полигонах института «Укрсельэнергопроект». Показано, что изолированную нейтраль сети 6–20 кВ с токами ОЗЗ до 10 А не следует заземлять через высокоомный резистор. В сетях 20 кВ следует применять компенсацию емкостных токов и ограничивать время существования ОЗЗ четырьмя часами. С целью уменьшения количества случаев ОЗЗ, на промежуточных опорах воздушных линий 20 кВ следует устанавливать стержневые изоляторы, а на анкерных – подвесные.

Ключевые слова: железобетонная опора, термическое поражение заземлителя, ток однофазного замыкания на землю

EXPERIMENTAL STUDY OF THERMAL SHOCK OF REINFORCED CONCRETE PILLARS BY CURRENTS SINGLE-PHASE GROUND FAULT

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Abstract. *The permissible duration of single-phase earth faults (SEF) in networks with isolated neutral and grounding currents up to 10 A in post-Soviet countries is not established. The purpose of the research is to normalize the permissible duration of the regime of ultrasound, which does not violate the thermal stability of the supports. The research was carried out by the staff of the UkrSilEnergoProject Institute. It is shown that the isolated neutral of the 6-20 kV network with the currents of the SEF to 10 A should not be grounded through a high-resistance resistor. In networks of 20 kV, capacitive currents compensation should be used starting from 5 amperes and the limitation of the lifetime of the SEF for four hours. In order to reduce the number of cases of ultrasound in the intermediate supports of a 20 kV overhead line, rod insulators shall be installed, and anchor-pendant on anchors.*

Keywords: reinforced concrete pillar, earthing switch's thermal shock, the current of single-phase ground fault

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MAPLE-MODELS OF MOVEMENT OF PARTS ON SURFACE SPHERE

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Abstract. *The main results of modeling the motion of a particle on a rough surface of a sphere in the function of an independent parameter are given* t *– time. In contrast to the line surfaces, analytical calculations of the motion of a particle on non-linear surfaces of rotation of the 2nd order are*

more complex. Therefore, only certain mathematical expressions that were obtained using developed maple models $Sphere\_t$.
Keywords: motion of a particle, rough surface, sphere, system of differential equations, trajectory, velocity

Topicality. Understanding the patterns of motion of a particle on different surfaces allows us to calculate the parameters of many working bodies of agricultural machines. The study of trajectory-kinematic characteristics of the motion of a particle requires the formation of a system of differential equations of the 2 nd order and its approximate solution, which is a rather labor-intensive process and practically impossible to implement without the use of modern packages of symbolic algebras.

Analysis of recent research and publications. In the work [1], the motion of a particle was investigated on a horizontal rough plane, which carries out translational displacements in a circle. In contrast to the line surfaces, analytical calculations of the motion of a particle on non-linear surfaces of rotation of the 2nd order are more complex. To study the motion of a particle for such surfaces, it is also advisable to develop maple models of motion [2, 3].

The purpose of the study is to develop for the environment of the symbolic algebra Maple a computer model of the motion of a particle on a rough surface of the sphere, which performs oscillatory displacements in space, and with the help of computational experiments to find out its trajectory-kinematic properties.

Materials and methods of research. The research was carried out using computer technology for the formation of the law of motion and analysis of trajectory and kinematic characteristics, comparing them with known results.

Research results and their discussion. Let's write the parametric equation of uv- coordinate grid of the sphere in the form:

$$R(u,v) = R[a\ \cos(u)\ \cos(v), a\ \sin(u)\cos(v), a\ \sin(v)], \quad (1)$$

where a – sphere radius;
$u \in [0; 2\pi]$, $v \in [-\pi/2; \pi/2]$ – independent curvilinear coordinates of the surface.

Coefficients of the 1-st quadratic shape of the sphere $R(u,v)$ equal to:

$$E = a^2\ \cos(v)^2,\ F = 0,\ G = a^2. \quad (2)$$

Since the coefficient $F = 0$, so the grid $R(u,v)$ of the sphere (1) is orthogonal.

