2}}, \quad (25)$$

where $\mu = 8\zeta(1+\zeta)\eta^2$.

From now on, we consider the shell geometry to be such that the condition

$$2\mu << 1. \quad (26)$$

This condition has the feature of containing characteristic geometric parameters.

Taking into account the caveat (26), we assume that

$$A_1 \cong 1 + \mu\,\xi(z) - \frac{1}{2}\mu^2\,\xi^2(z). \quad (27)$$

Now let us transform the relations (21) and (22) to calculate the values $\frac{1}{R_1}$ and $\frac{1}{R_2}$.

We have:

$$\frac{1}{R_1}=\frac{-\frac{8\delta}{l^2}}{\left[1+16\left(\frac{\delta}{l}\right)^2\left(\frac{2z}{l}-1\right)^2\right]^{\frac{3}{2}}}=-\frac{\mu}{R+\delta}\left[1+2\mu\,\xi(z)\right]^{-\frac{3}{2}}. \qquad (28)$$

Given the basic assumption (26), expression (28) is simplified:

$$\frac{1}{R_1}\cong-\frac{1}{R}\frac{\mu}{1+\zeta}\left[1-\zeta\,\mu\,\xi(z)+\frac{15}{2}\mu^2\,\xi^2(z)\right]. \qquad (29)$$

It is obvious that if $\delta\to 0$, then $\mu\to 0$, therefore, $\frac{1}{R_1}\to 0.$

According to (23)

$$\frac{1}{R_2}=-\frac{1}{f(z)\left[1+f'^2(z)\right]^{\frac{1}{2}}}=-\frac{1}{A_1A_2}.$$

Taking into account expressions (24) and (27), we obtain:

$$\frac{1}{R_2}=-\frac{1}{\left[1+\mu\,\xi(z)\right](R+\delta)\left[1-\xi(z)\right]}=$$

$$=-\frac{1}{R+\delta}\left[1-\mu\,\xi(z)+\mu^2\,\xi^2(z)-\ldots\right]\left[1+\xi(z)+\xi^2(z)+\ldots\right]= \qquad (30)$$

$$=-\frac{1}{R+\delta}\left[1+(1-\mu)\,\xi(z)+\left(1-\mu+\mu^2\right)\xi^2(z)+\ldots\right].$$

Finally, we have:

$$\frac{1}{R_2}\cong-\frac{1}{R(1+\zeta)}\left[1+\left(1-\mu+\mu^2\right)\xi(z)+(1-\mu)^2\,\xi^2(z)+\ldots\right].$$

If $\delta\to 0$, $\xi\to 0$ i $\mu\to 0$, then from the formula (26)

$$\frac{1}{R_2}\Rightarrow-\frac{1}{R}.$$

Formulas (24), (27), (28), and (30) are valid only for the fulfillment of condition (26).

Let us numerically evaluate the possible geometry of the shell. Let $\zeta=\frac{\delta}{R}=0{,}3.$ This is a sufficient convexity (or concavity) of the surface. Then.

$$2\,\mu=16\,\zeta\,(1+\zeta)\,\eta^2=16\cdot 0{,}3\cdot 1{,}3\eta^2=6{,}24\eta^2,\ \ \eta=\frac{R}{l}.$$

In order to fulfill conditions (30) and $2\mu << 1$, it is necessary to take relatively long shells with significant convexity (or curvature).

Let $\eta = 0,1$. Then

$$2\mu = 6,24 \cdot 0,01 \cong 0,06 \Rightarrow \mu \cong 0,030.$$

If $\eta = 0,2, \Rightarrow \eta^2 = 0,04, \Rightarrow \mu \cong 0,125$.

The last case is the most unfavorable. Let's analyze the values of the coefficients in expressions (24), (27), (28), and (30).

First of all

$$\xi(z) = \frac{\zeta}{1+\zeta}\left(\frac{2z}{l} - 1\right).$$

Then the maximum value of $\xi(z)$, if $\forall\, z \in [0, l]$ and $\zeta = 0,3$, will be equal to $\frac{0,3}{1,3} = 0,23$.

