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FINDING THE TRAJECTORIES OF THE MOVEMENT OF A MATERIAL PARTICLE ON THE INNER SURFACE OF A CONE WITH A VERTICAL AXIS WITH LATERAL FEED OF THE MATERIAL

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Abstract. *The movement of material particles along the inner surface of the cone takes place in cyclones, the designs of which can have both cylindrical and conical parts. Aerodynamic processes occurring in a cyclone are complex in nature, therefore they cannot be accurately modeled on the basis of theoretical approaches. A number of simplifications were introduced during the research: air resistance is not taken into account, since the particle is fed into the cone together with the air, although later their directions of movement do not coincide (the particle damps the speed and falls down, and the air along the central part along the axis of the cone rises up and goes out); the influence of particles on each other, their size, etc.*

The purpose of the article is to study the motion of a material particle entering the inner surface of a vertical cone with a given initial velocity.

If a material particle is directed with an initial velocity to the inner wall of the cone perpendicular to its generator, then its further motion will include both rotation around the axis of the cone and descent down under the action of its own weight. To find the trajectory of motion, a material point was taken as the vertex of the accompanying Frenet trihedron, which has three mutually perpendicular orthogonal planes. The second accompanying Darboux trihedron has a common orthogonal plane tangent to the trajectory with the Frenet trihedron.

The balance of the acting forces in the projections onto the orthogonal planes of the Darboux trihedron was considered. This made it possible to determine the projections of the curvature of the curve onto the corresponding orthogonal planes of the Darboux trihedron. The differential geometry apparatus made it possible to find them through the first and second quadratic forms of the surface, which allows avoiding cumbersome transformations.

Differential equations of motion of a material particle along the inner surface of a vertical cone were compiled. The equations were solved using the MatLab system.

The equation of motion of a particle along the inner surface of the cone was obtained. Analyzing the trajectory of the particle, we can conclude that it is significantly different from the trajectory of motion along the inner surface of the cylinder. The graphs of changes in velocity also show the difference between the motion of a particle along a cone and the same motion along a cylinder. If, upon entering the surface of a cylinder, the *particle damps its velocity to a certain limit, and then it begins to increase again, then during movement along a cone the velocity of the particle has a certain periodic character and approaches zero over time.*

In the absence of friction and air resistance, a material particle, after entering the inner surface of the cone at a certain angle to the generator (except zero), performs an oscillatory motion, alternately rising and falling along a trajectory in the form of a loop, moving for any length of time. Depending on the initial conditions, the particle can describe a finite number of branches of the loop, an infinite number of branches, move along a straight-line generator of the cone, or along an intermediate trajectory between a straight line and a loop.

In the presence of friction, the particle will descend to the top of the cone, with possible local rises, the magnitude of which will depend on the initial velocity and the angle of inclination of the generating cone. The velocity in such a motion will damp out, while also having an oscillatory character.

Key words: ***material particle, cone, Darboux trihedron, equation of motion of a particle***

Topicality. The movement of material particles along the inner surface of the cone takes place in cyclones, the designs of which can have both cylindrical and conical parts. The aerodynamic processes that occur in a cyclone are complex in nature, so they cannot be accurately modeled based on theoretical approaches [1]. But, as Academician P. M. Vasylenko wrote, “in many cases no the need to obtain accurate values, and if necessary, these quantities always can be verified and refined based on experimental data” [2]. Given this, in further theoretical calculations we will introduce a number of simplifications: we will not take into account air resistance, since the particle is fed into the cone together with the air, although in the future their directions of motion do not coincide (the particle damps the speed and falls down, and air in the central part along the axis of the cone rises up and goes out); the effect of particles on each other, their size, etc.

Analysis of recent research and publications. The movement of the processed material along cylindrical surfaces is considered in [3]. The article is devoted to the study of the movement of a material particle along the inner surface of a vertical cylinder with lateral material feed. [4]. However, the behavior of particles during lateral feeding onto the inner surface of a vertical cone is significantly different from the similar situation with a cylinder, which determined the direction of research.

The purpose of the study is to study the motion of a material particle entering the inner surface of a vertical cone with a given initial velocity.

Materials and methods of research. If a material particle is directed with an initial velocity v_o on the inner wall of the cone perpendicular to its generator, then its further motion will include both rotation around the axis of the cone and lowering down under the action of its own weight. To find the trajectory of motion, we will take the material point as the vertex of the accompanying Frenet trihedron , which has three mutually perpendicular orths (Fig. 1, *a*). The second accompanying *t n b* Darboux trihedron with orths *t N P* has in common with the trihedron Frenet orthogonal tangent to the trajectory. Orthogonals $\bar{P}$, $\bar{b}$, $\bar{N}$, $\bar{n}$ lie in the plane normal to the trajectory, and orthogonals $\bar{P}$ and $\bar{t}$ – in the plane tangent to the cone μ. Between orthogonals $\bar{P}$ and $\bar{b}$, $\bar{N}$ and $\bar{n}$ there is an angle ε (Fig. 1, *a*), which varies along the trajectory and is a function of its arc: $\varepsilon = \varepsilon(s)$.