The substitution of the sought expressions $u = u(t)$ and $v = v(t)$ of the particle motion law in the internal *u,v* – coordinates of the sphere $R(u,v)$ in equation (1) allows us to obtain the trajectory $r(t)$ of the particle on the surface in the general form:

$$r(t) = r[a \cos(v(t)) \cos(u(t)), a \cos(v(t)) \sin(u(t)), a \sin(v(t))], \quad (3)$$

from where the vector of the tangent $\tau(t)$ of the trajectory $r(t)$ and the velocity $V(t)$ of the particle are:

$$\tau(t)=\tau\begin{bmatrix}-a\left(\cos(u(t))\sin(v(t))\frac{d}{dt}v(t)+\cos(v(t))\sin(u(t))\frac{d}{dt}u(t)\right),\\ -a\left(\sin(v(t))\sin(u(t))\frac{d}{dt}v(t)-\cos(v(t))\cos(u(t))\frac{d}{dt}u(t)\right),\\ a\cos(v(t))\frac{d}{dt}v(t)\end{bmatrix}, \quad (4)$$

$$V(t)=|\tau(t)|=a\sqrt{\cos(v(t))^2\left(\frac{d}{dt}u(t)\right)^2+\left(\frac{d}{dt}v(t)\right)^2}. \quad (5)$$

The vector of normal ***N*** to the surface along the trajectory $r(t)$ of the particle is expressed by:

$$N(t)=N\left|a^2\cos(v(t))^2\cos(u(t)),a^2\cos(v(t))^2\sin(u(t)),a^2\cos(v(t))\sin(v(t))\right|. \quad (6)$$

The cosine of the angle η between the vector $G[0,0,-1]$ gravity $F_g=mg$ and the normal vector ***N*** of the surface ***R*** depends on:

$$\cos(\eta(t))=\cos(\widehat{G,N})=-\sin(v(t)). \quad (7)$$

The equation of the normal $n(t)$, the curvature $k(t)$ of the trajectory $r(t)$, the cosine of the angle ε between the vectors of the normal $n(t)$ and $N(t)$, which are included in the expressions of the centrifugal force F_C and the forces of the normal reactionF_N, are cumbersome (see geometry.com.ua), and therefore are not given here.

The equation of the force of the normal reaction F_N has a rather compact appearance:

$$F_N(t)=mg\cos(\eta)+mV^2k\cos(\varepsilon)=$$

$$m\left(a\left(\cos(v(t))^2\left(\frac{d}{dt}u(t)\right)^2+\left(\frac{d}{dt}v(t)\right)^2\right)-g\sin(v(t))\right), \quad (8)$$

which demonstrates that its value may be negative for $v(t)\in]-\pi/2;0]$, and therefore it is possible to separate the particle from the surface of the sphere in its upper part.

For the derivation of the law of motion of a particle on the inner surface of the sphere in projections on the orth ***u*** and ***v*** of the ***OuvN*** triangle, it is necessary to have their directions::

$$R'_u(t)=R'_u[-a\cos(v(t))\sin(u(t)),a\cos(v(t))\cos(u(t)),0], \quad (9)$$

$$R'_v(t) = R'_v\left[-a\ \sin(v(t))\ \cos(u(t)), -a\ \sin(v(t))\ \sin(u(t)), a\ \cos(v(t))\right]. \quad (10)$$

The acceleration vector ***w*** of a particle is expressed by:

$$w(t) = w\begin{bmatrix} -a\cos(u(t))\left(\cos(v(t))\left(\left(\frac{d}{dt}v(t)\right)^2 + \left(\frac{d}{dt}u(t)\right)^2\right) + a\sin(v(t))\left(\frac{d}{dt}v(t)\right)^2\right) + \\ +a\sin(u(t))\left(2\sin(v(t))\frac{d}{dt}u(t)\frac{d}{dt}v(t) - \cos(v(t))\frac{d^2}{dt^2}u(t)\right), \\ -a\sin(u(t))\left(\cos(v(t))\left(\left(\frac{d}{dt}v(t)\right)^2 + \left(\frac{d}{dt}u(t)\right)^2\right) + a\sin(v(t))\left(\frac{d}{dt}v(t)\right)^2\right) - \\ -a\cos(u(t))\left(2\sin(v(t))\frac{d}{dt}u(t)\frac{d}{dt}v(t) - \cos(v(t))\frac{d^2}{dt^2}u(t)\right), \\ -a\sin(v(t))\left(\frac{d}{dt}v(t)\right)^2 + \cos(v(t))\frac{d^2}{dt^2}v(t) \end{bmatrix}. \quad (11)$$

