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A PROBLEM OF NON – LINEAR DEFORMATION OF FIVE–LAYER CONICAL SHELLS WITH ALLOWANCE FOR DISCRETE RIBS

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Abstract. *In this paper, on the example of a five – layer conical shell, the problem of dynamic behavior of multiyear discrete reinforced conical shells of rotation is considered. The study is based on the geometrical nonlinear theory of shells and rods of the Tymoshenko type. The Reissner's variational principle is used for deductions of the motion equations. An efficient numerical method using Richardson type finite difference approximation for solution of problems on nonstationary behavior of multiplayer shells of revolution with allowance discrete rib is constructed. The method permit to realize solution of the investigated wave problems with the use of personal computers. For the case of axisymmetric vibrations, a detailed analysis of the stress-strain state of the fiver-layer reinforced conical shell was performed.*
Key words*. multilayered conical shells of revolution, geometrically nonlinear theory of shells and ribs, non-stationary loading, numerical method, nonstationary vibrations*

Non – linear deformation of multiplayer shells of revolution with allowance for discrete ribs have been considered by many authors. Particularly, a thorough review of the literature on this issue is set out in [1,2,3]. A significant contribution to the study of this problem was made by the staff of the Institute of Mechanics named after S.P. Tymoshenko, the main results of which are presented in [1,2]. Nonlinear axisymmetric oscillations of three-layer shells of rotation under pulse loads are considered in [6,7], where the calculation scheme is taken taking into account independent kinematic and static approximations to each layer. The study of nonstationary oscillations of five-layer cylindrical shells of rotation, taking into account the influence of discreteness, is additionally given in [3,5].

Problem statement. A problem of non – linear deformation of multiplayer conical shells with allowance for discrete ribs under non – stationary loading is considered. It is believed that the multilayer reinforced conical structure is loaded with an internal distributed non-stationary normal load by spatial and temporal coordinates.

When considering the axisymmetric oscillations of conical shells, a coordinate system is usually used, and the coordinate is calculated from the top of the cone. In some cases, in particular for truncated conical shells, it is more convenient to use a coordinate where the coordinate is subtracted from the rib of the shell.

The coefficients of the first quadratic shape and curvature of the coordinate surface are written as follows:

$$A_1 = 1, \quad A_2 = R_s, \; k_1 = 0, \quad k_2 = \cos\alpha / R_s,$$

where α – taper angle ; s_1 – flowing coordinate; $R_s = R_0 + s_1 \sin\alpha$.

The Reissner's variational principle is used for deductions of the motion equations. [8,9]:

- in a smooth area

$$\frac{1}{R}\frac{\partial}{\partial s}(RT_{11}) - \frac{\sin\alpha}{R}T_{22} = \rho h \frac{\partial^2 u_1}{\partial t^2}, \qquad (1)$$

$$\frac{1}{R}\frac{\partial}{\partial s}(R\overline{T}_{13}) - \frac{\cos\alpha}{R}T_{22} + P_3(s,t) = \rho h \frac{\partial^2 u_3}{\partial t^2},$$

$$\frac{1}{R}\frac{\partial}{\partial s}(RM_{11}) - \frac{\sin\alpha}{R}M_{22} - T_{13} = \frac{\rho h^3}{12}\frac{\partial^2 \varphi_1}{\partial t^2}.$$

$$(T_{11}, T_{22}, T_{13}) = \sum_k \int_z (\sigma_{11}^{kz}, \sigma_{22}^{kz}, \sigma_{13}^{kz}) dz; \qquad (2)$$

$$(M_{11}^*, M_{22}^*) = \sum_k \int_z (z\sigma_{11}^{kz}, z\sigma_{22}^{kz}) dz;$$

$$(M_{11}, M_{22}) = (M_{11}^*, M_{22}^*) \pm h_{ck}(T_{11}, T_{22});$$

$$\overline{T}_{13} = T_{13} + T_{11}\theta_1;$$

$$I_1 = \sum_k \rho_k h_k; \quad I_2 = \sum_k \pm \rho_k h_k h_{ck}; \quad I_3 = \sum_k \rho_k \frac{h_k^3}{12};$$

The relationship between the values of deformation and the values of generalized displacement vectors are written as:

$$\varepsilon_{11}^k = \frac{\partial u_1^k}{\partial s} + \frac{1}{2}[\theta_1^k]^2, \qquad (3)$$

$$\varepsilon_{22}^k = \frac{\sin\alpha}{R_k}u_1^k + \frac{\cos\alpha}{R_k}, \qquad \varepsilon_{13}^k = \varphi_1^k + \frac{\partial u_3^k}{\partial s_k},$$

$$\theta_1^k = \frac{\partial u_3^k}{\partial s_k}, \quad \kappa_{11}^k = \frac{\partial \varphi_1^k}{\partial s_k}, \quad \kappa_{22}^k = \frac{\sin\alpha}{R_k}\varphi_1^k.$$