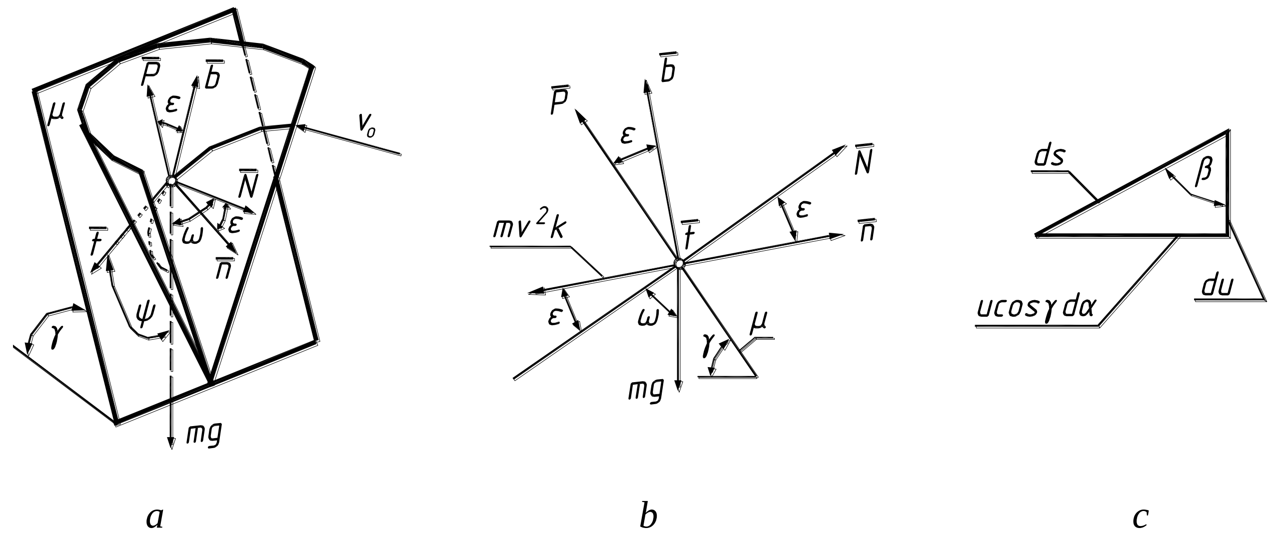

Fig. 1. Graphic illustrations for compiling differential equations of motion of a material particle along the inner surface of a cone with a vertical axis:
a – Frenet and Darboux trihedron of the trajectory of motion of a material particle;
b – decomposition of the acting forces in the normal plane of the trajectory;
c – to determine the differential of the trajectory arc

Research results and their discussion. Let us consider the equilibrium of the forces acting in the projections onto the orts of the Darboux trihedron . Let us project onto the ort $\bar{t}$ the forces that give the particle an acceleration $\frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v\frac{dv}{ds}$ (*t* - time; *s* – length trajectory arcs):

$$mv\frac{dv}{ds} = mg\cos\psi - fR, \tag{1}$$

where m is the mass of the particle; f – friction coefficient; R is the pressure of the particle on the surface of the cone; ψ- the angle between the gravity vector mg and the orthogonal line $\bar{t}$; $g = 9.81\ m/s^2$.

To determine the pressure R, we consider the forces acting in the normal plane. To do this, we choose the viewpoint so that the ordinate $\bar{t}$ projected into a point (Fig. 1,*b*).

Centrifugal force mv^2k ($k = k(s)$ – trajectory curvature) is directed along the orthogonal plane of the main normal $\bar{n}$ in the opposite direction. Its component in the projection onto the orthogonal plane $\overline{N}$ will cause a certain pressure. The other component – the projection of the weight force mg onto the orthogonal plane $\overline{N}$ – will increase the pressure on the surface, which is the main difference from a cylindrical surface, for which this component is zero. The pressure force will be written as the sum of two components:

$$R = mv^2k\cos\varepsilon + mg\cos\omega, \tag{2}$$

where ω *is* the angle between the gravity vector mg and the orthogonal line $\overline{N}$ (Fig. 1, *a*, *b*).

It should mentioned that in Fig. 1, *b* the vector of the force of gravity mg, unlike all other vectors, does not lie in the normal plane of the trajectory. Another component of the centrifugal force in the projection on the ort $\overline{P}$ acts in the tangential plane μ and balances the component of the force of gravity mg. Thus, the forces in the projection on the ort $\overline{P}$ can be written by the equation:

$$mv^2k\sin\varepsilon = mg\cos\varphi, \tag{3}$$

where φ is the angle between the gravity vector and the orthogonal point of $\overline{P}$ the Darboux trihedron.

