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# APPROXIMATION OF GROWTH INDICATORS AND ANALYSIS OF INDIVIDUAL GROWTH CURVES BY LINEAR DIMENSIONS OF TUBULAR BONES IN CHICKENS OF MEAT PRODUCTION DIRECTION DURING POSTNATAL PERIOD OF ONTOGENESIS

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***Abstract.*** *Medical and biological sciences, including morphology, now require the introduction of the latest information technologies and mathematical methods to process the obtained and accumulated research results. To study the growth dynamics of body weight in domestic birds, classical growth models, Gompertz, were used for the purpose of quantitative description of the growth processes in biological objects, in particular for the growth and development of birds – Von Bertalanffy, Richards, and hyperbolastic models.*

*The research material was tubular bones of the thoracic (humerus, ulna, and radius) and pelvic (femur, tibia, and tarsometatarsus) limbs in birds of meat production (broiler chickens and laying hens from the parent broiler flock of Cobb 500 strain) of different age groups during the postnatal period of ontogenesis.*

*An appropriate regression analysis of experimental data based on known growth models was performed to solve the goal of obtaining growth curves and identifying special points (extremes, inflections, etc.), to build a picture of the overall development of the body as a whole and individual bones of the extremities. The most biologically suitable growth models for describing the growth dynamics of the body as a whole and individual studied bones were determined.*

*The absence of a unified growth model of linear parameters of different tubular bones in meat-type chickens during the postnatal period of ontogenesis was established. This implies the need for a clear selection of growth models taking into account age, species, breed, keeping and feeding conditions of domestic birds.*

*The growth model that best describes the body weight dynamics of broiler chickens is the hyperbolastic growth model of the H3 type, and in laying hens from the parent broiler flock – the Brody growth model.*

***Keywords:*** *growth models, tubular bones, body weight, meat-type chickens*

## *Introduction*

The size and shape of living organisms are crucial for their survival and reproduction. Morphological adaptations of animals in terms of motor activity or feeding are classic examples in evolutionary biology (Cooney et al., 2016; Pigot et al., 2020). Instead, the dynamics of morphological transformations are mainly the result of the development during ontogenesis. Tubular bones, like most living tissues, are able to adapt their internal microstructure throughout life, and subsequently the associated mechanical properties to its specific mechanical and physiological environment in a process known as bone reconstruction (García-Aznar et al., 2005). Thus, morphological characteristics of tissues, in particular, tubular bones, affect the ability of animals to perform environmentally important tasks that are essential for their growth, survival, and suitability. At the same time, the thoracic and pelvic limbs of birds are specialized to perform various functions. Tubular bone growth and their relationship during ontogenesis are crucial to our understanding of morphological changes among species (Yan & Zhang, 2020).

Therefore, medical and biological sciences, including morphology, currently require the introduction of state-of-the-art information technology and mathematical methods to process the obtained and accumulated research results. Among the various methods in the assessment of morphological adaptations during ontogenesis is the methodology of selection of mathematical models and the formation of qualitative and quantitative patterns that describe the main features of the phenomenon under study. This is the stage where it is necessary to collect data on the structure and nature of this system's functioning, its properties, and manifestations. This stage ends with the creation of a qualitative model of the object (Tabatabai et al., 2007; Ahmadi & Golian, 2008; Dovhan et al., 2009).

## *Analysis of recent researches and publications*

There are different types of housing conditions in poultry. Therefore, it is necessary to use methods with optimal reliability to assess and predict bird health. One such accurate method is nonlinear mathematical models to determine the dynamics of growth processes, both individual organs and the body as a whole (Porter et al., 2010; Ramos et al., 2013).

At the same time, the relevance of individual selection of these models for different species of domestic animals is noted (Griebeler et al., 2013).

In particular, the linear dimensions of the tubular bones in broilers are more variable than body weight and bone structure. This is proved by the study of forced movement during the day, which affects the change in the length and width of the bones. At the same time, body weight and mineral density are maintained (Pinhasi et al., 2009).

Thus, the relative wing length and load on it affect the bird's ability to fly, and body weight is a contradictory sign of the ability to fly both within and among species (Foutz et al., 2007; Jones et al., 2019).

At the same time, comparing the length growth of the tubular bones of the wing and pelvic limb in birds, it was found that pelvic limb bones grow faster than the wings with a predominance of craniocaudal growth gradient. The study of the proportional bone growth by their length is more important in determining the degree of maturity than just the body mass index (Kaplan et al., 2016).

Classical Gompertz growth models (Cetin et al., 2007; Tjørve et al., 2017) were used to study the growth dynamics of bird body weight, in particular for bird growth and development (Tjørve et al., 2009), Bertalanfi (Tjørve et al., 2010; Zheng et al., 2020), Richards (E. Tjørve et al., 2010; Arando et al., 2021). However, hyperbolastic growth models designed to quantify the growth processes of biological objects are now widely used (Tabatabai et al., 2007).

At the same time, the criteria of compliance and flexibility of growth models were evaluated by comparing the root mean square error, the adjusted coefficient of determination, and the criteria of flexibility. All criteria were then reviewed in a pooled index to determine the most effective model for describing and predicting patterns of turkey body weight gain. The most suitable model for male growth was the logistic model, and for females – the Richards model (Arando et al., 2021).

Today, one of the problems in growing meat-type chickens is lameness, a phenotypic manifestation of different housing conditions. Meat-type chickens gain weight quickly and therefore develop limb disorders that lead to a high mortality rate in heavy birds. It should be noted that many abnormalities in bone development can be initiated at an early age, lameness appears only later due to conformational, environmental, or infectious problems. Some scientists have estimated the prediction of the development of this disease using mathematical models (Huff et al., 2006; Moraes et al., 2007).

The obtained growth rates should be adapted to the selective environmental factors of each bird species. However, the main forces of the growth and development are still unclear, especially when studying several features simultaneously. Therefore, the study of growth patterns can be best performed in poultry due to a sufficient number of indicators for calculation (Remeš et al., 2020).

Determining the parameters of the growth rate is important not only for the biological characteristics and productive qualities of different bird species but also to prevent possible violations of the integrity of the organism that occur at different stages of the postnatal ontogenesis.

***The purpose of the study*** is to select methods for determining approximation growth models and individual growth curves for body weight, length, sagittal and segmental diameters of the studied bones of the thoracic and pelvic limbs in broiler chickens and laying hens of the broiler parent flock.

## *Materials and methods of research*

The object of the study were broiler chickens of the Cobb 500 strain at the age of 1, 8, 15, 22, 28, 43, and 50 days and laying hens of the broiler parent flock of the same strain at the age of 1, 10, 51, 114, 175, 228, 350, and 410 days. In both age groups, the selection deadlines coincided with the technological cycle of use.

Broiler chickens were raised in the production enterprise CJSC Complex Agromars, a village of Gavrylivka (Kyiv region), and laying hens of the parent flock of broilers in the conditions of the production enterprise LLC Ruby Roses Agricol Co., LTD, a village of Morozivka (Kyiv region).

The studied poultry was kept in "floor" conditions according to the generally accepted technology for meat-type chickens. The feeding diet was balanced in terms of nutrients according to age.

After slaughter, the chickens were weighed on Casio HL-4 electronic scales and the bones of the thorax (wings) and pelvis were removed by standard methods.

The research material was tubular bones of the thoracic (humerus, ulna, and radius) and pelvic (femur, tibia, and tarsometatarsus) limbs of poultry meat production (broiler chickens and laying hens of the parent flock of crossbred Cobb 500 broilers of different age groups, postnatal period of ontogenesis.

First of all, the linear dimensions were determined according to the generally accepted scheme, which was tested on different mammal species and humans using a caliper U-10 (022504) with an accuracy of 0.05 mm. In total, 3150 measurements were performed according to the measurement scheme. Data on linear dimensions in millimeters were entered into osteological charts and subjected to statistical processing to obtain average values.

