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FORCED VIBRATIONS OF MULTILAYERED CYLINDRICAL SHELLS TAKING INTO ACCOUNT THE DISCRESIBILITY OF THE RIBS WITH NON-STEADY LOADS.

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Abstract. *This work considers the problem of nonstationary behavior of multilayered discretely reinforced cylindrical shells.By the way the problem is very important. Multiplayed shells with allowance for discrete ribs are widely used in engineering, industrial and public building, aviation and space technology, shipbuilding.*

In the framework of the Timoshenko type non – linear theory of shells and ribs nonstationary vibrations multilayered shells of revolution with allowance for discrete ribs are investigated. Reissner's variational principle for dynamical processes is used for deduction of the motion equations. An efficient numerical method with using Richardson type finite difference approximation for solution of problems on nonstationary behaviour of multiplayer shells of revolution with allowance for discrete ribs which permit to realize solution of the investigated wave problems with the use of personal computers, as well as bringing their solutions to receiving concrete numerical results in wide diapason of geometrical, physico–mechanical parameters of structures are elaborated. In particular three-layer discretely reinforced cylindrical shells were investigated.

Key words. *multilayered cylindrical shells of revolution, geometrically nonlinear theory of shells and ribs, stress–strained state, non-stationary loading, numerical method, nonstationary vibrations.*

Multilayer reinforced shell with regard to the discresibility of the ribs placement in non-stationary loads are widely used in various fields of modern technology such as: aviation and missile, machine building, shipbuilding, building. The complexity of the processes, which arise at the same time, cause the need to use modern numerical methods of solving dynamic problems of multilayer shell structures. One of the features of this subject is to account for the discresibility of placement when staging initial tasks.

Problem statement. As an example, the problem of determination of a three-layer reinforced cylindrical shell is considered. The three-layer elastic reinforced membrane structure consists of external and internal smooth shells, which are interconnected by filler, and discrete by the reinforcers of annular elements. It relies that for the calculation of stressed-deformed state (HC) of elastic structure the variant of geometrically nonlinear theory of rods and shells of type Timoshenko using hypotheses for the whole package is used. The reinforcer elements are regarded as a set of curved rods that are tightly connected to the shell. For calculation the variant of the theory of curved rods of type Timoshenko is accepted.

Using the variational principle of Reisner for dynamic processes [1,3] the following equations systems are obtained:

1) The equation of fluctuations of the proprietary multilayer shell in a smooth area between the corresponding discrete ribs.

$$\frac{\partial T_{11}}{\partial x} + P_1 = I_1 \frac{\partial^2 U_1}{\partial t^2} + I_2 \frac{\partial^2 \varphi_1}{\partial t^2}, \qquad (1)$$

$$\frac{\partial \overline{T}_{13}}{\partial x} + \frac{T_{22}}{R} + P_3 = I_1 \frac{\partial^2 U_3}{\partial t^2},$$

$$\frac{\partial M_{11}^*}{\partial x} - T_{13} + m_1 = I_2 \frac{\partial^2 U_1}{\partial t^2} + I_3 \frac{\partial^2 \varphi_1}{\partial t^2},$$

$$\overline{T}_{13} = T_{13} + T_{11}\Theta_1,\; M_{11}^* = M_{11} \pm h_{cm} T_{11};$$

2) Oscillation equation j ring ribs at tear points $x = x_j$ (points of design centers of weight cross section on the present the median surface of the smooth multilayer shell) [7,8]

$$[T_{11}]_j = \rho_j F_j \left(\frac{\partial^2 u_1}{\partial t^2} \pm h_{cj} \frac{\partial^2 \varphi_1}{\partial t^2} \right),$$

$$[\overline{T}_{13}]_j - \frac{T_{22j}}{R_j} = \rho_j F_j \frac{\partial^2 u_3}{\partial t^2}, \qquad (2)$$

$$[M_{11}]_j = \rho_j F_j \left[\pm h_j \left(\frac{\partial^2 u_1}{\partial t^2} \pm h_{cj} \frac{\partial^2 \varphi_1}{\partial t^2} \right) + \frac{I_{\kappa pj}}{F_j} \frac{\partial^2 \varphi_1}{\partial t^2} \right],$$

де

$$(T_{11}, T_{22}, T_{13}) = \sum_k \int_z \left(\sigma_{11}^{kz}, \sigma_{22}^{kz}, \sigma_{13}^{kz} \right) dz, \qquad (M_{11}) = \sum_k \int_z \left(z\sigma_{11}^{kz} \right) dz,$$

$$I_1=\sum_k \rho_k h_k,\quad I_2=\sum_k \pm\rho_k h_k h_{ck},\quad I_3=\sum_k \rho_k \frac{h_k}{12}.$$