On the obtained vectors R'_u, R'_v and w we find the cosines of the corresponding angles:

$$\cos\left(\widehat{w, R'_u}\right) = \frac{-2\sin(v(t))\frac{d}{dt}u(t)\frac{d}{dt}v(t) + \cos(v(t))\frac{d^2}{dt^2}u(t)}{w}, \quad (12)$$

$$\cos\left(\widehat{w, R'_v}\right) = \frac{\frac{d^2}{dt^2}v(t) + \sin(v(t))\cos(v(t))\left(\frac{d}{dt}u(t)\right)^2}{w}, \quad (13)$$

$$\cos\left(\widehat{G, R'_u}\right) = 0,\ \cos\left(\widehat{G, R'_u}\right) = -\cos(v(t)), \quad (14)$$

$$\cos\left(\widehat{\tau, R'_u}\right) = \frac{\cos(v(t))\frac{d}{dt}u(t)}{V},\ \cos\left(\widehat{\tau, R'_u}\right) = \frac{\frac{d}{dt}v(t)}{V}, \quad (15)$$

Substitution of the found expressions in the system of differential equations leads to their subsequent form:

$$\begin{cases} Qu \coloneqq -ma\left(2\ \sin(v(t))\frac{d}{dt}u(t)\ \frac{d}{dt}v(t) - \cos(v(t))\ \frac{d^2}{dt^2}u(t)\right) = \\ -\frac{mf\ \cos(v(t))\left(a\left(\frac{d}{dt}v(t)\right)^2 + a\ \cos(v(t))^2\left(\frac{d}{dt}u(t)\right)^2 - g\ \sin(v(t))\right)\frac{d}{dt}u(t)}{V} \\ Qv \coloneqq ma\left(\frac{d^2}{dt^2}v(t) + \sin(v(t))\cos(v(t))\ \left(\frac{d}{dt}u(t)\right)^2\right) = \\ -mg\ \cos(v(t)) - \frac{mf\ \frac{d}{dt}v(t)\left(a\left(\frac{d}{dt}v(t)\right)^2 - a\ g\sin(v(t))^2 + a\ \cos(v(t))^2\left(\frac{d}{dt}u(t)\right)^2\right)\frac{d}{dt}u(t)}{V} \end{cases} \quad (16)$$

The initial conditions for the solution of the resulting system of differential equations are:

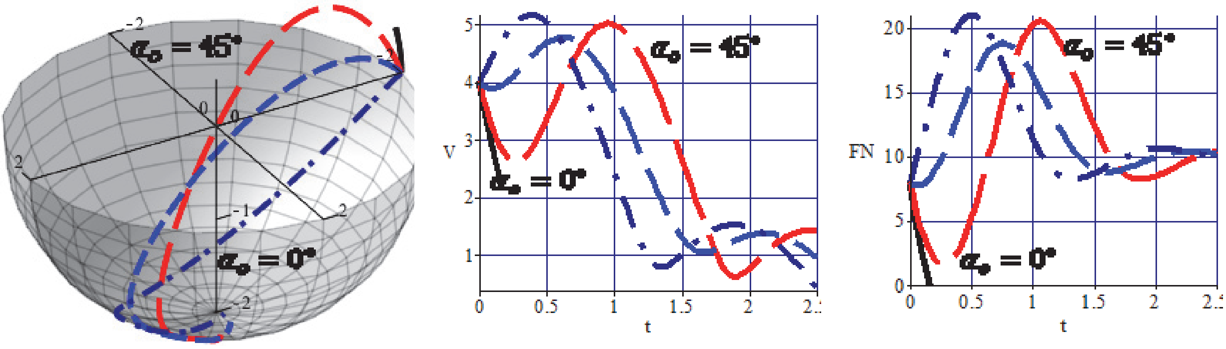
$$Oi := u(0) = u_0, \ \frac{d}{dt}u(0) = \frac{V_0 \sin(\alpha_0)}{a\cos(v_0)}, \ v(0) = v_0, \ \frac{d}{dt}v(0) = \frac{V_0 \cos(\alpha_0)}{a}, \tag{17}$$