To solve this goal, to obtain growth curves and identify special points (extrema, inflections, etc.) to build a picture of the overall development of the body as a whole and individual limb bones, a corresponding regression analysis of experimental data based on known growth models was performed. We determined the most biologically suitable growth models for describing the growth dynamics of the body as a whole and individual studied bones. The approximation of each of these functions was evaluated by the values of biologically determined parameters, in particular, the indicator in chicken hatching, the indicator in the asymptotics (i.e. the indicator for the adult organism) for biological reasons and the value of standard deviation (SD).

The above calculations were carried out by means of the Wolfram Mathematica 6.0 mathematical package using the Levenberg-Markart algorithm in order to solve the corresponding optimization problems.

In this work, the abbreviated notation of the selected growth functions in the form: $w\ (t)$ – body weight, $l\ (t)$ – bone length, $a\ (t)$ – sagittal diameter, $e\ (t)$ – segmental diameter, with the symbol $t$ affects a certain moment in the life (i.e. age) of the bird. The symbols $w$, $l$, $a$, and $e$ are also used in the indices to denote other growth characteristics (growth rates, inflection points, etc.). The generalized parameter ($w$, $l$, $a$, and $e$) is denoted by the symbol $p$.

It was also found necessary to establish the features of individual growth curves *w (t)* and for the tubular bones of the limbs in broiler chicken *a (t), e (t),* and *l (t)*, in particular, the age of maximum growth rate *(T)* and the maximum growth rate *(V)*, the current relative growth rate *(Q)*, the age position of the maximum relative growth rate *(C)* and the maximum relative growth rate *(N)*. If the parameters of the selected growth functions are available, the calculation of *T, V, N,* and *Q* becomes a trivial arithmetic problem.

## *Results of the research and their discussion*

To solve this goal, the selection of methods and their generalization was carried out. Growth curves (functions) for body weight, length, sagittal and segmental diameters of the studied bones of the thoracic and pelvic limbs of broiler chickens and laying hens of the parent broiler flock were determined by the selected method.

The generalized values of the obtained biologically significant results of approximation of growth parameters of tubular bones for broiler chickens are given in Tables 1–3.

From the indicators shown in Table 1, it follows that they can be used to select the most successful models to describe *l (t)* for the corresponding bones in broiler chickens. The defined models in the specified Table are marked in bold.

By pointing to the indicators in Table 2, you can turn the most distant models for the description of *a (t)* for the type of bones in chicken broilers. The model values in the table are also indicated in bold.

Using the indicators presented in Table 3, models can be used to describe the growth of *e (t)* of tubular bones in broiler chickens. The models selected for the description in the table are also marked in bold.

In addition, according to the principle of analogy, an approximation of the growth indices for broilers was carried out, the results of that for hatched, the definitive mass of broilers, and the root mean square error are presented in Table 4.

The results of Table 5 are followed by the preference of the model H3 to describe *w (t)* in broilers.

However, an approximation of the corresponding experimental data of laying hens was performed. The same model functions were tested as in the case of broiler chickens (Tables 5–7).

From the indicators given in Table 5, it follows that they can be used to select the most successful models to describe *l (t)* for the corresponding tubular bones in the hens of the parent flock. The defined models in the specified the table are marked in bold.

From the indicators given in Table 6, you can choose the most successful models to describe *a (t)* of the corresponding bones in chickens of the parent flock. The defined models in the specified Table are also marked in bold.

From the indicators given in Table 7, you can choose the most successful models to describe *e (t)* of the corresponding bones in chickens of the parent flock. The defined models in the specified Table are also marked in bold.

The mentioned model functions were also tested to approximate the weight gain of laying hens. Biologically significant results of these tests are presented in Table 8.

For the convenience of further analyzes of all types, the selected model functions are summarized together with the corresponding parameters in a separate Table 9.

The information in Table 9 is the basic information for further studies of growth processes in tubular bones of experimental birds.

The analysis of the data given in

### 1. Biologically determined indicators of different approximation of growth models for l (t) of tubular bones in broiler chickens, mm

| Selected growth models | The name of the bone | | | | | |
|---|---|---|---|---|---|---|
| | Humerus | | | Ulna | | |
| | $l_\infty$ | $l_0$ | *SD* | $l_\infty$ | $l_0$ | *SD* |
| Von Bertalanffy | 141.717 | 30.404 | 3.319 | 74.003 | 11.453 | 2.230 |
| Brody | 540.200 | 30.335 | 3.469 | **80.234** | **8.503** | **1.659** |
| Gompertz | 89.017 | 28.933 | 3.883 | 72.262 | 12.460 | 2.609 |
| Logistic | 99.442 | 30.448 | 3.110 | 70.702 | 13.427 | 3.126 |
| Richards | 82.730 | 27.927 | 5.705 | 71.663 | 10.258 | 2.502 |
| Weibull | 68.426 | 34.481 | 1.548 | 90.348 | 3.926 | 1.527 |
| H1 | 92.677 | 30.503 | 3.016 | 79.549 | 5.532 | 1.375 |
| H2 | 68.093 | 34.279 | 1.365 | 99.547 | 1.111 | 1.525 |
| H3 | **70.056** | **33.910** | **1.635** | 94.255 | 3.150 | 1.504 |
| | Radius | | | Femur | | |
| | $l_\infty$ | $l_0$ | *SD* | $l_\infty$ | $l_0$ | *SD* |
| Von Bertalanffy | 64.716 | 9.038 | 7.850 | 74.003 | 11.453 | 2.230 |
| Brody | **67.864** | **5.288** | **7.446** | 80.234 | 8.503 | 1.659 |
| Gompertz | 63.795 | 10.206 | 8.138 | 72.262 | 12.460 | 2.609 |
| Logistic | 62.978 | 11.270 | 8.514 | 70.702 | 13.427 | 3.126 |
| Richards | 69.692 | 2.575 | 7.402 | **71.663** | **10.258** | **2.502** |
| Weibull | 68.703 | 4.497 | 7.441 | 140.623 | 14.628 | 4.305 |
| H1 | 65.874 | 4.806 | 7.090 | 91.167 | 14.549 | 4.022 |
| H2 | 71.262 | 2.539 | 7.549 | 116.966 | 11.932 | 4.249 |
| H3 | 68.727 | 4.474 | 7.441 | 118.318 | 14.984 | 4.285 |
| | Tarsometatarsus | | | Tibia | | |
| | $l_\infty$ | $l_0$ | *SD* | $l_\infty$ | $l_0$ | *SD* |
| Von Bertalanffy | 94.781 | 20.528 | 1.747 | 117.503 | 10.927 | 16.258 |
| Brody | 113.119 | 19.463 | 2.165 | 132.080 | 4.537 | 14.775 |
| Gompertz | 90.265 | 20.971 | 1.588 | **114.094** | **12.916** | **17.784** |
| Logistic | **85.788** | **21.563** | **1.405** | 111.685 | 14.447 | 19.596 |
| Richards | 92.225 | 20.813 | 1.654 | 131.979 | 4.578 | 14.775 |
| Weibull | 83.206 | 23.838 | 1.469 | 131.060 | 4.888 | 14.774 |
| H1 | 83.023 | 21.891 | 1.281 | 126.765 | 4.147 | 14.698 |
| H2 | 82.247 | 22.552 | 1.301 | 145.129 | 0.644 | 14.896 |
| H3 | 74.250 | 74.250 | 1.288 | 113.474 | 8.257 | 15.631 |

**Note:** The growth models highlighted in bold type are the most acceptable in terms of the biological change $p_0$, $p\infty$ and the accuracy of the description of the experimental data.