On the rupture lines, the equations of oscillations are written in the form

$$\sum_{i=1}^{2} T_{11}^{i\pm} = \rho_j F_j \frac{\partial^2 u_{1j}}{\partial t^2}, \quad \sum_{i=1}^{2} \overline{T}_{13}^{i\pm} = \rho_j F_j \frac{\partial^2 u_{3j}}{\partial t^2}, \qquad (4)$$

$$\sum_{i=1}^{2} (M_{11}^{i\pm} \mp h_j T_{11}^{i\pm}) = \rho_j I_{\text{кр}j} \frac{\partial^2 \varphi_{1j}}{\partial t^2}.$$

In equations (4) of the quantity $T_{11}^{i\pm}$, $\overline{T}_{13}^{i\pm} = T_{11}^{i\pm} + \theta_{11}^{i\pm}, M_{11}^{i\pm}$ $(i = 1, 2)$ correspond to the forces-moments acting on - and a discrete element on the line of rupture $s_i = s_{ij}$. Equations of oscillations (1) - (4) are supplemented by the corresponding boundary and initial conditions.

Numerical algorithm. To build a numerical algorithm for solving nonstationary problems in the theory of inhomogeneous multilayer shells, the integra-interpolation method of constructing difference schemes [8] for hyperbolic equations is used. Due to the initial formulation of the problems, the numerical solution is sought in the smooth region of the elastic structure (for a multilayer shell between the edges) and on the lines of location of the corresponding edges.

An approach based on finding approximate Richardson solutions is used to construct more efficient algorithms [8]. Moreover, with a fixed difference step in time coordinate, a sequence of approximate approximations in spatial coordinate is used. In this case, the extrapolation procedure is formed according to the formulas [3,8]

$$\tilde{\overline{U}}_{l(\Delta s)}^n = \frac{4}{3}\overline{U}_{l(\Delta s/2)}^n - \frac{1}{3}\overline{U}_{l(\Delta s)}^n, \qquad (5)$$

where $\overline{U}_{l(\Delta s/2)}^n$ i $\overline{U}_{l(\Delta s)}^n$ - numerical solutions of the equations of oscillations according to discrete steps in the spatial coordinate $\Delta s/2$ i Δs, $s = A_1\alpha_1$.

It is easy to show that the difference equations (5) approximate the original equations of oscillations (1) in the smooth region with the fourth order of coordinate accuracy.

The results of calculations. As a numerical example, the problem of dynamic deformation of a five-layer conical shell with rigidly clamped ends under the action of an internal distributed load was considered. $P_3(s,t)$. Boundary conditions at $s = s_0$, $s = s_N$ have the form: $u_1 = u_3 = \varphi_1 = 0$.

The non-stationary impulse load was set in the form

$$P_3(s,t) = A \cdot \sin\frac{\pi t}{T}\left[\eta(t) - \eta(t-T)\right],$$

Where A – load amplitude, T – load duration. The calculations relied $A = 10^6$ Па; $T = 50 \cdot 10^{-6}$с. Calculations were performed with the following geometric and physico-mechanical parameters:

$$E_1^1 = E_1^3 = E_1^5 = E_j = 7 \cdot 10^{10}\text{Па}; \; E_1^1 / E_1^{зап} = 10 \div 1000;$$

$$\nu_1^1 = \nu_1^3 = \nu_1^5 = 0{,}3; \; \nu_1^{зап} = 0{,}4;$$

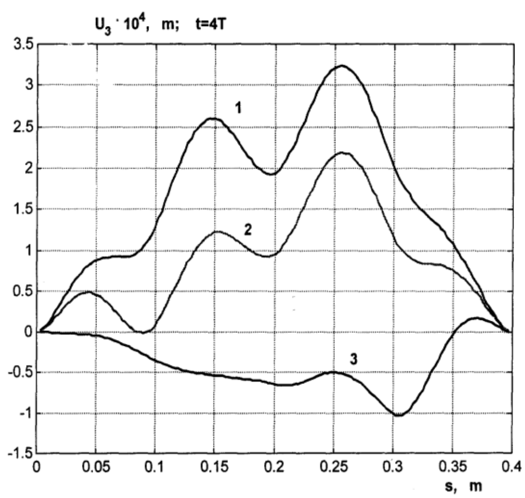
$$\rho_1 = \rho_3 = \rho_5 = \rho_j = 2{,}7 \cdot 10^3 \text{кг/м}^3; \; \rho_1 / \rho_{зап} = 7;$$

$$h = h_1 + h_2 + h_3 + h_4 + h_5; \; h_1 = h_3 = h_5 = 10^{-3}\text{м}; \; h_2 = h_4; \; h_2 / h_1 = 3;$$