where: α_0 – the angle between the velocity vector $V(t)$ and u – coordinate lines of the sphere R; V_0 – initial particle velocity;
$[u_0, v_0]$ - its position at the moment $t_0 = 0$.

The law of motion of a particle along a rough interior surface of a sphere in projections on the *T* and *P* axis of the accompanying triangular *OTPN* is determined by the system of differential equations of the following form:

$$\begin{cases} m\tau = a\left(\cos(v(t))^2 \frac{d}{dt}u(t)\frac{d^2}{dt^2}u(t) + \frac{d}{dt}v(t)\left(-\sin(v(t))\cos(v(t))\left(\frac{d}{dt}u(t)\right)^2 + \frac{d^2}{dt^2}v(t)\right)\right) = \\ -f\left(a\left(\frac{d}{dt}v(t)\right)^2 + a\cos(v(t))^2\frac{d}{dt}u(t) - g\sin(v(t))\right)\sqrt{\left(\frac{d}{dt}v(t)\right)^2 + \cos(v(t))^2\left(\frac{d}{dt}u(t)\right)^2} + \cos(v(t))\frac{d}{dt}v(t) \\ m\tau = a\left(\cos(v(t))\frac{d}{dt}u(t)\frac{d^2}{dt^2}v(t) - \cos(v(t))\frac{d}{dt}v(t)\frac{d^2}{dt^2}u(t) + \sin(v(t))\frac{d}{dt}u(t)\left(2\frac{d}{dt}v(t)\right)^2 + \cos(v(t))^2\left(\frac{d}{dt}u(t)\right)^3\right) = \\ -g\cos(v(t))^2\frac{d}{dt}u(t) \end{cases} \tag{18}$$

The sought *u(t)* and *v(t)* dependencies of the systems of differential equations (16) – (18) can be found only approximate. Fig. 1 shows the trajectories $r(t)$ of the particle, the graphs of its velocity *V(t)* and the forces of the normal reaction $F_N(t)$ in the sphere of radius $a = 2$ different values of the angle $\alpha_0 = 0°$,45°,$90°$,$135°$ throwing a particle with constant values of its initial position $u_0 = \pi$, $v_0 = 0$ on the sphere, initial velocity $V_0 = 4$ м/с and coefficient of friction $f = 0.3$.

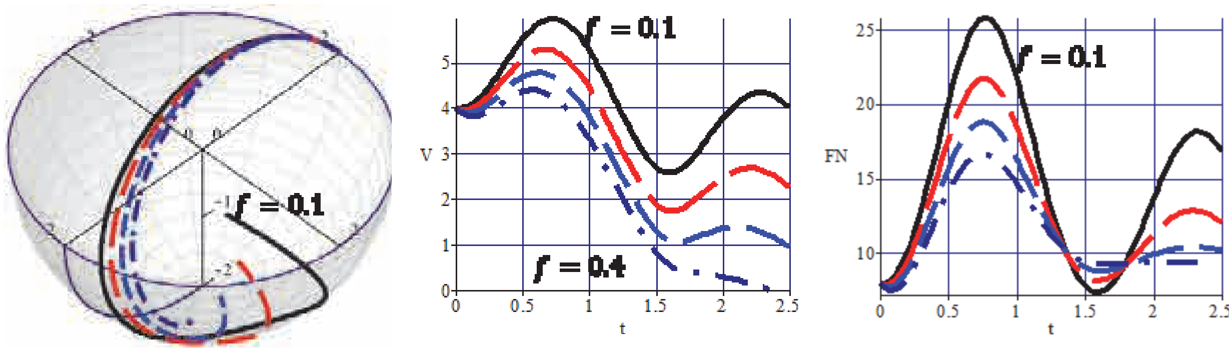


a) b) c)