## 2. Biological values of the indicators of the approximate of growth models of a (t) of tubular bones in broiler chickens, mm

| Selected growth models | The name of the bone | | | | | |
|---|---|---|---|---|---|---|
| | Humerus | | | Ulna | | |
| | $a_\infty \times 10$ | $a_0 \times 10^2$ | $SD \times 10^4$ | $a_\infty \times 10$ | $a_0 \times 10^3$ | $SD \times 10^3$ |
| Von Bertalanffy | 8.747 | 10.331 | 9.024 | 6.387 | 13.077 | 1.661 |
| Brody | 9.664 | 6.756 | 11.195 | 7.052 | 0 | 2.355 |
| Gompertz | 8.511 | 11.498 | 8.270 | 6.330 | 29.534 | 1.694 |
| Logistic | 8.334 | 12.574 | 7.641 | 6.299 | 39.148 | 1.726 |
| Richards | 8.534 | 11.380 | 8.346 | 6.431 | 3.001 | 1.654 |
| Weibull | 8.046 | 16.559 | 7.395 | 6.309 | 6.191 | 1.562 |
| H1 | 8.158 | 13.123 | 6.911 | 6.350 | 13.605 | 1.616 |
| H2 | **8.082** | **15.192** | **7.013** | 6.415 | 14.650 | 1.710 |
| H3 | 8.242 | 15.333 | 7.664 | 8.560 | ≈ 0 | 2.445 |
| | Radius | | | Femur | | |
| | $a_\infty \times 10$ | $a_0 \times 10^2$ | $SD \times 10^4$ | $a_\infty \times 10$ | $a_0 \times 10^3$ | $SD \times 10^4$ |
| Von Bertalanffy | 4.245 | 6.097 | 2.303 | 9.075 | 108.711 | 4.173 |
| Brody | 5.203 | 5.334 | 2.748 | 10.010 | 66.739 | 3.097 |
| Gompertz | 4.035 | 6.401 | 2.129 | **8.838** | **122.809** | **5.172** |
| Logistic | 3.875 | 6.739 | 1.963 | 8.649 | 134.699 | 6.519 |
| Richards | 4.036 | 6.399 | 2.130 | 10.302 | 55.026 | 3.070 |
| Weibull | 3.573 | 9.030 | 1.624 | 35.224 | 50.607 | 7.014 |
| H1 | 3.726 | 7.132 | 1.786 | 21.631 | 36.929 | 3.134 |
| H2 | 3.593 | 8.415 | 1.654 | 11.836 | 4.752 | 3.050 |
| H3 | **3.513** | **7.638** | **1.537** | 10.742 | 71.479 | 3.011 |
| | Tarsometatarsus | | | Tibia | | |
| | $a_\infty \times 10$ | $a_0 \times 10^2$ | $SD \times 10^3$ | $a_\infty \times 10$ | $a_0 \times 10^2$ | $SD \times 10^4$ |
| Von Bertalanffy | 6.832 | 8.890 | 1.303 | 8.488 | 12.654 | 11.519 |
| Brody | 7.634 | 6.480 | 1.411 | 9.324 | 8.899 | 8.618 |
| Gompertz | 6.631 | 9.740 | 1.269 | 8.265 | 13.922 | 12.936 |
| Logistic | 6.475 | 10.566 | 1.242 | **8.065** | **15.067** | **14.607** |
| Richards | 6.641 | 9.700 | 1.271 | 14.327 | ≈ 0 | 6.194 |
| Weibull | 6.283 | 12.941 | 1.234 | 18.160 | ≈ 0 | 6.052 |
| H1 | 6.371 | 9.890 | 1.213 | 16.706 | 4.9068 | 6.054 |
| H2 | **6.345** | **11.547** | **1.229** | 18.215 | ≈ 0 | 6.328 |
| H3 | 9.279 | 6.880 | 1.418 | 18.162 | ≈ 0 | 6.052 |

**Note:** The growth models highlighted in bold type are the most acceptable in terms of the biological change $p^0$, $p^\infty$ and the accuracy of the description of the experimental data.

### 3. Biological values of indicators of the approximate of growth models for *e (t)* of tubular bones in broiler chickens, mm

| Selected growth models | The name of the bone | | | | | |
|---|---|---|---|---|---|---|
| | Humerus | | | Ulna | | |
| | $e_{\infty}\times 10$ | $e_0\times 10^2$ | $SD\times 10^4$ | $e_{\infty}\times 10$ | $e_0\times 10^2$ | $SD\times 10^4$ |
| Von Bertalanffy | 6.731 | 7.848 | 13.302 | 4.824 | 7.762 | 6.534 |
| Brody | 7.125 | 4.612 | 16.506 | 5.014 | 5.679 | 7.823 |
| Gompertz | 6.620 | 8.833 | 11.954 | 4.766 | 8.399 | 6.018 |
| Logistic | 6.537 | 9.808 | 10.660 | 4.715 | 9.169 | 5.432 |
| Richards | 6.612 | 8.824 | 11.981 | 4.767 | 8.386 | 6.219 |
| Weibull | 6.292 | 17.515 | 6.265 | 4.574 | 14.016 | 3.974 |
| H1 | 6.449 | 10.819 | 8.794 | 4.636 | 9.837 | 4.835 |
| H2 | 6.298 | 16.879 | 5.645 | **4.570** | **13.596** | **3.893** |
| H3 | **6.268** | **12.453** | **3.608** | 5.975 | 6.23 | 8.336 |
| | Radius | | | Femur | | |
| | $e_{\infty}\times 10$ | $e_0\times 10^2$ | $SD\times 10^4$ | $e_{\infty}$ | $e_0\times 10^2$ | $SD\times 10^4$ |
| Von Bertalanffy | 3.178 | 6.317 | 9.398 | 0.967 | 11.097 | 4.637 |
| Brody | 3.506 | 5.970 | 10.157 | 1.046 | 6.300 | 6.055 |
| Gompertz | 3.143 | 6.457 | 9.047 | 0.946 | 12.636 | 4.536 |
| Logistic | 3.024 | 6.727 | 8.571 | **0.930** | **13.983** | **4.771** |
| Richards | 3.113 | 6.457 | 9.039 | 0.949 | 1.2347 | 4.530 |
| Weibull | **2.835** | **10.517** | **5.364** | 1.003 | 8.701 | 5.355 |
| H1 | 2.983 | 6.824 | 8.085 | 0.939 | 8.718 | 4.395 |
| H2 | 2.841 | 10.392 | 5.393 | 0.956 | 10.439 | 4.556 |
| H3 | 4.253 | 6.251 | 10.139 | 1.147 | 6.772 | 6.230 |
| | Tarsometatarsus | | | Tibia | | |
| | $e_{\infty}$ | $e_0\times 10^2$ | $SD\times 10^4$ | $e_{\infty}\times 10$ | $e_0\times 10^2$ | $SD\times 10^4$ |
| Von Bertalanffy | 1.148 | 1.943 | 5.537 | 16.918 | 2.355 | 13.934 |
| Brody | 1.314 | 1.678 | 5.584 | 208.251 | 2.332 | 15.063 |
| Gompertz | 1.106 | 2.043 | 5.768 | 14.001 | 2.368 | 13.433 |
| Logistic | 1.068 | 2.148 | 6.262 | 12.140 | 2.386 | 12.867 |
| Richards | 1.203 | 1.838 | 5.466 | 16.496 | 2.340 | 13.955 |
| Weibull | 1.194 | 1.926 | 5.456 | 9.148 | 2.905 | 8.457 |
| H1 | 1.113 | 1.417 | 5.943 | 10.904 | 2.414 | 12.243 |
| H2 | 1.197 | 1.494 | 5.565 | 9.084 | 2.827 | 9.500 |
| H3 | **1.567** | **1.546** | **5.595** | **9.168** | **2.935** | **8.557** |

**Note:** The growth models highlighted in bold type are the most acceptable in terms of the biological change $p^0$, $p^{\infty}$ and the accuracy of the description of the experimental data.

