Арнаута Н. В., Савчук С. Г. Дібрівна Е. І.

UDC 539.3

NUMERICAL SOLUTION OF THE DYNAMIC PROBLEM OF AXISYMMETRIC VIBRATIONS OF REINFORCED SHELLS

N. V. ARNAUTA, Candidate of Physics and Mathematics,
Associate Professor at the Department of Higher and Applied Mathematics
S. G. SAVCHUK
National University of Live and Environmental Sciences of Ukraine
E. I. DIBRIVNA, Candidate of Pedagogical
Separate division National University of Life and Environmental Sciences Ukraine"Irpen Economic College"
E-mail: arnauta_nata@ukr.net

https://doi.org/10.31548/dopovidi2022.06.011

Abstract. *The article investigates the dynamic behavior of discretely reinforced ribbed shells of revolution under non-stationary loads. To obtain a numerical solution, the finite difference method is used. The correctness of the formulation of the problems is achieved by using the well-known equations of the theory of shells and rods of the Tymoshenko type, which are approximations of the original equations of the three-dimensional theory of elasticity. Numerical algorithms for approximate solutions of the original equations are based on the use of the integro-interpolation method of constructing difference schemes. When constructing difference diagrams, kinematic quantities refer to difference points with integer indices, and the values of deformations and forces–moments refer to difference points with half-integer indices.*

Keywords: *shells of revolution, non-stationary loads, numerical methods*

Problem statement. The problem of non-stationary axisymmetric oscillations of a Tymoshenko-type cylindrical shell supported by an external elastic ring under the action of an internal pressure pulse is considered.

The correctness of the formulation of problems is achieved by using the well-known equations of the theory of shells and rods of the Tymoshenko type, which are approximations of the original equations of the three-dimensional theory of elasticity.

In the case of axisymmetric vibrations of reinforced shells of revolution, taking into account the discrete location of the ribs, the vibration equations can be written in the following form [1, 2, 6]

- in a smooth area

$$\frac{1}{A_2}\frac{\partial}{\partial s}\left(A_2 T_{11}\right) - \Psi T_{22} + k_1 \overline{T}_{13} + P_1 = \rho h \frac{\partial^2 u_1}{\partial t^2}, \qquad (1)$$

$$\frac{1}{A_2}\frac{\partial}{\partial s}\left(A_2 \overline{T}_{13}\right) - k_1 T_{11} - k_2 T_{22} + P_3 = \rho h \frac{\partial^2 u_3}{\partial t^2},$$

$$\frac{1}{A_2}\frac{\partial}{\partial s}\left(A_2 M_{11}\right)-\Psi M_{22}-T_{13}+m_1=\rho\frac{h^3}{12}\frac{\partial^2\varphi_1}{\partial t^2},$$

$$\bar{T}_{13}=T_{13}+T_{11}\theta_1,\ \Psi=\frac{1}{A_2}\frac{dA_2}{ds};$$

- on break lines $s=s_j$

$$[T_{11}]_j=\rho_j F_j\left(\frac{\partial^2 u_1}{\partial t^2}\pm h_{cj}\frac{\partial^2\varphi_1}{\partial t^2}\right),\qquad [\bar{T}_{13}]_j-\frac{T_{22j}}{R_j}=\rho_j F_j\frac{\partial^2 u_3}{\partial t^2},$$

(2)

$$[M_{11}]_j=\rho_j F_j\left[\pm h_{cj}\frac{\partial^2 u_1}{\partial t^2}+\left(\frac{I_{krj}}{F_j}+h_{cj}^2\right)\frac{\partial^2\varphi_1}{\partial t^2}\right].$$

Equations (1), (2) are supplemented with appropriate boundary and initial conditions. In the oscillation equations (1), (2) the relationship between the values of effort-moments and the corresponding deformations is taken according to [1, 2, 6].