Fig. 1. The trajectorie $r(t)$ of the particle, graphs of its velocity $V(t)$ and the forces of the normal reaction $F_N(t)$ for different values of the angle α_0 of its throw

Since the graph of the normal reaction $F_N(t)$ of the particle thrown at an angle $\alpha_0 = 0°$ (perpendicular to the parallels of the sphere up) over the time interval $t \cong 0.2c$ goes to zero, then at that moment the particle will break off from the surface of the sphere. All other particles will approach the lower pole of the sphere under the zigzag-like law – their velocity decreases to zero, and the magnitude of the normal reaction $F_N(t)$ to the value of mg. Moreover, all of them will not stop at the point of the pole of the sphere, but in its vicinity, whose radius depends on the value of the coefficient of friction *f* and the radius of the sphere *a*. Of all the particles, the fastest stop is that which is thrown towards the lower pole of the sphere. Although the arc of the trajectory of a particle thrown at an angle $\alpha_0 = 45°$ was in the upper part of the sphere, the particle from the surface did not tear off due to centrifugal force.

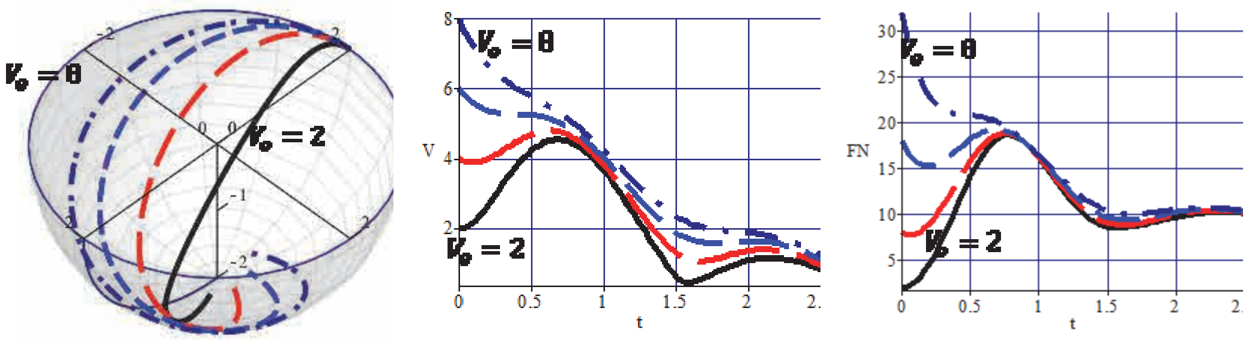
Figure 2 shows the trajectory-kinematic characteristics of the particle, depending on the coefficient of friction $f = 0.1, 0.2, 0.3, 0.4$ at a given angle $\alpha_0 = 90°$ of its throw. The particle with a friction coefficient *f* = 0.4 will stop at the first through the interval *t*≈2.3 s. Regardless of the initial velocity V_0 and the coefficient of friction *f*, the trajectories of all particles thrown at an angle $\alpha_0 = 90°$ will not cross the equator of the sphere.



a) b) c)

Fig. 2. Trajectories $r(t)$ of the particle, graphs of its velocity $V(t)$ and forces $F_N(t)$ of the normal reaction, depending on the coefficient of friction f