By substituting (2) into (1) and adding equation (3), we obtain a system of equations, which after reduction by mass m takes the form:

$$\begin{cases} v\dfrac{dv}{ds} = g\cos\psi - f(v^2k\cos\varepsilon + g\cos\omega); \\ v^2k\sin\varepsilon = g\cos\varphi\ . \end{cases} \tag{4}$$

The expressions $k \cos \varepsilon = k_n$ *and* $k \sin \varepsilon = k_r$ in differential geometry are called, respectively, the normal and geodesic curvatures of a curve on the surface [5]. These expressions are projections of the curvature of the curve onto the corresponding orthogonals of the Darboux trihedron . The apparatus of differential geometry makes it possible to find them through the first and second quadratic forms of the surface, which allows avoiding cumbersome transformations when finding expressions separately for the curvature k and the sine and cosine of the angle ε. This approach to finding the normal and geodesic curvatures of a trajectory is shown in [6]. In this article, we will consistently define all the expressions included in system (4). Since the trajectory lies on the surface of a cone, the expressions for the angles ε, ψ, φ, ω, the velocity v and the curvature k of the trajectory must be expressed in terms of one of its parameters. We write the parametric equations of the cone as follows:

$$X = u\cos\gamma\cos\alpha; \qquad Y = u\cos\gamma\sin\alpha; \qquad Z = u\sin\gamma, \tag{5}$$

where α is the angle of rotation of the surface point around the *OZ axis*; u – length of the straight-line generator of the cone – variable parameters; γ– the angle of inclination of the generating cone to the horizontal plane (Fig. 1, a) – a constant value.

We express the partial derivatives and the arc differential of the trajectory by the equations:

$$\begin{aligned} &X_\alpha = -u\cos\gamma\sin\alpha; \qquad Y_\alpha = u\cos\gamma\cos\alpha; \qquad Z_\alpha = 0; \\ &X_u = \cos\gamma\cos\alpha; \qquad Y_u = \cos\gamma\sin\alpha; \qquad Z_u = \sin\gamma; \\ &ds^2 = du^2 + u^2\cos^2\gamma\, d\alpha^2. \end{aligned} \tag{6}$$

As can be seen from the expression of the differential of the arc (6), geometrically it can be represented as the hypotenuse of an elementary right-angled triangle (Fig. 1,c). From Fig. 1,c we can write:

$$du = u\cos\gamma \cdot \operatorname{ctg}\beta \cdot d\alpha, \qquad where \;\; u = ae^{\cos\gamma \int \operatorname{ctg}\beta \cdot d\alpha}, \tag{7}$$

where β is the angle between the trajectory and the generator of the cone; a is the integration constant, which affects the distance of the point from the origin (the vertex of the cone) along the straight line generator.

Having set a certain dependence $\beta = \beta(\alpha)$, we thereby set a line on the cone. After substituting (7) into (5), we can write the parametric equations of the line:

$$x = a\cos\gamma e^{\cos\gamma\int \mathrm{ctg}\beta\cdot d\alpha}\cos\alpha; \qquad y = a\cos\gamma e^{\cos\gamma\int \mathrm{ctg}\beta\cdot d\alpha}\sin\alpha;$$
$$z = a\sin\gamma e^{\cos\gamma\int \mathrm{ctg}\beta\cdot d\alpha}. \tag{8}$$

Our task is to find such a dependence $\beta = \beta(\alpha)$, at which the line (8) would be a trajectory, i.e. satisfy the system (4). To do this, we will find the first and second derivatives with respect to the parameter α of equations (8), which are needed to determine the curvature k of the trajectory and the angles $\varepsilon,\ \psi,\ \varphi, \omega$:

$$x' = a\cos\gamma e^{\cos\gamma\int \mathrm{ctg}\beta\cdot d\alpha}(\cos\gamma\,\mathrm{ctg}\beta\cos\alpha - \sin\alpha);$$
$$y' = a\cos\gamma e^{\cos\gamma\int \mathrm{ctg}\beta\cdot d\alpha}(\cos\gamma\,\mathrm{ctg}\beta\sin\alpha + \cos\alpha);$$
$$z' = a\sin\gamma\cos\gamma\,\mathrm{ctg}\beta\cdot e^{\cos\gamma\int \mathrm{ctg}\beta\cdot d\alpha};$$
$$x'' = a\cos\gamma e^{\cos\gamma\int \mathrm{ctg}\beta\cdot d\alpha}\left[\frac{\cos^2\gamma\cos^2\beta - \beta'\cos\gamma - \sin^2\beta}{\sin^2\beta}\cos\alpha - 2\cos\gamma\,\mathrm{ctg}\beta\sin\alpha\right];$$
$$y'' = a\cos\gamma e^{\cos\gamma\int \mathrm{ctg}\beta\cdot d\alpha}\left[\frac{\cos^2\gamma\cos^2\beta - \beta'\cos\gamma - \sin^2\beta}{\sin^2\beta}\sin\alpha + 2\cos\gamma\,\mathrm{ctg}\beta\cos\alpha\right]; \tag{9}$$
$$z'' = a\cos\gamma e^{\cos\gamma\int \mathrm{ctg}\beta\cdot d\alpha}\,\frac{\cos^2\beta\cos\gamma - \beta'}{\sin^2\beta}.$$

Substituting (9) into the well-known formula [5] , after transformations and simplifications we will find trajectory curvature:

$$k = \frac{\sin\beta}{a\cos\gamma e^{\cos\gamma\int \mathrm{ctg}\beta\cdot d\alpha}}\sqrt{(\beta' + \cos\gamma)^2 + \sin^2\gamma\sin^2\beta}. \tag{10}$$