The spatial interval is considered $s_0\le s\le s_N$, $s=A_1\alpha_1$. To construct a numerical algorithm, the interval is divided into N equal parts in discrete steps $\Delta s=s/N$. A difference grid is introduced at integer and half-integer nodes. The components of the generalized displacement vectors of the skin and ribs are taken down to integer nodes of the difference grid

$$u_1,\ u_3,\ \varphi_1\to(u_1)_l^n,\ (u_3)_l^n,\ (\varphi_1)_l^n.$$

The construction of a difference scheme is carried out using the integro-interpolation method for constructing finite difference schemes [1 - 3]. We consider oscillation equations (1) in the region

$$\left\{s_{l-1/2}\le s\le s_{l+1/2},\ t_{n-1/2}\le t\le t_{n+1/2}\right\}.$$

The integration of the left and right parts of equations (1) is carried out over the corresponding areas, which are considered.

$$\int_{t_{n-1/2}}^{t_{n+1/2}}\int_{s_{l-1/2}}^{s_{l+1/2}}\left[\frac{1}{A_2}\frac{\partial}{\partial s}\left(A_2 T_{11}\right)-\Psi T_{22}+k_1\bar{T}_{13}+P_1\right]dsdt= \qquad (3)$$

$$=\rho h\int_{t_{n-1/2}}^{t_{n+1/2}}\int_{s_{l-1/2}}^{s_{l+1/2}}\frac{\partial^2 u_1}{\partial t^2}dsdt;$$

$$\int_{t_{n-1/2}}^{t_{n+1/2}}\int_{s_{l-1/2}}^{s_{l+1/2}}\left[\frac{1}{A_2}\frac{\partial}{\partial s}\left(A_2\bar{T}_{13}\right)-k_1 T_{11}-k_2 T_{22}+P_3\right]dsdt=$$

$$= \rho h \int_{t_{n-1/2}}^{t_{n+1/2}} \int_{s_{l-1/2}}^{s_{l+1/2}} \frac{\partial^2 u_3}{\partial t^2} dsdt;$$

$$\int_{t_{n-1/2}}^{t_{n+1/2}} \int_{s_{l-1/2}}^{s_{l+1/2}} \left[\frac{1}{A_2} \frac{\partial}{\partial s}\left(A_2 M_{11}\right) - \Psi M_{22} - T_{13} \right] dsdt =$$

$$= \rho \frac{h^3}{12} \int_{t_{n-1/2}}^{t_{n+1/2}} \int_{s_{l-1/2}}^{s_{l+1/2}} \frac{\partial^2 \varphi_1}{\partial t^2} dsdt.$$

After the integration operation in (3), we obtain the difference relations [1, 2]. Approximation of vibration equations on discontinuity lines $s = s_j$ built in a similar way..

Numerical algorithm. The constructed numerical algorithms for solving problems in the theory of shells with local inhomogeneities were worked out on test calculations, and also checked for practical convergence.

A comparative analysis of the dynamic behavior of a reinforced shell under short-term loading is carried out according to the analytical solution presented in the work [4], and a numerical algorithm for solving the equations of motion of reinforced cylindrical shells according to the equations (1), (2). The problem of non-stationary axisymmetric vibrations of a Timoshenko-type cylindrical shell, supported by an external elastic ring, under the action of an internal pressure impulse is considered. The equations of motion of such a construction have the following form

Results of calculations. The case of a structure with hinged fastening at the ends is considered. The boundary conditions for this case have the following form for $x = 0$, $x = L$: $T_{11} = 0$, $M_{11} = 0$, $U_3 = 0$.

The initial conditions are zero. The calculations were carried out for shells reinforced in the middle part by a ring (the center of gravity is at the point $x = L/2$). The shell and ring parameters are as follows: $L = 1{,}5$ м; $R = 0{,}6$ м; $h = 3 \cdot 10^{-2}$ м; $H = 0{,}1$ м; $F = 0{,}015$ м^2. The load was given as: $P_3 = A\left[\eta(\tau) - \eta(\tau - \tau_0)\right]$; $\tau = tc/L$; $c^2 = E/\left[\rho(1-\nu^2)\right]$; $A = 10^3$ н/м^2; $\tau_0 = 0{,}5$.

On fig. 1 shows a comparison of the results of calculations of this problem with the results of work [4]. The solid line shows the dependences of the quantity U_3 from the value x according to the methodology of this work, dashed – according to analytical calculation [4]. Curve 1 corresponds to time $\tau = 0{,}8$; curve $2 - \tau = 0{,}4$.