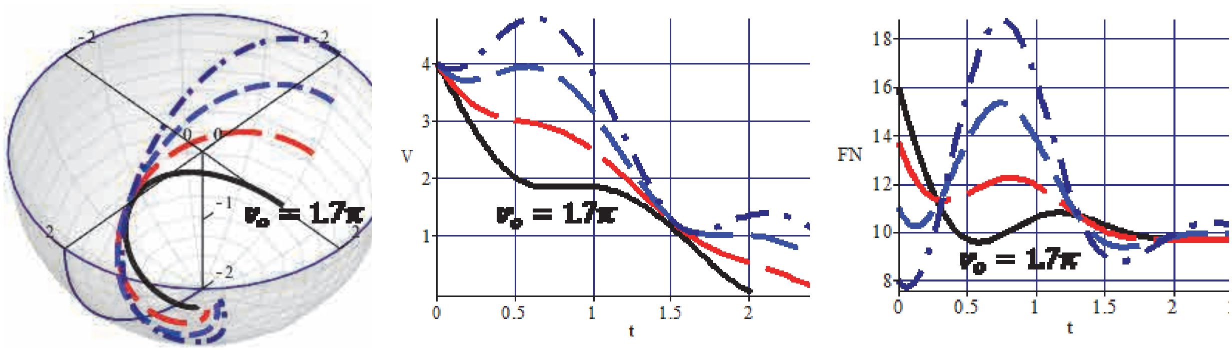
Let the particles have a different initial velocity $V_0 = 2, 4, 6, 8$ м/с, but the same values $\alpha_0 = 90°$ and $f = 0.3$. According to the graphs of the velocities *V(t)* and the forces $F_N(t)$ of the normal particle reaction, it can be seen (Fig. 3a) that, regardless of their initial velocity V_0 they begin to converge through the interval *t*≈2.5 s, although at the beginning of the movement *t* = 0 there was a significant difference in their values.



a) b) c)

Fig. 3. Trajectories $r(t)$, graphs of velocity $V(t)$ and forces of the normal reaction $F_N(t)$ depending on the initial velocity V_0

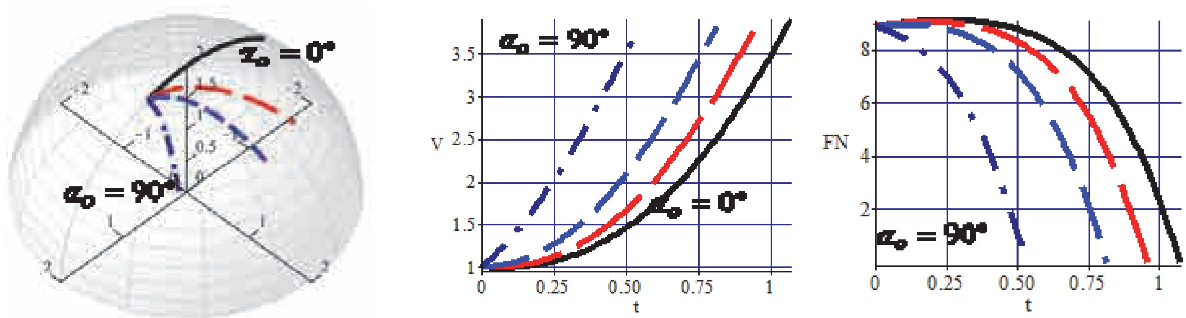
If the particle is thrown at the same initial velocity $V_c = 4$ м/с in different places on the meridian of the spheres in the direction of the parallels, then their trajectories $r(t)$ converge in the vicinity of a certain point (Fig. 4, a). The fastest stop is the particle that was the lowest (Fig. 4, b). For this particle on the sphere, the force of the normal reaction $F_N(t)$ at the beginning of its movement is greatest due to the greater curvature of the trajectory (Fig. 4, c).



a) b) c)

Fig. 4. The trajectories $r(t)$, the graphs of velocity $V(t)$ and the forces of the normal reaction $F_N(t)$ depending on its initial position on the surface of the sphere

Figure 5 shows the trajectories $r(t)$ of motion of the particle on the outer surface of the sphere of radius $a = 2$, the graphs of velocity $V(t)$, the forces of the normal reaction $F_N(t) = F_G(t) + F_C(t)$, the gravity forces $F_G(t) = mg\cos(\widehat{G,N})$, the centrifugal force $F_C(t) = mV^2k\cos(\widehat{n,N})$ and the curves $k(t)$ depending on the angle $\alpha_n = 0°$, 15°, $30°$, $90°$ of the particle with constant values of its initial position $u_o = \pi$, $v_o = 0.4\pi$, initial velocity $V_o = 1$ м/с and friction coefficient $f = 0.3$.