### 4. Summary table of results of approximation of growth data of body weight in broiler chickens, g

| Growth model | $w_\infty$ | $w_0$ | *SD* |
|---|---|---|---|
| Von Bertalanffy | 3656.92 | 0.43 | 2922.63 |
| Brody | $1.79\times10^8$ | $< 0$ | 16500.90 |
| Gompertz | 3133.92 | 18.50 | 2125.19 |
| Logistic | 2983.12 | 28.10 | 1993.54 |
| Richards | 3124.92 | 18.33 | 2125.20 |
| Weibull | 263.95 | 51.39 | 2592.46 |
| H1 | 2660.35 | 40.00* | 2372.43 |
| H2 | 2765.47 | 40.00 | 2138.05 |
| H3 | **2813.69** | 40.00 | **2304.35** |

**Note:** * When approximated by hyperbolastic functions, it was taken into account that the average body weight of broiler chickens at the time of hatching is at the level of 40 g. The growth models highlighted in bold type are the most acceptable in terms of the biological change $p^0$, $p^\infty$ and the accuracy of the description of the experimental data.

Table 9 shows the acceptability of the hyperbolastic model of type H3 for the formation of body weight because this model gives the most biologically probable indicators of $p_0$ and $p$ under the lowest value of $R_2$.

Different parametric models are acceptable for the studied linear parameters of tubular bones, which indicates the need to select growth models for each case in the study of different species of animals (Zheng et al., 2020).

For the analysis of individual growth curves, information on the characteristics of individual growth curves, in particular, the values of $p\infty$, $p_0$, *T, V,* and *N*, may be of some practical interest. We have performed a comparative analysis of these characteristics for different *p* of a particular bone (Yan & Zhang, 2020).

In order to obtain the values of *T, V,* and *N* for *p* bones, the known technology of finding the inflection points of analytical functions was used. Therefore, the search for inflection points is actually reduced to finding the roots of the second derivatives of the growth function from Table 9.

For relatively simple growth functions (Weibull, Gompertz), which have a simple form of the second derivative, this problem was solved analytically (Tjørve et al, 2017). For other growth functions (hyperbolastic), the problem of finding these roots was solved by numerical methods of the Wolfram Mathematica® 6.0 package. The results of these calculations for *T, V,* and *N* are presented in Table 10.

Dashes indicate no inflection point. In this case, the absolute growth rate has the maximum value during hatching and gradually decreases with age.

From Table 10, it is seen that all tubular bones of laying hens are characterized by the absence of the inflection point on the growth curves *a (t)*, which causes the absence of a period of intensive growth in the growth of sagittal diameter at the beginning of the postnatal period of ontogenesis.

That is, the maximum growth rate of this diameter is at the beginning of the postnatal period and in subsequent age periods only decreases. This increase in

## 5. Biologically determined indicators of different approximation of growth models for l (t) of tubular bones in laying hens, mm

| Selected growth models | The name of the bone | | | | | |
|---|---|---|---|---|---|---|
| | Humerus | | | Ulna | | |
| | $l_\infty$ | $l_0$ | $SD \times 10^2$ | $l_\infty$ | $l_0$ | $SD \times 10^3$ |
| Von Bertalanffy | 8.267 | 1.123 | 5.237 | 8.710 | 1.362 | 17.389 |
| Brody | 8.253 | 1.650 | 4.732 | 8.660 | 1.775 | 29.392 |
| Gompertz | 8.247 | 1.797 | 4.570 | 8.637 | 1.895 | 36.218 |
| Logistic | 8.238 | 1.947 | 4.428 | 8.607 | 2.000 | 46.305 |
| Richards | 8.249 | 1.757 | 4.615 | 8.769 | $< 10^{-3}$ | 9.627 |
| Weibull | 8.227 | 2.258 | 4.341 | 8.897 | < 0 | 4.541 |
| H1 | 8.226 | 2.401 | 4.364 | **8.895** | **0.248** | **4.536** |
| H2 | **8.225** | **2.178** | **4.364** | 8.980 | 0.078 | 4.178 |
| H3 | 8.508 | < 0 | 9.495 | 9.005 | 0.692 | 4.100 |
| | Radius | | | Femur | | |
| | $l_\infty$ | $l_0 \times 10$ | $SD \times 10^3$ | $l_\infty$ | $l_0$ | $SD \times 10^4$ |
| Von Bertalanffy | 7.490 | 6.436 | 10.251 | 9.664 | 1.155 | 53.749 |
| Brody | 7.477 | 12.329 | 7.909 | 9.622 | 1.700 | 11.180 |
| Gompertz | 7.470 | 13.836 | 7.369 | 9.603 | 1.853 | 15.645 |
| Logistic | 7.463 | 15.203 | 7.150 | 9.579 | 1.993 | 42.677 |
| Richards | 7.471 | 13.620 | 7.434 | 9.618 | 1.737 | 10.814 |
| Weibull | **7.462** | **15.650** | **6.980** | 9.613 | 1.797 | 9.501 |
| H1 | 7.463 | 10.071 | 7.160 | 9.618 | 0.785 | 10.717 |
| H2 | 7.462 | 15.437 | 7.223 | 9.623 | 1.590 | 13.372 |
| H3 | 8.282 | 2.573 | 101.612 | **9.613** | **1.640** | **9.051** |
| | Tarsometatarsus | | | Tibia | | |
| | $l_\infty$ | $l_0$ | $SD \times 10^3$ | $l_\infty$ | $l_0$ | $SD \times 10^2$ |
| Von Bertalanffy | 8.703 | 0.755 | 5.262 | 13.080 | 1.818 | 7.135 |
| Brody | 8.679 | 1.389 | 3.033 | 13.042 | 2.526 | 4.548 |
| Gompertz | 8.668 | 1.552 | 2.987 | 13.002 | 2.939 | 2.789 |
| Logistic | 8.656 | 1.695 | 3.688 | 13.002 | 2.939 | 2.789 |
| Richards | 8.673 | 1.488 | 2.932 | 13.026 | 2.718 | 3.693 |
| Weibull | 7.171 | 1.500 | 2.803 | **12.960** | **3.555** | **2.167** |
| H1 | 8.673 | 0.667 | 2.954 | 13.023 | 1.299 | 3.971 |
| H2 | 8.674 | 1.425 | 3.094 | 12.962 | 3.326 | 2.230 |
| H3 | **8.671** | **1.487** | **2.802** | 13.097 | 1.277 | 22.192 |

**Note:** The growth models highlighted in bold type are the most acceptable in terms of the biological change $p^0$, $p^\infty$ and the accuracy of the description of the experimental data.