Derivatives (9) completely determine the direction of the principal normal $\bar{n}$. The coordinates of its direction vector are also found by well-known formulas [5]. After reducing the principal normal vector to the unit vector, its projection can be written by the equations:

$$n_x = -\frac{\sin\beta}{\sqrt{(\beta' + \cos\gamma)^2 + \sin^2\gamma\sin^2\beta}}\left[\mathrm{ctg}\beta(\beta' + \cos\gamma)\sin\alpha + (1 + \beta'\cos\gamma)\cos\alpha\right];$$

$$n_y = \frac{\sin\beta}{\sqrt{(\beta' + \cos\gamma)^2 + \sin^2\gamma\sin^2\beta}}\left[\mathrm{ctg}\beta(\beta' + \cos\gamma)\cos\alpha - (1 + \beta'\cos\gamma)\cos\alpha\right]; \quad (11)$$

$$n_z = -\frac{\beta'\sin\gamma\sin\beta}{\sqrt{(\beta' + \cos\gamma)^2 + \sin^2\gamma\sin^2\beta}}.$$

The direction vector of the Horta $\bar{t}$ is the first derivatives (9). The direction of the vectors $\overline{N}$ and $\overline{P}$ is found as the vector product of two vectors: for $\overline{N}$ - the vector product of two vectors tangent to the coordinate lines of the cone (these are the first and second lines of (6)) ; for $\overline{P}$ - the vector product of the vector $\bar{t}$ and the found vector $\overline{N}$ Omitting the operations for finding vectors, we write down the finished results (vectors reduced to unit vectors):

$$N_x = -\sin\gamma\cos\alpha; \qquad N_y = -\sin\gamma\sin\alpha; \qquad N_z = \cos\gamma;$$
$$P_x = -\cos\gamma\sin\beta\cdot\cos\alpha - \cos\beta\sin\alpha; \quad (12)$$
$$P_y = -\cos\gamma\sin\beta\cdot\sin\alpha + \cos\beta\cos\alpha; \qquad P_z = -\sin\gamma\sin\beta.$$

Knowing the coordinates of the vectors, we find expressions for the required angles between them (the coordinates of the gravity vector will be {*0, 0, - 1*}):

$$\cos\varepsilon = \frac{\sin\gamma\sin\beta}{\sqrt{(\beta' + \cos\gamma)^2 + \sin^2\gamma\sin^2\beta}}; \ \sin\varepsilon = \frac{\beta' + \cos\gamma}{\sqrt{(\beta' + \cos\gamma)^2 + \sin^2\gamma\sin^2\beta}}; \quad (13)$$
$$\cos\varphi = -\sin\gamma\sin\beta; \qquad \cos\psi = -\sin\gamma\cos\beta; \qquad \omega = \gamma.$$

We have found all the expressions for the angles and curvature of the trajectory included in system (4) through the dependence $\beta = \beta(\alpha)$. We will do the same for the arc differential *ds* included in the first equation of system (4). Substituting *du* from (7) into the arc differential expression (6), we obtain:

$$ds = \frac{a\cos\gamma}{\sin\beta}e^{\cos\gamma\int\mathrm{ctg}\beta\cdot d\alpha}d\alpha. \quad (14)$$

We substitute the arc differential expression from (14), the angle expressions from (13) and the curvature from (10) into system (4). After simplifications, we obtain a system of two differential equations, which includes two unknown functions: $\beta = \beta(\alpha)$ and $v = v(\alpha)$:

$$\begin{cases} v\dfrac{dv}{d\alpha} = -ug\sin\gamma\cos\gamma \operatorname{ctg}\beta - f\left(v^2\sin\gamma\sin\beta + ug\dfrac{\cos^2\gamma}{\sin\beta}\right); \\ v^2 = \dfrac{ug\sin\gamma\cos\gamma}{\beta' + \cos\gamma}, \qquad \text{де} \qquad u = ae^{\cos\gamma\int ctg\beta d\alpha}. \end{cases} \tag{15}$$

Let's introduce new variable $w = \int \operatorname{ctg}\beta \cdot d\alpha$, from where:

$$w' = \operatorname{ctg}\beta; \quad \beta' = -\frac{w''}{1+w'^2}; \qquad \sin\beta = \frac{1}{\sqrt{1+w'^2}}. \tag{16}$$

By substituting expressions (16) into system (15), we will give its form, convenient for integration into the environment *MatLab* using the *SimuLink* package:

$$\begin{cases} v' = -\dfrac{ag}{v}w'\sin\gamma\cos\gamma\cdot e^{w\cos\gamma} - f\left(\dfrac{v\sin\gamma}{\sqrt{1+w'^2}} + \dfrac{ag}{v}\cos^2\gamma\sqrt{1+w'^2}\cdot e^{w\cos\gamma}\right); \\ w'' = \cos\gamma\left(1+w'^2\right)\left(1 - \dfrac{ag}{v^2}\sin\gamma\cdot e^{w\cos\gamma}\right). \end{cases} \tag{17}$$