Maximum values of other parameters:

$$A_1 \cong 1 + 0,125 \cdot 0,23 - \frac{1}{2}(0,125)^2 (0,23)^2 \cong 1 + 0,03; \; A_2 = 1,3\,R\,(1 - 0,23);$$

$$\frac{1}{R_1} \cong \frac{-1,25}{1,3\,R}\left[1 - 3 \cdot 0,125 \cdot 0,23 + 7,5 \cdot 0,016 \cdot 0,053\right] \cong$$

$$\cong -\frac{1,25}{13R}\left[1 - 0,375 \cdot 0,23 + \ldots\right] \cong -\frac{1,25}{13R}(1 - 0,086);$$

$$\frac{1}{R_2} = -\frac{1}{1,3R}\left[1 + (1 - 0,125 + 0,0156) \cdot 0,23 + (1 - 0,25 + 0,0156) \cdot 0,053\right] =$$

$$= -\frac{1}{1,3R}(1 + 0,205 + 0,04).$$

Thus, the above numerical calculations give us the right to assert that it is possible to set such geometric dimensions of the shell that in the parameters A_1, A_2, $\frac{1}{R_1}$ and $\frac{1}{R_2}$, we can neglect the terms after the third term.

Conclusion

These results enable the solution of a broad range of applied problems. For instance, comparative calculations can be performed for a classical circular float shell. There are various methods to determine the coordinate functions, and one of them has been analyzed here. This option has the advantage of allowing the selection of a side surface geometry that meets the conditions of a specific technical problem.

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Мельник В. М., Косова В. П., Бойко Г. В., Остапенко Ж. І., Павленко В. П.

ДИФЕРЕНЦІАЛЬНІ РІВНЯННЯ ОБОЛОНКИ АПАРАТУ З ДОВІЛЬНОЮ ГЕОМЕТРІЄЮ ЛІНІЇ МЕРИДІАНУ

В. М. Мельник, В. П. Косова, Г. В. Бойко, Ж. І. Остапенко, В. П. Павленко

Анотація. *Узагальнюється теорія шляхом опису оболонкової частини у вигляді поверхні з довільним геометричним окресленням і діючими чинниками загального виду. Будується математична модель та формулюються граничні умови для визначення координатних функцій деформації оболонкової частини апарату за будь-якої структури зовнішніх збурень. Узагальнюється методика обчислень пружних деформацій її поверхні з довільним окресленням лінії меридіану. Найважливішим на етапі аналізу природи явища з наступним обранням шляху боротьби з негативним впливом розглядаємих чинників на прилади інерціальної навігації постає обчислення координатних функцій деформації оболонкової частини апарату під дією просторового збурення. Доведено, що некоректності, або зайві спрощення, призведуть до похибок інтегрування рівнянь оболонки, отже – до похибок обчислення координатних функцій деформації поверхні і викривлення змісту явища. Будуються рівняння для визначення парціальних частот. З'ясовано, що коливальні процеси на поверхні поплавця певним чином діють один на одний за всіма напрямками. Тож, для конкретних масогабаритних модифікацій ДУСУ можна визначити ступінь їх впливу. Закладені наукові засади для глибокого аналізу динаміки оболонкової частини апарату за натурних умов, з одного боку, та виявлена можливість для аргументованого порівняльного аналізу з класичною циліндричною модифікацією поплавця – з другого. З'явилася можливість для вирішення задач оптимізації масогабаритних характеристик приладу. Будуються теоретичні засади розв'язання задач підвищення точності і надійності поплавкових приладів (і систем інерціальної навігації в цілому) на підгрунті пасивних методів звукоізоляції та їх поєднання з іншими методами – активними, автокомпенсаційними тощо*.

Ключові слова: *оболонкова частина, граничні умови, математичне моделювання, довільна геометрія оболонки, координатні функції*