## 6. Biologically determined indicators of different approximation of growth models for a (t) of tubular bones in laying hens, mm

| Selected growth models | The name of the bone | | | | | |
|---|---|---|---|---|---|---|
| | Humerus | | | Ulna | | |
| | $a_\infty\times10$ | $a_0\times10^2$ | $SD\times10^3$ | $a_\infty\times10$ | $a_0\times10$ | $SD\times10^5$ |
| Von Bertalanffy | 9.337 | 22.541 | 2.063 | 7.651 | 1.804 | 13.060 |
| Brody | 9.270 | 25.324 | 2.193 | 7.604 | 2.043 | 13.886 |
| Gompertz | 12.556 | 26.329 | 2.269 | 7.584 | 2.133 | 16.285 |
| Logistic | 9.199 | 27.529 | 2.414 | 7.554 | 2.250 | 22.757 |
| Richards | 9.504 | 0 | 1.933 | 7.633 | 1.900 | 12.690 |
| Weibull | 9.752 | 0 | 1.876 | 7.626 | 1.948 | 12.476 |
| H1 | 10.978 | 4.242 | 1.870 | 7.630 | 1.013 | 11.959 |
| H2 | 10.000 | 1.475 | 1.849 | 7.650 | 1.724 | 13.899 |
| H3 | **9.937** | **19.698** | **1.809** | **7.598** | **1.703** | **7.161** |
| | Radius | | | Femur | | |
| | $a_\infty\times10$ | $a_0\times10^2$ | $SD\times10^5$ | $a_\infty\times10$ | $a_0\times10^3$ | $SD\times10^4$ |
| Von Bertalanffy | 4.403 | 19.742 | 14.229 | 8.847 | 240.409 | 5.624 |
| Brody | **3.679** | **12.836** | **14.870** | 8.802 | 267.164 | 7.194 |
| Gompertz | 4.325 | 20.343 | 17.401 | 8.780 | 276.452 | 8.059 |
| Logistic | 4.280 | 20.765 | 20.272 | 8.740 | 286.197 | 9.743 |
| Richards | 4.465 | 19.367 | 12.435 | 8.953 | 3.208 | 3.646 |
| Weibull | 26.010 | 7.303 | 4.094 | 9.078 | 0 | 3.350 |
| H1 | 7.813 | 10.784 | 2.988 | 9.770 | 39.013 | 3.720 |
| H2 | > 100 | 2.381 | 3.909 | 9.244 | 18.202 | 3.474 |
| H3 | 16.318 | ≈ 0 | 3.369 | **9.400** | **197.614** | **3.713** |
| | Tarsometatarsus | | | Tibia | | |
| | $a_\infty\times10$ | $a_0\times10^4$ | $SD\times10^4$ | $a_\infty\times10$ | $a_0\times10$ | $SD\times10^4$ |
| Von Bertalanffy | 6.859 | 1664.399 | 3.725 | 8.177 | 2.444 | 13.955 |
| Brody | 6.801 | 1878.231 | 5.165 | 8.163 | 2.648 | 12.260 |
| Gompertz | 6.776 | 1957.349 | 5.949 | 8.156 | 2.725 | 11.575 |
| Logistic | 6.737 | 2043.843 | 7.359 | 8.143 | 2.859 | 10.540 |
| Richards | 7.027 | 0 | 1.964 | 8.157 | 2.719 | 11.635 |
| Weibull | 7.241 | 0 | 1.546 | 8.210 | 1.096 | 19.261 |
| H1 | 8.207 | 316.356 | 1.412 | 8.138 | 2.902 | 10.165 |
| H2 | **7.664** | **5.159** | **1.432** | 8.113 | 3.201 | 9.538 |
| H3 | 7.408 | 1432.926 | 1.861 | **8.129** | **2.785** | **8.073** |

**Note:** The growth models highlighted in bold type are the most acceptable in terms of the biological change *p0, p∞* and the accuracy of the description of the experimental data.

the sagittal diameter can be explained by the predominance of factors inhibiting the growth of sagittal diameter of tubular bones over the factors of its acceleration. Moreover, this prevalence is observed throughout the postnatal period of ontogenesis (experimental) laying hens of the parent broiler herd (Remeš et al., 2020).

In addition to the sagittal diameter, we can assume that this behavior is also

### 7. Biologically determined indicators of different approximation of growth models for e (t) of tubular bones in laying hens, mm

| Selected growth models | The name of the bone | | | | | |
|---|---|---|---|---|---|---|
| | Humerus | | | Ulna | | |
| | $e_\infty\times10$ | $e_0\times10^3$ | $SD\times10^3$ | $e_\infty\times10$ | $e_0\times10$ | $SD\times10^4$ |
| Von Bertalanffy | 7.504 | < 0 | 7.269 | 4.898 | 1.293 | 6.992 |
| Brody | 7.504 | < 0 | 7.269 | 4.892 | 1.480 | 6.854 |
| Gompertz | 7.504 | 7.335 | 7.269 | 4.889 | 1.542 | 6.818 |
| Logistic | 7.504 | 19.154 | 7.269 | 4.883 | 1.628 | 6.807 |
| Richards | 7.504 | ≈ 0 | 7.269 | 4.889 | 1.542 | 6.819 |
| Weibull | 7.504 | ≈ 0 | 7.269 | **4.885** | **1.684** | **6.760** |
| H1 | **7.504** | **44.660** | **7.269** | 4.885 | 1.278 | 6.801 |
| H2 | 7.504 | 68.999 | 7.269 | 4.883 | 1.627 | 6.824 |
| H3 | 8.037 | ≈ 0 | 8.834 | 4.851 | 1.471 | 6.947 |
| | Radius | | | Femur | | |
| | $e_\infty\times10$ | $e_0\times10^4$ | $SD\times10^4$ | $e_\infty\times10$ | $e_0\times10^2$ | $SD\times10^4$ |
| Von Bertalanffy | 3.725 | 1211.982 | 12.068 | 9.385 | 22.152 | 7.051 |
| Brody | 3.679 | 1283.649 | 14.870 | 9.350 | 25.761 | 8.368 |
| Gompertz | 3.663 | 1313.890 | 16.311 | 9.333 | 26.881 | 9.065 |
| Logistic | 3.635 | 1361.934 | 19.325 | 9.304 | 28.003 | 1.025 |
| Richards | 4.271 | 3.126 | 6.334 | 9.443 | 0 | 5.642 |
| Weibull | 5.465 | 0 | 6.240 | 9.495 | ≈ 0 | 5.284 |
| H1 | **4.249** | **4205.523** | **6.155** | **9.508** | **4.343** | **5.145** |
| H2 | 5.678 | 19.083 | 6.291 | 9.576 | 2.891 | 5.515 |
| H3 | 5.085 | 5082.130 | 51.317 | 9.495 | ≈ 0 | 5.284 |
| | Tarsometatarsus | | | Tibia | | |
| | $e_\infty$ | $e_0\times10^3$ | $SD\times10^4$ | $e_\infty\times10$ | $e_0\times10^2$ | $SD\times10^3$ |
| Von Bertalanffy | 1.041 | 303.583 | 37.973 | 9.119 | 16.462 | 2.685 |
| Brody | 1.035 | 330.393 | 58.633 | 9.083 | 20.543 | 2.702 |
| Gompertz | 1.032 | 340.334 | 70.144 | 9.065 | 21.777 | 2.724 |
| Logistic | 1.026 | 352.342 | 93.784 | 9.040 | 23.046 | 2.770 |
| Richards | 1.061 | 3.573 | 10.822 | 9.115 | 17.063 | 2.685 |
| Weibull | 1.078 | 9.305 | 7.581 | 9.110 | 17.127 | 2.685 |
| H1 | **1.077** | **75.400** | **7.611** | 9.115 | 7.365 | 2.686 |
| H2 | 1.107 | 36.377 | 7.606 | 9.117 | 16.370 | 2.691 |
| H3 | 1.115 | 269.22 | 9.726 | **9.115** | **16.497** | **2.684** |

**Note:** The growth models highlighted in bold type are the most acceptable in terms of the biological change p0, p∞ and the accuracy of the description of the experimental data.

### 8. Summary table of results of approximation of growth data of body weight in laying hens, g

| Growth model | $w_\infty$ | $w_0$ | $SD$ |
|---|---|---|---|
| Von Bertalanffy | 243.33 | 5245.90 | 57732.70 |
| Brody | 47.01 | **6537.61** | **33784.10** |
| Gompertz | 352.748 | 4913.25 | 86110.70 |
| Logistic | 0 | 7870.24 | 31754.24 |
| Richards | 315.98 | 5031.62 | 73448.55 |
| Weibull | ≈ 0 | 8560.02 | 31317.92 |
| H1 | 50.00 | 63808.20 | 35682.28 |
| H2 | 50.00 | 6899.78 | 38404.96 |
| H3 | 50.00 | 7952.67 | 32249.47 |

**Note:** The growth models highlighted in bold type are the most acceptable in terms of the biological change p0, p∞ and the accuracy of the description of the experimental data.