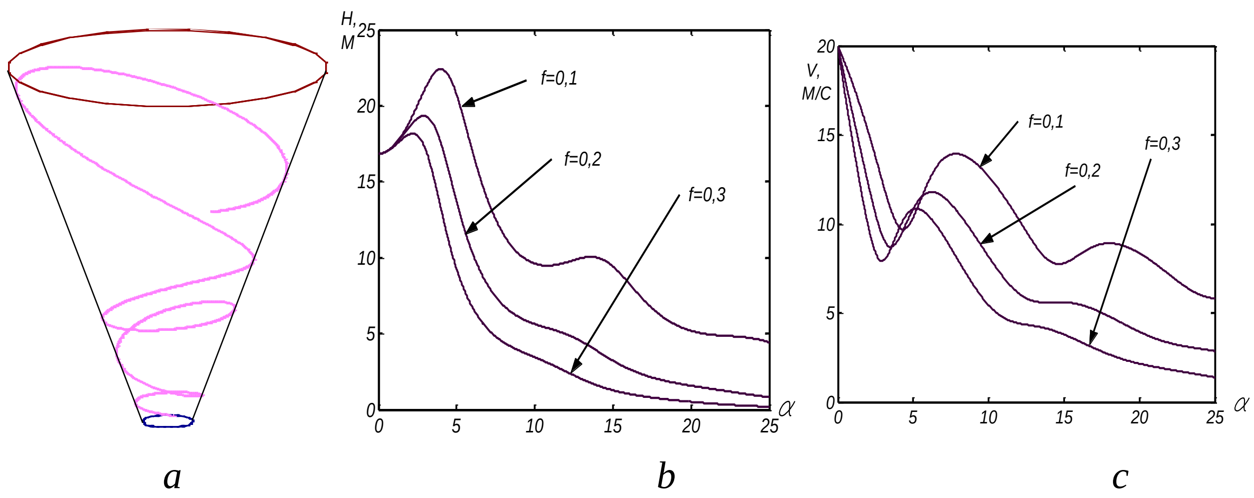

Fig. 2. Graphs of dependences on the angle α , constructed as a result of integrating the system (17) under the given initial conditions *a=50; γ=70 ⁰;* *v ₒ =20 m/s:*

a – trajectory of motion of a material particle at *f =0.1;*
b – dependence of the height of particle lift with different friction coefficients;
c – graphs of changes in particle velocities with different friction coefficients

Numerical integration system (17) was found dependencies $v=v(\alpha)$ and $w=w(\alpha)$. The model that performs integration, three integrators are included, at the input whose need specify three constants integration. They define weekend conditions: point on the surface, direction movement material particles in it and the initial speed. Substitution found dependencies $w = \int ctg\beta \cdot d\alpha$ in (8) gives the trajectory of the particle's motion along the internal surface of the cone under given initial conditions. Analyzing the trajectory of the particle, we can conclude that it is significantly different from the trajectory of movement along the inner surface of the cylinder [4]. Fig. 2 shows the graphs - the result of integrating the system (17). Fig. 2,a shows that a particle entering the inner surface of the cone at an angle of 90^{0} to its generator, begins to move up, and then descends down. Fig. 2,b shows the graphs of the change in height depending on the angle of rotation α, from which it is clear that with a decrease in the friction coefficient, the height of the rise increases. In addition, during the downward movement of the particle there may be local rises. The graphs of the change in speeds (Fig. 2,c) also show the difference between the movement of a particle along a cone and the same movement along a cylinder. If, upon entering the surface of the cylinder, the particle damps its velocity to a certain limit, and then it begins to increase again [4], then during movement along the cone, the velocity of the particle has a certain periodic character and approaches zero over time. It is interesting to study the regularity of the particle movement along the inner surface of the cone at $f=0$, i.e., when its surface is absolutely smooth. In this case, the first equation of system (17) takes the form:

$$\frac{dv}{d\alpha} = -\frac{ag}{v}\frac{dw}{d\alpha}\sin\gamma\cos\gamma\cdot e^{w\cos\gamma} \quad \text{або} \quad vdv = -ag\sin\gamma\cos\gamma\cdot e^{w\cos\gamma}dw. \qquad (18)$$

After integrating the right and left sides of equation (18), we have:

$$v^2 = -2ag\sin\gamma\cdot e^{w\cos\gamma} + c \qquad \text{або} \qquad v^2 = c - 2gH, \qquad (19)$$

where c is the integration constant; H is the height of the particle descent, which according to the last equation (8) is equal to the coordinate z taken with the opposite sign (since the direction of the *Oz axis* does not coincide with the direction of the particle descent). The second equality (19) expresses Galileo's law, according to which the velocity of the particle does not depend on the shape of the trajectory in the absence of friction (depends only on the initial velocity v_o and the height H of the particle) [2]. Substituting (19) into the second equation of system (17), we have:

$$w'' = \cos\gamma\left(1 + w'^2\right)\frac{3ag\sin\gamma \cdot e^{w\cos\gamma} + c}{2ag\sin\gamma \cdot e^{w\cos\gamma} + c}. \tag{20}$$