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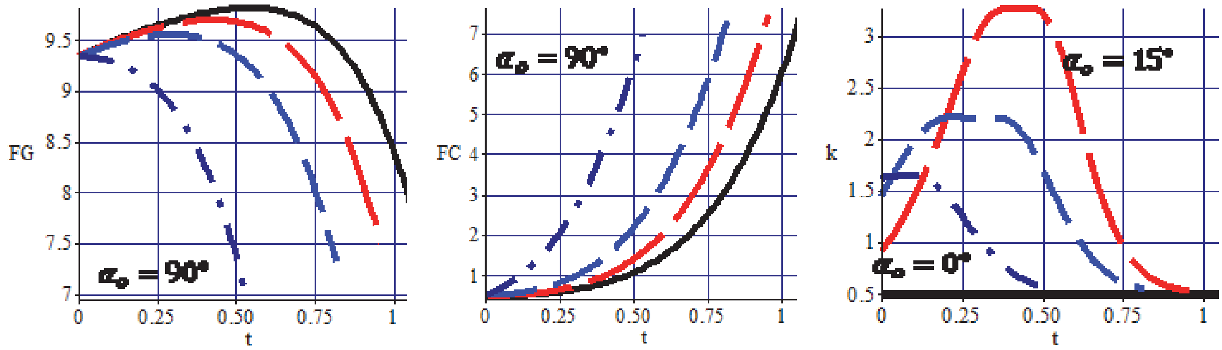

d) e) f)

Fig. 5. The trajectories $r(t)$, the graphs of velocity $V(t)$ and the forces of the normal reaction $F_N(t)$, its components of gravity $F_G(t)$, the centrifugal force $F_C(t)$ and the curvature $k(t)$

Conclusions and perspectives. The particles thrown perpendicular to the parallels of the sphere upwards after a certain moment of time will break off from the surface of the sphere, and all other particles will approach the lower pole of the sphere under the zigzag-based law. Moreover, all of them will not stop at the point of the pole of the sphere, but in its vicinity, whose radius depends on the value of the coefficient of friction f and the radius of the sphere a. Of all the particles, the fastest stop is that which is thrown towards the lower pole of the sphere.

Regardless of the initial velocity V_0 and the coefficient of friction *f*, the trajectories of all particles thrown at an angle $\alpha_0 = 90°$ will not cross the equator of the sphere. If the particle is thrown at the same initial velocity $V_0 = 4$ м/с in different places on the meridian of the spheres in the direction of the parallels, then their trajectories $r(t)$ converge in the vicinity of a certain point. The fastest part will be stopped, which was the lowest.

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MAPLE-МОДЕЛІ РУХУ ЧАСТИНКИ ПО ПОВЕРХНІ СФЕРИ

А. В. Несвідомін

Анотація. *Наведено основні результати моделювання руху частинки по шорсткій поверхні сфери у функції незалежного параметра t – часу. На відміну від лінійчатих поверхонь, аналітичні викладки руху частинки по нелінійчатих поверхнях обертання 2-го порядку є більш складними. Тому наводяться тільки окремі математичні вирази, які були одержані за допомогою розроблених maple-моделей* **Sphere_t***.*

Ключові слова: рух частинки, шорстка поверхня, сфера, система диференціальних рівнянь, траєкторія, швидкість

MAPLE-МОДЕЛИ ДВИЖЕНИЯ ЧАСТИЦЫ ПО ПОВЕРХНОСТИ СФЕРЫ

А. В. Несвидомин

Аннотация. *Приведены основные результаты моделирования движения частицы по шероховатой поверхности сферы в функции независимого параметра t – времени. В отличие от линейчатых поверхностей, аналитические выкладки движения частицы по нелинейчатых поверхностях вращения 2-го порядка являются более сложными. Поэтому приводятся только отдельные математические выражения, которые были получены с помощью разработанных maple-моделей Sphere_t.*

Ключевые слова: движение частицы, шероховатая поверхность, сфера, система дифференциальных уравнений, траектория, скорость