In the equations (1) – (2) the following designations are introduced: x, t – dimensional and timeline coordinates, respectively, R – normalized radius of the median surface of a multilayer shell; ρ_k, ρ_j – density of materials according k layer shell and j edges; h_k - the thickness of the corresponding shell layers, h_{ck} - distance from original median surface to middle surface k layer; h_{cj} – distance from original median surface to center line of gravity of cross section j edges; x_j – coordinate the contact line j edges with a multilayer shell; R_j, F_j, I_{kpj} – geometric parameters j edges. In the notation for the values of effort and moments it relies on $\sigma_{11}^{kz}, \sigma_{22}^{kz}, \sigma_{13}^{kz}$ – thickness voltage respectively k layer at $-\frac{h_k}{2} \le z \le \frac{h_k}{2}$, $k=\overline{1,n}$.

Numerical algorithm. In order to construct a numerical solution of non-stationary problems, the theory of heterogeneous multilayer shells is used integro – interpolation method of construction of difference schemes [2,4] for hyperbolic equations. Due to initial setting of problems the numerical solution is searched in smooth area of elastic structure (for multilayer shell between ribs) and on the lines of corresponding ribs.

To build more efficient algorithms, an approach based on finding approximate solutions to Richardson is applied. Moreover, with a fixed differential step on the timeline coordinate, the sequence of approximate approximations by spatial coordinate. In this case, the extrapolation procedure is formed according to formulas [5,6]

$$\tilde{\overline{U}}^{n}_{l(\Delta s)}=\frac{4}{3}\overline{U}^{n}_{l(\Delta s/2)}-\frac{1}{3}\overline{U}^{n}_{l(\Delta s)}, \tag{4}$$

where $\overline{U}^{n}_{l(\Delta s/2)}$ and $\overline{U}^{n}_{l(\Delta s)}$ - numerical solutions of vibration equations according to discrete steps on the dimensional coordinate $\Delta s/2$ і Δs, $s=A_1\alpha_1$.

Easy to show that differencing equation (4) approximated the output equation of oscillation (2) in a smooth area with the fourth order of accuracy by the coordinate x.

Results of calculations. As a numerical example, the problem of determination of a three-layer reinforced cylindrical shell is considered, taking into account the discresibility of the elements representing a set of discrete annular ribs, in the internal normal sturgeon pulse load. It was relied that the edges of the shells rigidly pinched. The initial conditions are zero.

Axial oscillations of the three-layer cylindrical reinforced shells taking into account the discrecibility of the ribs placement were considered in the following geometric and physical-mechanical parameters.

$$h = h_1 + h_2 + h_3; \quad h_1 = h_3 = 1 \cdot 10^{-3} \text{ м}; \quad h_2 / h_1 = 3;$$

$$R/h = 20; \; L/h = 80; \; L/R = 4; \; h_j/h = 2; \; F_j = h_j h;$$

$$E_1^1 = E_1^3 = E_j = 7 \cdot 10^{10} \text{ Pa}; \; E_1^1 / E^{зап} = 10 \div 1000;$$

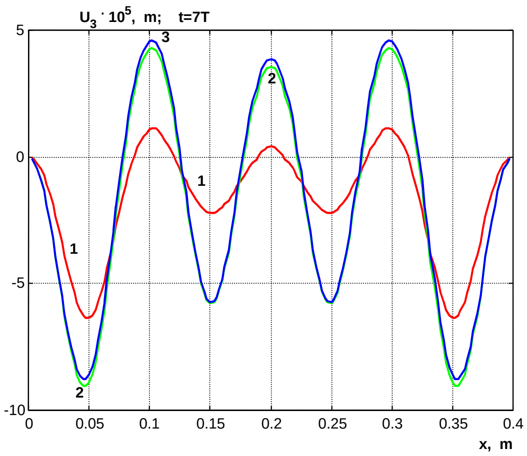
$$\nu_1^1 = \nu_1^3 = 0.3; \; \nu_1^{зап} = 0{,}4; \; \rho_1 / \rho_{зап} = 7; \; \rho_1 = \rho_3 = \rho_j = 2.7 \cdot 10^3 \text{kg/m}^3.$$

Normal pulse load was assignment in the form:

$$P_3 = A \cdot \sin \frac{\pi t}{T} [\eta(t) - \eta(t - T)],$$

where A – amplitude load; T – load duration. The calculations relied $A = 10^6$ Pa; $T = 50 \cdot 10^{-6}$ sec. Reinforcing elements are located in points $x_j = 0{,}25 L j$, $j = \overline{1{,}3}$. Numerical results can be characterized by the stress-deformation state of the three-layer elastic structure of the cylindrical type at any time in the investigated time interval in accordance with the above productions.