In the differential equation (20) there is no independent variable α, therefore, by substitution

$$w' = p; \qquad w'' = p\frac{dp}{dw} \tag{21}$$

equation (20) reduces to a first-order differential equation:

$$p\frac{dp}{dw} = \cos\gamma\left(1 + p^2\right)\frac{3ag\sin\gamma \cdot e^{w\cos\gamma} + c}{2ag\sin\gamma \cdot e^{w\cos\gamma} + c}. \tag{22}$$

After separating variables and integrating (22), we obtain:

$$\frac{2abg(1 + p^2)}{2ag \cdot e^{w\cos\gamma} + c} = e^{2w\cos\gamma}, \tag{23}$$

where b –another constant of integration. Substituting in (23) instead of p its value $p=w'$ from (21) leads to the expression, which, unfortunately, cannot be integrated in elementary terms functions :

$$\frac{dw}{\sqrt{b \cdot e^{3w\cos\gamma} + \dfrac{bc}{2ag\sin\gamma}e^{2w\cos\gamma} - 1}} = d\alpha. \tag{24}$$

However, when $c=0$, integration of expression (24) becomes possible. In this case, we lose the generality of the solution and obtain only partial result:

$$\alpha = \frac{2\operatorname{Arctg}\left(\sqrt{b \cdot e^{3w\cos\gamma} - 1}\right)}{3\cos\gamma}. \tag{25}$$

Let us find from (25) the dependence $w = w(\alpha)$:

$$w = \frac{1}{3\cos\gamma}\ln\frac{1}{b\cos^2\left(\frac{3}{2}\alpha\cos\gamma\right)}, \text{ отже } e^{w\cos\gamma} = \frac{1}{\sqrt[3]{b}\cos^{2/3}\left(\frac{3}{2}\alpha\cos\gamma\right)}. \tag{26}$$

By substituting second expression from (26) into (8), we obtain the trajectory movement particles on the inside surface of the cone in the absence of friction:

$$x = \frac{a\cos\gamma\cos\alpha}{\sqrt[3]{b}\cos^{2/3}\left(\frac{3}{2}\alpha\cos\gamma\right)};\ y = \frac{a\cos\gamma\sin\alpha}{\sqrt[3]{b}\cos^{2/3}\left(\frac{3}{2}\alpha\cos\gamma\right)};\ z = \frac{a\sin\gamma}{\sqrt[3]{b}\cos^{2/3}\left(\frac{3}{2}\alpha\cos\gamma\right)}. \quad (27)$$

Analyzing equation (27), we can conclude that at a certain value of the angle α the denominators become zero, and the trajectory of the particle along the cone goes to infinity. In Fig. 3, *a*, a limited section of the trajectory is constructed using equations (27) for different values of the integration constant *b*. To obtain the general solution ' We integrate system (17) numerically using the *SimuLink* package at *f =0*. By changing the direction of motion at the initial point, the trajectories shown in Fig. 3, *b-d* were obtained. As can be seen from these figures, a particle in the absence of friction and air resistance performs an oscillatory motion, alternately rising and falling along the surface of the cone, describing a certain number of branches. The particle can return to its original position, describing a finite number of branches, or not return, describing an infinite number of branches and moving for an arbitrary long time. This is the main difference in the motion of a particle compared to a cylinder [4]. As can be seen from Fig. 3,b-d, at the same initial velocity, the particle rises along the surface of the cone by the same amount regardless of the number of branches describing it (this also follows from Galileo's law). In the presence of friction, the particle will fall down, although the oscillatory nature of the motion may persist (especially at a small coefficient of friction). Fig. 3,e shows the trajectory of motion at *f =0.01* , from which it can be seen that the oscillations of the particle damp out over time. Analyzing Fig. 3,a-d, which show the trajectories of the particle in the absence of friction, at first it is difficult to understand the connection of Fig. 3,a with the others, since there is no loop in it.