A comparative analysis of the dynamic behavior of the reinforced shell under impulsive loading was also carried out according to the analytical solution presented in the work [5], and a numerical algorithm for solving the equations of motion of reinforced shells according to the equations (1) – (2) (case of a spherical reinforced shell). The problem of non-stationary axisymmetric vibrations of a Timoshenko-type spherical shell, supported by an external elastic ring, under the action of internal impulse loading is considered..

The initial conditions are zero. The calculations were carried out for a shell reinforced with a ring along the greatest parallel ($\Theta = \pi / 2$).The shell and ring parameters are as follows: $R = 0{,}3$ м; $h = 3 \cdot 10^{-2}$ м; M=160 кг, where M – mass of the ring. The load was given as: $P_3 = A \cdot exp(-\omega\tau)$; $\tau = t \cdot c / R$; $c^2 = E / \left[\rho(1-\nu^2)\right]$; $A = 0{,}981 \cdot 10^5$ н/м^2; $\omega = 0{,}3$.

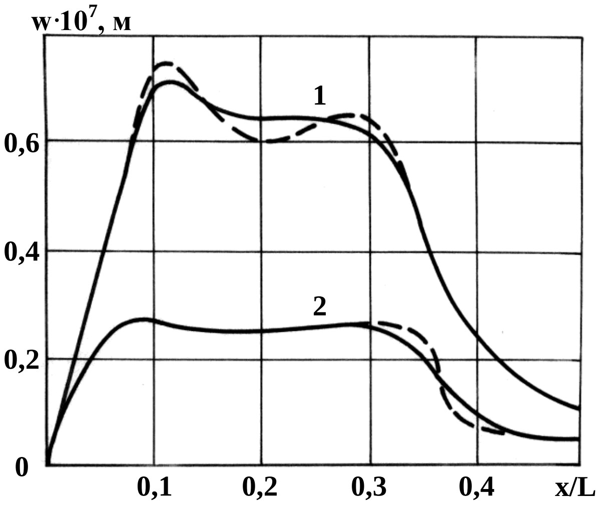

Fig. 1

The reliability of the results obtained in the work is determined by the rigor and correctness of the statements of the initial problems; theoretical substantiation of the finite-difference schemes used; controlled accuracy of numerical calculations; conducting test calculations; compliance of the established regularities with the general properties of oscillations of thin-walled structural elements.

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ЧИСЕЛЬНИЙ РОЗВ'ЯЗОК ДИНАМІЧНОЇ ЗАДАЧІ ОСЕСИМЕТРИЧНИХ КОЛИВАНЬ ПІДКРІПЛННИХ ОБОЛОНОК

Н. В. Арнаута, С. Г. Савчук, Е. І. Дібрівна

Анотація *Достовірність одержаних в роботі результатів визначається строгістю і коректністю постановок вихідних задач; теоретичним обґрунтуванням скінченно – різницевих схем, які використовуються; контрольованою точністю чисельних розрахунків; проведенням тестових розрахунків; відповідністю встановлених закономірностей загальним властивостям коливань тонкостінних елементів конструкцій.*

Коректність постановки задач досягається використанням відомих рівнянь теорії оболонок і стержнів типу Тимошенка, які являються апроксимацією вихідних рівнянь тривимірної теорії пружності. При виводі рівнянь отримано рівняння коливань багатошарової оболонки в гладкій області, та рівняння коливань підкріплюючих ребристих елементів (поперечні ребра). Неважко показати, що вказані рівняння по класифікації рівнянь в частинних похідних є рівняннями гіперболічного типу, які є апроксимацією коливальних рівнянь тривимірних пружних тіл і достатньо коректно відтворюють хвильові процеси в неоднорідних оболонкових структурах з врахуванням просторових розривів.

Чисельні алгоритми наближених розв'язків вихідних рівнянь базуються на використанні інтегро–інтерполяційного методу побудови різницевих схем. При побудові різницевих схем кінематичні величини відносяться до різницевих точок з цілими індексами, а величини деформацій та зусиль–моментів відносяться до різницевих точок з напівцілими індексами. Чисельний алгоритм базується на використанні окремих скінченно–різницевих співвідношень в гладкій області та на лініях просторових розривів з другим порядком точності по просторовим та часовій координатам.

Ключові слова: *оболонки обертання, нестаціонарні навантаження, чисельні методи*