On pic.1 presented depending on the magnitude of deflection U_3 by coordinates *x* depending on the physical-mechanical parameters of the filler. Curve 1 responds to the case $E_1^1 / E_1^{зап} = 10$; 2 - curve – case $E_1^1 / E_1^{зап} = 100$; 3 - curve – case $E_1^1 / E_1^{зап} = 1000$.



Pic.1 Dependence of the magnitude U_3 from the spatial coordinates

Calculations were made in the time interval $0 \le t \le 40T$, moreover, the dependence on the figure corresponds to the time of achieving the maximum values of these values t=7T. Bringing dependence to investigate the impact of values $E_1^1/E_1^{зап}$ and reinforcing ribs on the tense-the deformed state of a multilayer heterogeneous structure. Note that curves 2 and 3 on the presented figures almost coincide, itindicates that starting with the value of $E_1^1/E_1^{зап} = 100$ numerical results indicate slight influence of the aggregate properties of the filler on the strained – deformed state of the inhomogeneous structure.

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ВИМУШЕНІ КОЛИВАННЯ БАГАТОШАРОВИХ ЦИЛІНДРИЧНИХ ОБОЛОНОК З ВРАХУВАННЯМ ДИСКРЕТНОСТІ РОЗМІЩЕННЯ РЕБЕР ПРИ НЕСТАЦІОНАРНИХ НАВАНТАЖЕННЯХ.

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Анотація. *Ця робота присвячена дослідженню динамічної поведінки багатошарових підкріплених циліндричних оболонок обертання з врахуванням дискретності розміщення ребер при нестаціонарних навантаженнях. На основі варіаційного принципу Рейсснера отримано рівняння коливань та відповідні природні граничні умови багатошарових підкріплених циліндричних оболонок обертання з врахуванням дискретності розміщення ребер в рамках геометрично нелінійної теорії в квадратичному наближенні оболонок та стержнів типу Тимошенка. Розвинено ефективний чисельний метод, який* *базується на застосуванні інтегро – інтерполяційного методу побудови скінчено – різницевих схем по просторовій координаті та явній скінчено–різницевій схемі типу "хрест" по часовій координаті із використанням апроксимації Річардсона по просторовій координаті, створено чисельні алгоритми розв'язку динамічних задач поведінки багатошарових підкріплених оболонкових структур при осесиметричних нестаціонарних навантаженнях. Розглянуто конкретну задачу динамічної поведінки тришарової циліндричної оболонки з врахуванням дискретності розміщення ребер для випадку осесиметричних коливань.*

Ключові слова: *багатошарові циліндричні оболонки, геометрично нелінійна теорія оболонок та ребер, напружено–деформований стан, нестаціонарні навантаження, чисельні методи, нестаціонарні коливання.*

ВЫНУЖДЕННЫЕ КОЛЕБАНИЯ МНОГОСЛОЙНЫХ ЦИЛИНДРИЧЕСКИХ ОБОЛОЧЕК С УЧЕТОМ ДИСКРЕТНОСТИ РАСПОЛОЖЕНИЯ РЕБЕР ПРИ НЕСТАЦИОНАРНЫХ НАГРУЗКАХ

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Аннотация. *Данная робота посвящена исследованию нестационарных осесимметричных колебаний многослойных подкрепленных цилиндрических оболочек вращения с учетом дискретности расположения ребер. На основе вариационного принципа Рейсснера получены уравнения нелинейных колебаний и естественные граничные условия многослойных подкрепленных оболочек вращения с учетом дискретности расположения ребер в рамках гипотез типа Тимошенко для оболочек и стержней. Для представленных уравнений колебаний неоднородных по толщине оболочечных структур развит эффективный численный метод решения динамических задач многослойных оболочек вращения. В основе численного алгоритма лежит интегро – интерполяционный метод построения конечно – разностной схемы по пространственной координате и явная конечно – разностная схема типа «крест» по временной координате с использованием аппроксимации Ричардсона по пространственной координате. В частном случае, для многослойных цилиндрических оболочек проведено теоретическое исследование условий устойчивости явных по временной координате конечно – разностных схем. Разработаны алгоритмы и программы, которые позволяют реализовать решение рассматриваемых волновых задач на ПК, а также доведение решений до получения конкретных числовых результатов. На основе развитого численного метода решена задача осесимметричных колебаний трьохслойной подкрепленных цилиндрической оболочки с учетом дискретности расположения ребер (в широком диапазоне изменения геометрических и физико–механических параметров при осесиметрическом нагружении.*

Ключевые слова: *оболочки вращения, геометрически нелинейная теория оболочек и ребер, напряженно–деформированное состояние, нестационарные колебания, численные методы*