### 9. The most acceptable approximations of the growth of tubular bones and body weight in broiler chickens and laying hens, mm

| The name of the bone | *l* (*t*) | *a* (*t*) | *e* (*t*) |
|---|---|---|---|
| Broiler chickens<br>$w(t)$: H3: $M = 2713.69$, $\alpha = 2673.69$, $\beta = 0.000181414$, $\gamma = 2.41066$, $\theta = 0.00166361$ | | | |
| Humerus | H3:<br>$M = 70.0562$<br>$\alpha = 36.1457$<br>$\beta = 0.000633271$<br>$\gamma = 2.119$<br>$\theta = 7.21822\times10^{-6}$ | H2:<br>$M = 0.808165$<br>$\alpha = 4.90089$<br>$\beta = 0.11606$<br>$\gamma = 1.0387$ | H3:<br>$M = 0.625752$<br>$\alpha = 0.501221$<br>$\beta = 6.97168\times10^{-6}$<br>$\gamma = 3.72442$<br>$\theta = 0.02516$ |
| Ulna | Brody:<br>$B = 0.894021$<br>$k = 0.0351153$<br>$M = 80.234$ | - | H2:<br>$M = 0.458035$<br>$\alpha = 2.68738$<br>$\beta = 0.0487252$<br>$\gamma = 1.50633$ |
| Radius | Brody:<br>$B = 0.922084$<br>$K = 0.0461969$<br>$M = 67.8641$ | H3:<br>$M = 0.351316$<br>$\alpha = 0.274938$<br>$\beta = 0.0000517819$<br>$\gamma = 2.86125$<br>$\theta = 0.018792$ | Weibull:<br>$k = 0.0000316808$<br>$m = 0.105182$<br>$M = 0.179191$<br>$\beta = 3.26981$ |
| Femur | Ричардс:<br>$B = 0.570888$<br>$k = 0.0571839$<br>$M = 71.6627$<br>$\mu = 2.29763$ | Gompertz:<br>$L = 1.97354$<br>$M = 0.883751$<br>$\alpha = 0.0694911$ | Logistic:<br>$k = 0.0879022$<br>$M = 0.929591$<br>$\mu = 2.73291$ |
| Tibia | Gompertz:<br>$L = 2.17857$<br>$M = 114.094$<br>$\alpha = 0.0683775$ | Logistic:<br>$k = 0.068346$<br>$M = 0.806512$<br>$\mu = 2.4203$ | H3:<br>$M = 0.914752$<br>$\alpha = 0.624297$<br>$\beta = 0.00064504$<br>$\gamma = 2.13112$<br>$\theta = 0$ |

| | | | |
|---|---|---|---|
| Tarsometatarsus | Logistic:<br>k = 0.0551342<br>M = 85.7876<br>$\mu = 1.99219$ | H2:<br>M = 0.634489<br>$\alpha = 5.09938$<br>$\beta = 0.194285$<br>$\gamma = 0.936812$ | H3:<br>M = 1.56447<br>$\alpha = 1.41006$<br>$\beta = 0.0112256$<br>$\gamma = 0$<br>$\theta = 0.0208725$ |
| Laying hens<br>*w* (*t*): Brody: *M* = 6537.61, *B* = 0.992808, *k* = 0.00333459 | | | |
| Humerus | H2:<br>M = 8.22537<br>$\alpha = 3.15005$<br>$\beta = 0.00510514$<br>$\gamma = 1.02602$ | H3:<br>M = 0.99373<br>$\alpha = 0.796746$<br>$\beta = 7.30852\times10^{-7}$<br>$\gamma = 2.14373$<br>$\theta = 0.0173553$ | H1:<br>M = 0.750404<br>$\alpha = 15.8027$<br>$\beta = 0.200098$<br>$\theta = 0.180303$ |
| Ulna | H1:<br>M = 8.89477<br>$\alpha = 36.3884$<br>$\beta = 0.00721397$<br>$\theta = 0.800526$ | H3:<br>M = 0.759775<br>$\alpha = 0.589478$<br>$\beta = 9.01315\times10^{-9}$<br>$\gamma = 3.58625$<br>$\theta = 0.0164941$ | Weibull:<br>M = 0.320103<br>k = 0.00709267<br>m = 0.168373<br>$\beta = 1.27428$ |
| Radius | Weibull:<br>M = 5.89785<br>k = 0.00656149<br>m = 1.56503<br>$\beta = 1.34392$ | Von Bertalanffy:<br>B = 0.296024<br>k = 0.00986984<br>M = 0.367936 | H1:<br>M = 0.42479<br>$\alpha = 9.10076$<br>$\beta = 0.00558144$<br>$\theta = 0.467032$ |
| Femur | H3:<br>M = 9.61272<br>$\alpha = 7.97292$<br>$\beta = 0.0233235$<br>$\gamma = 1.06994$<br>$\theta = -0.0114563$ | H3:<br>M = 0.94003<br>$\alpha = 0.742415$<br>$\beta = 0$<br>$\gamma = 0.669368$<br>$\theta = 0.216069$ | H1:<br>M = 0.950838<br>$\alpha = 20.8933$<br>$\beta = 0.00873147$<br>$\theta = 0.832502$ |
| Tibia | Weibull:<br>k = 0.00164194<br>m = 3.55469<br>M = 9.40693<br>$\beta = 1.60955$ | H3:<br>M = 0.812858<br>$\alpha = 0.534372$<br>$\beta = 1.64534\times10^{-8}$<br>$\gamma = 3.73593$<br>$\theta = 0.0139368$ | H3:<br>M = 0.911457<br>$\alpha = 0.74649$<br>$\beta = 0.0263728$<br>$\gamma = 0.998023$<br>$\theta = -0.00499139$ |
| Tarsometatarsus | H3:<br>M = 8.67074<br>$\alpha = 7.18375$<br>$\beta = 0.00864064$<br>$\gamma = 1.24041$<br>$\theta = 0.00171335$ | H2:<br>M = 0.766396<br>$\alpha = 1684.22$<br>$\beta = 6.68533$<br>$\gamma = 0.107594$ | H1:<br>M = 1.07727<br>$\alpha = 13.2873$<br>$\beta = 0.00474761$<br>$\theta = 0.669921$ |

observed in the growth and development of the segmental diameter of the radial, femoral, tibial, and metatarsal bones, in which *T* is quite close to 0.

Only for the humerus and ulna, the growth curve of segmental diameter has a significant (quite distant from 0) inflection point, which may be due to the influence of torsional loads, which have a fairly high level of action in these bones.

However, a completely different picture is observed regarding the distribution of inflection points in the tubular bones of broiler chickens, where all inflections (except $T_l$ of ulna and $T_e$ of tarsometatarsus) are reliable. In addition, the $T_e$ of the humerus and ulna in chickens is larger than the corresponding parameters of laying hens.

