UDC 514.18

CONSTRUCTING OF SPHERICAL CURVES IN THE FUNCTION OF THE NATURAL PARAMETER

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Parametric equations of the curves in the function of the natural parameter have the particular importance for solving many problems, especially in the differential geometry. Such curves on the sphere are interesting, because spherical curves are a special class of curves. The objective of this article is to develop the approaches of the constructing of space curves on the surface of the sphere, which have the parametric equations in the function of the natural parameter.

The parametric equations of the sphere of radius, which is equal by one, are:

$$X=\sin u\cdot\cos v;\qquad Y=\sin u\cdot\sin v;\qquad Z=\cos u, \tag{1}$$

where u and v – independent variables.

The length of the lines on the surface of the sphere can be determined by first quadratic form:

$$dS^2=du^2+\sin^2 u\cdot dv^2. \tag{2}$$

<u>The first approach.</u> To set the line at the surface of the sphere (1), we should associate independent variables u and v. The first approach is based on constructing curves by means of integrating the quadratic form (2).

Let the independent variable is u, so the functional dependence is: $v=v(u)$. Let's divide left and right side on the equation (2) to du^2:

$$(ds/du)^2=(du/du)^2+\sin^2 u\cdot(dv/du)^2, \tag{3}$$

where $ds/du=\sqrt{1+\sin^2 u\cdot(dv/du)^2}$. Impediment in obtaining of analytical expressions of the length of arc is integrating expression under the square root. It can be avoided by equating the left side of equation (3) to the function *f(u)*, so that it is integrated, and then find the dependence *u=u(s)*. It allows to find the curve in the function of the natural parameter on the surface of the sphere.

<u>The second approach.</u> Another approach to constructing of the curves on the surface of sphere in the natural parameter can be formulated if the surface is attributed to isometric coordinate system. Parametric equations of the sphere of radius which is equal by one in this case can be written:

$$X=\operatorname{sech}u\cdot\cos v;\qquad Y=\operatorname{sech}u\cdot\sin v;\qquad Z=\operatorname{tgh}u. \tag{4}$$

First fundamental form of the sphere looks like:

$$dS^2 = \operatorname{sech}^2 u(du^2 + dv^2). \tag{5}$$

Constructing of the spherical curve in the function of the natural parameter can be realized by the linking of the independent variables *u* and *v* by means of a third variable *t*. Then integration of the first quadratic form allows to get the dependence *t=t(s)*, and the equation of the curve in the internal coordinates can help you find the parametric equation of the curve in the function of the length of own arc.

<u>A third approach</u> to constructing of the curves in the surface of sphere which can be described by the parametric equations in the function of the natural parameter is based on the equations of surface of rotation:

$$X = f \cdot \cos v; \quad Y = f \cdot \sin v; \quad Z = \psi, \tag{6}$$

where $f=f(s)$ and $\psi=\psi(s)$ – some functions.

Ley's put a condition that the surface of rotation is sphere. Then should the equality be performed:

$$x'^2 + y'^2 + z'^2 = 1. \tag{7}$$

By the substituting (6) into (7) we can obtain:

$$\psi = \sqrt{1 - f^2}. \tag{8}$$

The parametric equations of the sphere are:

$$X = f \cdot \cos v; \quad Y = f \cdot \sin v; \quad Z = \sqrt{1 - f^2}. \tag{9}$$

Setting of the function $f=f(s)$ will provide definitive spherical curve in the function of the natural parameter.

By the developed approaches we can get new spherical curves which are described by the parametric equations in the function of the length of own arc.