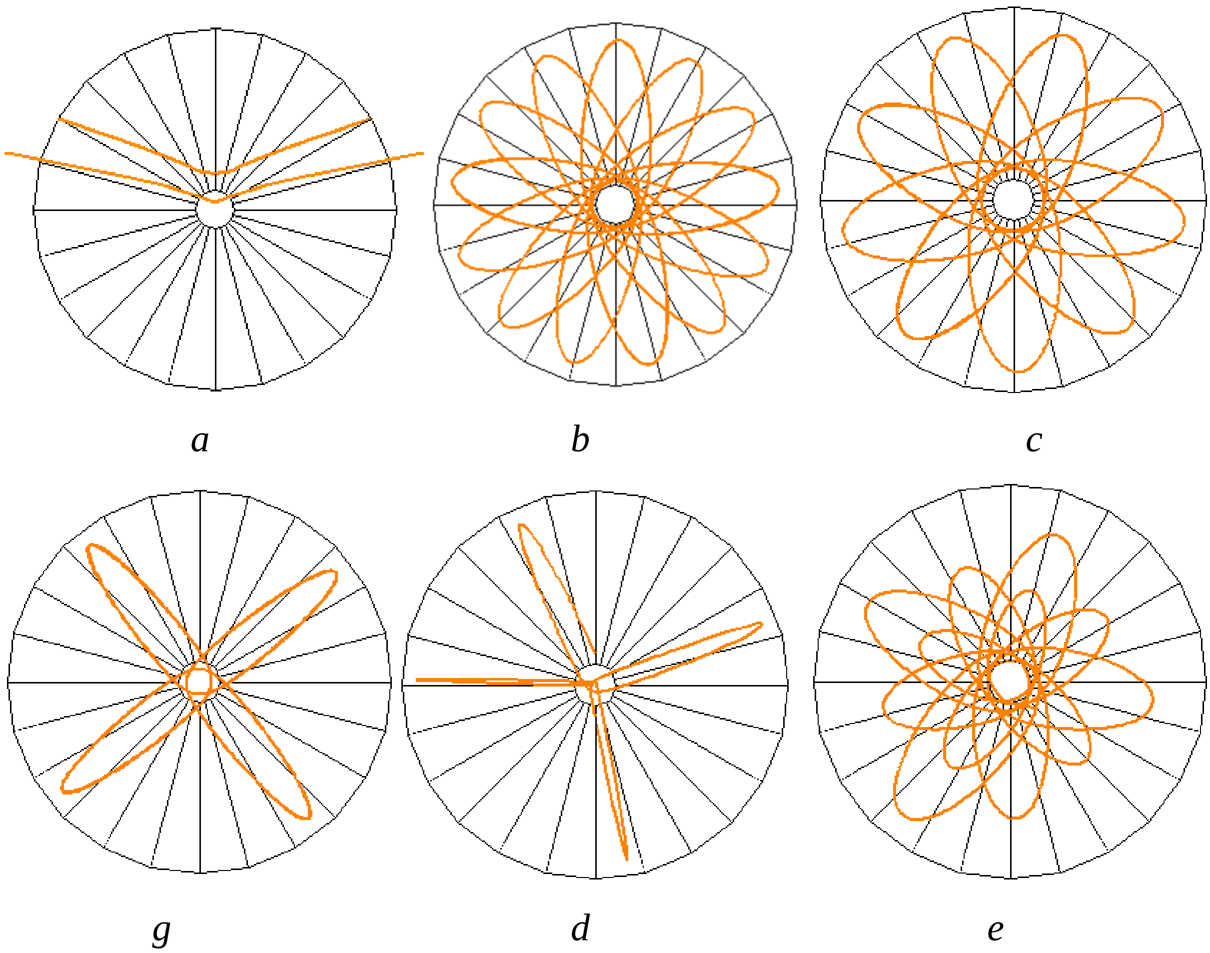

Fig. 3. Trajectories of particle motion along the inner surface of the cone at ***v_o =20 v/s; γ=45^0; a =50*** **and different directions movement in the initial point (view) from above)**:

a – trajectory constructed according to equations (27), which are a partial solution system (17) at *f=0; b* – particle describes 12 branches; *c* – particle describes 9 branches; *d* – particle describes 4 branches; *d* – the loop degenerates into a line close to the straight-line generator of the cone; *e* – particle trajectory at *f =0.01* (at *f =0* under the same other conditions is shown in Fig. 3, *c*)

However, knowledge of the features of the motion particles on unfolded surfaces with the same inclination of the generating surfaces to the horizontal plane will help in this. The fact is that the trajectory of the particle movement can be a straight line – the generator of the cone [7]. In this case $\beta=0$ – *const* and the system (17) cannot be integrated, since the expression $w' =$ ctg β turns into infinity. When trying to provide in the initial point movement direction by an angle β close to zero, the integration of system (17) stops, but from Fig. 3e it is seen that the loop is so narrow that it approaches a straight line. Thus, the partial result obtained from equations (27) and depicted in Fig. 3a is a transitional trajectory between the loop and a straight line. Of course, the particle cannot move upwards for an infinitely long time (according to equations (19) the initial velocity must also be infinitely large), but with a limited value of the initial velocity such a trajectory is quite probable. It is worth drawing attention to the fact that such a transitional trajectory was constructed thanks to the analytical dependencies . It is not possible to detect it during numerical integration. This indicates that no matter what modern computers and software products we use, numerical methods, in our opinion, will not be able to replace analytical studies, but will only be able to successfully supplement them.