The presence of inflection *a (t)* of tubular bones in chickens, in contrast

## 10. Individual characteristics of the development of individual tubular bones in experimental birds birds

| The name of the bone | $l\ (t)$ | $a\ (t)$ | $e\ (t)$ |
|---|---|---|---|
| Broiler chickens$w\ (t)$: $T = 27.737$, $V = 72.654$, $N = 5.882\times10^{-2}$, C = 5.58461 | | | |
| Humerus | $T = 23.904 V = 9.977\times10^{-1} N = 2.047\times10^{-2}$<br>C = 19.2213 | $T = 16.033 V = 2.162\times10^{-2} N = 5.119\times10^{-2}$<br>C = 5.16865 | $T = 19.556$<br>$V = 2.153\times10^{-2}$<br>$N = 5.046\times10^{-2}$<br>C = 0 |
| Ulna | - | $T = 6.605 V = 3.043\times10^{-2} N = 1.996\times10^{-1}$<br>C = 0.675032 | $T = 16.396 V = 1.458\times10^{-2} N = 5.060\times10^{-2}$<br>C = 11.6958 |
| Radius | - | $T = 21.158$<br>$V = 8.23938\times10^{-3} N = 3.81449\times10^{-2}$<br>C = 0 | $T = 21.257 V = 9.55721\times10^{-3} N = 4.90442\times10^{-2}$<br>C = 17.9777 |
| Femur | $T = 4.745 V = 1.952 N = 1.013\times10^{-1}$<br>C = 0 | $T = 9.783 V = 2.259\times10^{-2} N = 6.949\times10^{-2}$<br>C = 0 | $T = 11.437 V = 2.551\times10^{-2} N = 6.435\times10^{-2}$<br>C = 0 |
| Tibia | $T = 11.388 V = 2.870 N = 6.838\times10^{-2}$<br>C = 0 | $T = 12.933 V = 1.689\times10^{-2} N = 4.836\times10^{-2}$<br>C = 0 | $T = 23.333 V = 1.780\times10^{-2} N = 3.251\times10^{-2}$<br>C = 16.3312 |
| Tarsometatarsus | $T = 12.502 V = 1.400 N = 3.671\times10^{-2}$<br>C = 0 | $T = 15.326 V = 1.520\times10^{-2} N = 4.872\times10^{-2}$<br>C = 0 | - |
| Laying hens $w\ (t)$: there is no inflection point | | | |
| Humerus | $T = 27.8238 V = 0.0937657 N = 0.0213174$<br>C = 7.76348 | - | $T = 13.9236 V = 0.0305553 N = 0.084365$<br>C = 0 |
| Ulna | $T = 1.31197 V = 0.313971 N = 0.514416$<br>C = 0 | - | $T = 14.5581$<br>$V = 0.00486297$<br>$N = 0.0211101$<br>C = 0 |
| Radius | $T = 15.2725$<br>$V = 0.102825$<br>$N = 0.0354976$<br>C = 5.61676 | $T < 0$ | $T = 0.388981 V = 0.0190749 N = 0.386832$<br>C = 0 |
| Femur | $T = 7.02549 V = 0.123033 N = 0.0501829$<br>C = 1.64339 | - | $T = 0.983014 V = 0.047113 N = 0.547511$<br>C = 0 |
| Tibia | $T = 29.3819 V = 0.133631 N = 0.0204947$<br>C = 2.77722 | - | - |
| Tarsometatarsus | $T = 10.6745 V = 0.123758 N = 0.0462142$<br>C = 15.8117 | - | $T = 0.646768$V = $0.0559817 N = 0.50977$<br>C = 0 |

**Note:** Units: T – day, V – g/day in the case of w (t) or mm/day in other cases, N – day-1, C – day.

to laying hens, may be explained by the conditions of keeping these birds that change the biological patterns of ontogenetic growth, which are manifested in chickens of the parent broiler flock.

Broilers are kept in conditions aimed at maintaining the maximum rate of weight gain. In this case, this rate of mass growth may occur due to the suboptimal functioning of other parts of the body. Therefore, the factors influencing the diametrical development of bones in the case of broiler chickens are more unpredictable, which may cause an abnormally large acceleration of diametrical bone growth in broiler chickens during the 6th–23rd day of the postnatal period.

The pattern of growth of the bone length of the extremities is more stable. Both broiler chickens and laying hens have $T_l$ (except for the radial bone). Given the proximity to 0 $T_l$ of the ulna in laying hens, the patterns of distribution of acceleration and inhibition factors in increasing the length of the tubular bones of broiler chickens and laying hens can be considered qualitatively similar. The exclusion of the radial bone from the general picture may be explained by the increased growth rate of those tubular bones in the wing that perceive the force of torsional, bending loads in broiler chickens against the background of limited overall metabolism, which causes virtually no factors accelerating radial bone growth. This bone mainly performs the auxiliary function of the lever of movements, which holds the muscle groups and has no significant longitudinal loads (Blom & Clas, 2004).

Analysis of the $V_l$ values of the respective bones in broiler and laying hens indicates an increased growth rate of tubular bone length in chickens compared to laying hens, due to housing conditions that contribute to a rapid weight gain of birds. However, a similar pattern is observed for the values of $V_e$ of ulna and humerus.

Maximum relative velocities do not give a clear comparative picture between broiler chickens and laying hens, which may be due to the behavior of the growth curve of the bird's body weight. Thus, for broilers, it was found that $T_w \approx 28$ days, and in the general case indicates a later period of inhibition of development of the whole organism than individual tubular bones.

However, for laying hens, no inflection points were found at all in the body weight growth curve, which may also be caused by the conditions of the bird, aimed at the optimal physiological functioning of all parts of the body.

## Conclusions and future perspectives

The main purpose of the selection of mathematical models is the formation of qualitative and quantitative patterns of age development of the studied objects. The growth models used in the studies may be of practical value for assessing the dynamics of changes in body weight and linear parameters of tubular bones. Therefore, the most acceptable growth models for these parameters should be recommended for widespread use in poultry farming to increase poultry meat productivity.

The performed approximation analysis makes it possible to analytically determine the age periods of the prevalence of one parameter over another and comparative analysis between different tubular bones allows to analyze their functional activity at different age periods.

In addition, a comparative analysis of similar parameters between broiler chickens and laying hens allows us to draw some conclusions about the influence of housing conditions and genetics on the development of similar tubular bones in chickens.

The lack of a unified growth model of the linear parameters of different tubular bones in meat-type chickens during the postnatal period of ontogenesis indicates the need for a clear selection of growth models taking into account age, species, breed, housing and feeding conditions. The growth model that best describes the body weight dynamics of broiler chickens is the hyperbolastic growth model of the H3 type, and the laying hens of the parent broiler flock are the Brody growth model.

In the future, we plan to compare the growth rate of body weight, as an integral characteristic of the development of the organism, with the growth rate of individual tubular bones.

## References

Ahmadi, H., & Golian, A. (2008). Non-linear hyperbolastic growth models for describing growth curve in classical strain of broiler chicken. Research Journal of Biological Siences, 3(11), 1300-1304. doi: rjbsci.2008.1300.1304

Arando, A., González-Ariza, A., Lupi, T. M., Nogales, S., León, J. M., & Navas-González, F. J. (2021). Comparison of non-linear models to describe the growth in the Andalusian turkey breed. Italian Journal of Animal Science, 20(1), 1156-1167. doi: 10.1080/1828051X.2021.1950054

Blom, J., & Clas, L. (2004). A comparative study of growth, skeletal development and eggshell composition in some species of birds. Journal of Zoology, 262(4), 361-369. doi: 10.1017/S0952836903004746

Cetin, M., Sengul, T., Sogut, B., & Yurtseven, S. (2007). Comparison of growth models of male and female Partridges. Journal of Biological Science, 7(6), 964-968. doi: 10.3923/jbs.2007.964.968

Cooney, C. R., Seddon, N., & Tobias, J. A. (2016). Widespread correlations between climatic niche evolution and species diversification in birds. Journal Animal Ecology, 85(4), 869-878. doi: 10.1111/1365-2656.12530

Dovhan, Y. P., Hladkykh, V. Yu., & Shevchuk, Yu. H. (2009). Zastosuvannia metodu matematychnoho modeliuvannia z metoiu prohnozuvannia stanu miazovo-venoznoi pompy homilky [Application of the method of mathematical modeling in order to predict the state of the muscle-venous pump of the lower leg]. Visnyk morfolohii, 15, 184-188.