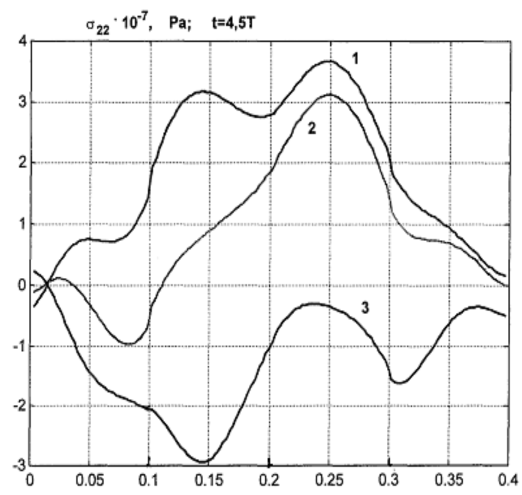
$$R_0 = 0{,}1\text{м}; \; h / h_j = 9/20; \; F_j = 10^{-4}\text{м}^2;$$

$$\alpha_1 = \pi/3; \; \alpha_2 = \pi/4; \; \alpha_3 = \pi/6$$

Discrete reinforcing elements are located at points $s_j = 0{,}25 s_N j, \quad j = \overline{1{,}3}$



Pic. 1 Dependence of size U_3 *from the spatial coordinate* s *from the spatial coordinate t=4T*



Pic.2 Dependence of size σ_{22} from the spatial coordinate s at the time t=4,5T

The obtained numerical results allow the analysis of the stress-strain state of a five-layer reinforced elastic structure of conical type at any time (calculations were performed at $0 \le t \le 40T$). In particular, in pic. 1 and pic. 2 shows the dependences of the quantities u_3 and the stress from the spatial coordinate depending on the magnitude of the angles of the taper at the time t=4T i t=5,5T.

The curve with index 1 corresponds to the angle of taper $\alpha_1 = \pi/3$; curve with index 2 – $\alpha_2 = \pi/4$, curve with index 3 – $\alpha_3 = \pi/6$. The case is considered $E_1^1 / E_1^{зап} = 100$. Based on the presented graphic material, you can visually determine the effect of the conicity of the structure on the antisymmetry of the distribution of values u_3 i σ_{22} by spatial coordinate (as a partial case, for a cylindrical shell in the case $\alpha = 0$ there is a symmetrical pattern about the axis s).

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ЗАДАЧА НЕЛІНІЙНОЇ ДЕФОРМАЦІЇ П'ЯТИШАРОВИХ КОНІЧНИХ ОБОЛОНОК З ВРАХУВАННЯМ ДИСКРЕТНОСТІ РОЗМІЩЕННЯ РЕБЕР

Н. В. Арнаута

Анотація. *У цій роботі, на прикладі п'ятишарової конічної оболонки, розглянута задача динамічної поведінки багатошарових дискретно підкріплених* *конічних оболонок обертання. В основі дослідження покладена геометрично нелінійна теорія в квадратичному наближенні оболонок та стержнів типу Тимошенка. За допомогою варіаційного принципу Рейсснера одержуються рівняння коливань з відповідними початковими граничними умовами для багатошарових конічних оболонок обертання з врахуванням дискретності розміщення ребер. Маємо чисельний алгоритм розв'язку шуканої динамічної задачі на основі інтегро – інтерполяційного методу побудови скінчено – різницевих схем по просторовій координаті та явній скінчено–різницевій схемі типу "хрест" по часовій координаті із використанням апроксимації Річардсона по просторовій координаті. Для випадку осесиметричних коливань проведено детальний аналіз напружено–деформованого стану п'ятишарової підкріпленої конічної оболонки з врахуванням дискретності розміщення ребер.*

Ключові слова: *багатошарові конічні оболонки обертання, геометрично нелінійна теорія оболонок та ребер, напружено–деформований стан, нестаціонарні навантаження, чисельні методи, нестаціонарні коливання*

ЗАДАЧА НЕЛИНИЙНОЙ ДЕФОРМАЦИИ ПЯТИСЛОЙНЫХ КОНИЧЕСКИХ ОБОЛОЧЕК С УЧЕТОМ ДИСКРЕТНОСТИ РАСПОЛОЖЕНИЯ РЕБЕР

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Аннотация. *В данной роботе, на примере пятислойной конической оболочки, рассмотрена задача динамического поведения многослойных подкрепленных конических оболочек вращения с учетом дискретности расположения ребер. В основе исследования положена геометрически нелинейная теория Тимошенко оболочек и стержней. С помощью вариационного принципа Рейсснера составлены уравнения колебаний и начальные граничные условия многослойных подкрепленных оболочек вращения с учетом дискретности расположения ребер. Построены численные алгоритмы решения динамических задач на основе интегро – интерполяционный метод построения конечно – разностной схемы по пространственной координате и явная конечно – разностная схема типа «крест» по временной координате с использованием аппроксимации Ричардсона по пространственной координате. При осесиметрических колебаниях проведен детальный анализ напружено – деформированного состояния пятислойной конической оболочки с учетом расположения дискретности расположения ребер.*

Ключевые слова: *многослойные конические оболочки вращения, геометрически нелинейная теория оболочек и ребер, напряженно–деформированное состояние, нестационарные колебания, численные методы*