If necessary, you can determine the time of movement of a particle along the surface of the cone:

$$v = \frac{ds}{dt} = \frac{ds}{d\alpha}\frac{d\alpha}{dt}, \qquad \text{where } t = \int \frac{s'}{v} d\alpha. \tag{28}$$

Substituting the arc derivative from (14) into (28) taking into account (16), we obtain:

$$t = a\cos\gamma \int \frac{\sqrt{1+w'^2}}{v} e^{w\cos\gamma} d\alpha, \tag{29}$$

where $v = v\ (\alpha)$ – dependence of the particle velocity change, which is determined by numerical integration of system (17). Thus, we can find the main parameters of the particle motion: the trajectory given by the equations $x = x\ (\alpha)$; $y = y\ (\alpha)$ and $z = z\ (\alpha)$; speed $v = v\ (\alpha)$; time of motion $t = t\ (\alpha)$. All these dependences in the form of graphs can be obtained using the *SimuLink package MatLab* environment. *MatLab* allows you to not only build a trajectory, but also do it in the form of animation using the *comet 3 command*. However such The trajectory, although constructed correctly, is unreliable. will reflect movement particles about the curve, since dependencies *x, y, z* are functions of the angle *α*, not of the time *t*. Possibilities *MatLab* allows us to solve this problem as well. With the help of functions interpolations *interp 1* you can reassign the equation of the trajectory *x, y, z* depending on the new independent variable - time *t* , that is, in fact, eliminate the parameter *α*. Reproducing using the *comet 3 command* movement particles along a cone, equation trajectories which are functions of time, we will get a real picture of such a movement on the surface of the cone.

Conclusions and perspectives. In the absence of friction and air resistance, a material particle, after entering the inner surface of the cone at a certain angle to the generator (except zero), performs an oscillatory motion, alternately rising and falling along a trajectory in the form of a loop, moving for any length of time. Depending on the initial conditions, the particle can describe a finite number of branches of the loop, an infinite number of branches, move along a straight-line generator of the cone or along an intermediate trajectory between a straight line and a loop. In the presence of friction, the particle will descend down to the top of the cone, while local rises are possible, the magnitude of which will depend on the initial velocity and the angle of inclination of the generators of the cone. The velocity during such a motion will decay, while also having an oscillatory character.

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ЗНАХОДЖЕННЯ ТРАЄКТОРІЙ РУХУ МАТЕРІАЛЬНОЇ ЧАСТИНКИ ПО ВНУТРІШНІЙ ПОВЕРХНІ КОНУСА ІЗ ВЕРТИКАЛЬНОЮ ВІССЮ ПРИ БОКОВІЙ ПОДАЧІ МАТЕРІАЛУ

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Анотація. *Рух матеріальних частинок по внутрішній поверхні конуса має місце у циклонах, конструкції яких можуть мати як циліндричні, так і конічні частини. Аеродинамічні процеси, які відбуваються у циклоні, мають складний характер, тому їх не можна точно змоделювати на основі теоретичних підходів При дослідженнях було введено ряд спрощень: не враховується опір повітря, оскільки частинка подається у конус разом із повітрям, хоча у подальшому їх напрями руху не збігаються (частинка гасить швидкість і опускається вниз, а повітря по центральній частині вздовж осі конуса піднімається вгору і виходить назовні); вплив частинок одна на одну, їх розмір тощо.*

Метою статті є дослідження руху матеріальної частинки, яка вступає на внутрішню поверхню вертикального конуса із заданою початковою швидкістю.

Якщо матеріальну частинку спрямувати з початковою швидкістю на внутрішню стінку конуса перпендикулярно його твірній, то подальший її рух включатиме як обертання навколо осі конуса, так і опускання вниз під дією сили власної ваги. Для знаходження траєкторії руху прийнято матеріальну точку за

вершину супровідного тригранника Френе, який має три взаємно перпендикулярні орти. Другий супровідний тригранник Дарбу має спільний із тригранником Френе орт дотичної до траєкторії.

Було розглянуто рівновагу діючих сил в проекціях на орти тригранника Дарбу. Це дало змогу визначити проекції кривини кривої на відповідні орти тригранника Дарбу. Апарат диференціальної геометрії дав можливість їх знайти через першу і другу квадратичні форми поверхні, що дозволяє уникнути громіздких перетворень.

Складено диференціальні рівняння руху матеріальної частинки по внутрішній поверхні вертикального конуса. Рівняння розв'язані за допомогою системи MatLab.

Отримане рівняння руху частинки по внутрішній поверхні конуса. Аналізуючи траєкторію руху частинки, можна зробити висновок, що вона суттєво відрізняється від траєкторії руху по внутрішній поверхні циліндра. Графіки зміни швидкостей теж показують відмінність руху частинки по конусу від такого ж руху по циліндру. Якщо при вступі на поверхню циліндра частинка гасить свою швидкість до певної межі, а потім вона знову починає зростати, то під час руху по конусу швидкість частинки має певний періодичний характер і з часом наближається до нуля.

За відсутності тертя і опору повітря матеріальна частинка після вступу на внутрішню поверхню конуса під певним кутом до твірної (крім нуля) здійснює коливальний рух, почергово піднімаючись і опускаючись по траєкторії у вигляді петлі, рухаючись при цьому як завгодно довго. Залежно від початкових умов частинка може описувати кінцеву кількість віток петлі, нескінченну кількість віток, рухатися по прямолінійній твірній конуса або по проміжній траєкторії між прямою лінією і петлею.

За наявності тертя частинка буде опускатися вниз до вершини конуса, при цьому можливі локальні підйоми, величина яких залежатиме від початкової швидкості і кута нахилу твірних конуса. Швидкість при такому русі буде затухати, маючи при цьому також коливальний характер.

Ключові слова: ***матеріальна частинка, конус, тригранник Дарбу, рівняння руху частинки***