Foutz, T. L., Griffin, A. K., Halper, J. T., & Rowland G. N. (2007). Effects of increased physical activity on juvenile avian bone. Transactions of the ASABE, 50(1), 213-219. doi: 10.13031/2013.22402

García-Aznar, J. M., Rueberg, T. & Doblare, M. (2005). A bone remodelling model coupling microdamage growth and repair by 3D BMU-activity. Biomechanics and Modeling in Mechanobiology, 4(2-3), 67-147. doi: 10.1007/s10237-005-0067-x.

Griebeler, E. M., Klein, N., & Sander, P. M. (2013). Aging, maturation and growth of sauropodomorph dinosaurs as deduced from growth curves using long bone histological data: An assessment of methodological constraints and solutions. Plos One, 8(6), e67012. doi: 10.1371/journal.pone.0067012

Huff, G. R., Huff, W. E., Rath, N. C., Balog, J. M., Anthony, N., & Nestor, K. (2006). Stress-induced Colibacillosis and turkey osteomyelitis complex in turkeys selected for increased body weight. Poultry Science, 85(2), 266-272. doi: 10.1093/ps/85.2.266

Jones,T. M., Benson, T. J., & Ward, M. P. (2019). Does the size and developmental stage of traits at fledging reflect juvenile flight ability among songbirds? Functional Ecology, 34(4), 799-810. doi: 10.1111/1365-2435.13513

Kaplan, S., Narinc, D., & Gurcan, E. K. (2016). Genetic parameter estimates of weekly body weight and Richard's growth curve in Japanese quail. Europian Poultry Science, 80, 1-10. doi: 10.1399/eps.2016.165

Moraes, V. M. B., Malheiros, R. D., Furlan, R. L. Bruno, L. D. G., Malheiros, E. B., & Macari, M. (2007). Effect of environmental temperature during the first week of brooding on broiler chick body weight, viscera and bone development. Brazil Journal Poultry Science, 4(1), 1-8. doi: 10.1590/S1516-635X2002000100003

Pigot, A. L., Sheard, C., Miller, E. T., Bregman, T. P., Freeman, B. G., Roll, U., ... & Tobias, J. A. (2020). Macroevolutionary convergence connects morphological form to ecological function in birds. Nature Ecology & Evolution, 4, 230-239. doi: 10.1038/s41559-019-1070-4

Pinhasi, R., Shaw, P., White, B., & Ogden, A. R. (2009). Morbidity, rickets and long-bone growth in post-medieval Britain a cross-population analysis. Annals of Human Biology, 33(3), 372-389. doi: 10.1080/03014460600707503

Porter, T., Kebreab, E., Darmani, K. H., Lopez, S., Strathe A. B., & France, J. (2010). Flexible alternatives to the Gompertz equation for describing growth with age in turkey hens. Poultry Science, 89(2), 371-378. doi: 10.3382/ps.2009-00141

Ramos, S. B., Caetano, S. L., Savegnago, R. P., Nunes, B. N., Ramos, A. A., & Munari, D. P. (2013). Growth curves for ostriches (Struthio camelus) in a Brazilian population. Poultry Science, 92(1), 277-282. doi: 10.3382/ps.2012-02380

Remeš, V., Matysioková, B., & Vrána, J. (2020). Adaptation and constraint shape the evolution of growth patterns in passerine birds across the globe. Frontiers in Zoology, 17, 29. doi: 10.1186/s12983-020-00377-7

Tabatabai, M. A., Bursac, Z., Wiliams, D. K., & Singh, K. P. (2007). Hyperbolastic survival model. Theoretical Biology and Medical Modelling, 4(40), 325-335. doi: 10.1186/1742-4682-4-40

Tjørve, K. M. C., & Tjørve, E. (2017). The use of Gompertz models in growth analyses, and new Gompertz-model approach: An addition to the Unified-Richards family. Plos One, 12(6), e0178691. doi: 10.1371/journal.pone.0178691

Tjørve, K. M. C., García-Peña, G. E., & Székely, T. (2009). Chick growth rates in charadriiformes: Comparative analyses of breeding climate, development mode and parental care. Journal of Avian Biology, 40(5), 553-558. doi: 10.1111/j.1600-048X.2009.04661.x

Tjørve, K. M. C., & Tjørve, E. (2010). Shapes and functions of bird-growth models: How to characterize chick postnatal growth. Zoology, 113(6), 326-333. doi: 10.1016/j.zool.2010.05.003

Tjørve, E., & Tjørve, K. M. C. (2010). A unified approach to the richards-model family for use in growth analyses: Why we need only two model forms. Journal of Theoretical Biology, 267(3), 417-425. doi: 10.1016/j.jtbi.2010.09.008

Yan, J., & Zhang, Z. (2020). Post-hatching gr owth of the limbs in an altricial bird species. Veterinary Medicine and Science, 7(1), 210-218. doi: 10.1002/vms3.357

Zheng, N., Cadigan, N., & Morgan, M. J. (2020). A spatiotemporal Richards-Schnute growth model and its estimation when data are collected through length-stratified sampling. Environmental and Ecological Statistics, 27, 415-446. doi: 10.1007/s10651-020-00450-8

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***Ткачук С. А., Пасніченко О. С., Савчук Л. Б. (2021). АПРОКСИМАЦІЯ РОСТОВИХ ПОКАЗНИКІВ ТА АНАЛІЗ ІНДИВІДУАЛЬНИХ РОСТОВИХ КРИВИХ ЗА ЛІНІЙНИМИ ПРОМІРАМИ ТРУБЧАСТИХ КІСТОК КУРЕЙ М'ЯСНОГО НАПРЯМУ ПРОДУКТИВНОСТІ В ПОСТНАТАЛЬНОМУ ПЕРІОДІ ОНТОГЕНЕЗУ.***
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***Анотація.*** *Медико-біологічні науки, у тому числі й морфологія, нині потребують впровадження найсучасніших інформаційних технологій та математичних методів для обробки отриманих і накопичених результатів дослідження. Для вивчення ростової динаміки маси тіла свійської птиці застосовували класичні ростові моделі – Гомпертца, зокрема для росту та розвитку птиці – Берталанфі, Річардса та гіперболастичні, для кількісного описання ростових процесів біологічних об'єктів.*

*Матеріалом досліджень були трубчасті кістки грудної (плечова, ліктьова та променева) та тазової (стегнова, великогомілкова та заплесно-плеснова) кінцівок птиці м'ясного напряму продуктивності (курчат-бройлерів та курей-несучок батьківського стада бройлерів кросу Cobb-500) різних вікових груп постнатального періоду онтогенезу.*

*Для вирішення поставленої мети щодо отримання ростових кривих і виявлення особливих точок (екстремумів, перегинів тощо), для побудови картини сукупного розвитку тіла загалом та деяких кісток кінцівок, проведено відповідний регресійний аналіз експериментальних даних на базі відомих ростових моделей. Водночас визначалися найбільш придатні з біологічної точки зору ростові моделі для описання динаміки росту тіла загалом та деяких досліджуваних кісток.*

*Встановлено відсутність уніфікованої ростової моделі лінійних параметрів різних трубчастих кісток курей м'ясного напряму продуктивності в постнатальному періоді онтогенезу. З цього слідує необхідність чіткого підбору ростових моделей із врахуванням віку, виду, породи, умов утримання та годівлі свійської птиці.*

*Ростовою моделлю, що найкраще описує динаміку маси тіла курчат-бройлерів, є гіперболастична ростова модель типу H3, а курей-несучок батьківського бройлерного стада – ростова модель Броді.*

***Ключові слова:*** *ростові моделі, трубчасті кістки, маса тіла, кури мясного напряму продуктивності*