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STUDY OF THE MOTION OF A MATERIAL PARTICLE ON THE INTERNAL SURFACE OF A STATIONARY INCLINED CYLINDER

S. F. Pylypaka, doctor of technical sciences, professor

A. V. Nesvidomin, Candidate of Technical Sciences, docent

National University of Life and Environmental Sciences of Ukraine

E-mail: a.nesvidomin@gmail.com

Abstract. *Cylindrical surfaces as working bodies of agricultural machines have quite wide application.*

The motion of a material particle on cylindrical surfaces is considered in the works of academicians of the Ukrainian Academy of Sciences P. M. Vasylenko and P. M. Zaika. However, P. M. Vasylenko failed to integrate the differential equations of motion and he considers approximate solutions. A similar problem was solved by P. M. Zaika, considering the motion of a particle on the inner surface of an inclined cylinder rotating around its own axis.

The purpose of the study is to find the kinematic parameters of the motion of a material particle on the inner surface of a stationary inclined cylinder under the action of its own weight under different initial conditions.

The article investigates the motion of a material particle on the inner surface of an inclined cylinder under the action of its own weight. Cases are considered when the particle moves accelerated, decelerated and at a constant speed. The system of differential equations is solved by numerical methods. The results were visualized.

It was established that the motion of a material particle along the inner surface of an inclined cylinder under the action of its own weight in the presence of friction can be divided into three cases: 1) the angle of inclination of the generating cylinder to the horizon is greater than the angle of friction; 2) the angle of inclination of the generating cylinder to the horizon is equal to the angle of friction; 3) the angle of inclination of the generating cylinder to the horizon is less than the angle of friction. If we exclude the rectilinear motion along the lower generating cylinder, then all three cases are characterized by oscillatory motion, which over time acquires certain signs of stability, in particular, by decreasing the amplitude. In this case, in the first case, the particle velocity increases over time, in the second - it stabilizes and becomes constant, in the third - it decreases until the particle stops completely. In the absence of friction, in all three cases, the oscillations will continue indefinitely with an increase in their period.

Key words: ***material particle, cylinder, equation of particle motion, force of its own weight***

Topicality. Cylindrical surfaces as working bodies of agricultural machines have quite wide application. Although they mainly perform rotational motion during the working process, the study of the motion of a material particle on a stationary surface is interesting from a cognitive point of view.

Analysis of recent research and publications. The motion of a material particle on cylindrical surfaces was considered in the works of academicians of the Ukrainian Academy of Sciences P.M. Vasylenko [1] and P.M. Zaika [2]. When studying the motion of a particle on the surface of an inclined stationary cylinder, P.M. Vasylenko considered the so-called inertial motion, that is, motion in which the weight of the particle is not taken into account. In this case, the particle would move along a geodesic curve, which for a cylinder is a helical line. In real conditions, the weight of the particle cannot be ignored, although it is quite possible to achieve motion whose trajectory is close to the geodesic line: for this, it is necessary to give the particle a sufficiently high speed [3]. Taking into account the weight of the particle, P.M. Vasylenko again simplifies the problem by neglecting the friction force. However, in this case, he is unable to integrate the differential equations of motion and he considers approximate solutions [1, p. 232-234]. Later, with the advent of computing technology and the possibility of numerical integration, a similar problem was solved by P.M. Zaika [2] (in the indicated work, the motion of a particle along the inner surface of an inclined cylinder rotating around its own axis was considered).

The purpose of the study is to find the kinematic parameters of the motion of a material particle along the inner surface of a stationary inclined cylinder under the action of its own weight under different initial conditions.

Materials and methods of research. Parametric equations of a cylinder with a vertical axis, and also after rotating it by an angle ε from the vertical position, we write accordingly:

$$\begin{aligned} X &= R\cos\alpha; & \quad X &= R\cos\varepsilon\cos\alpha + u\sin\varepsilon; \\ Y &= R\sin\alpha; & \quad Y &= R\sin\alpha; \\ Z &= u; & \quad Z &= -R\sin\varepsilon\cos\alpha + u\cos\varepsilon, \end{aligned} \tag{1}$$

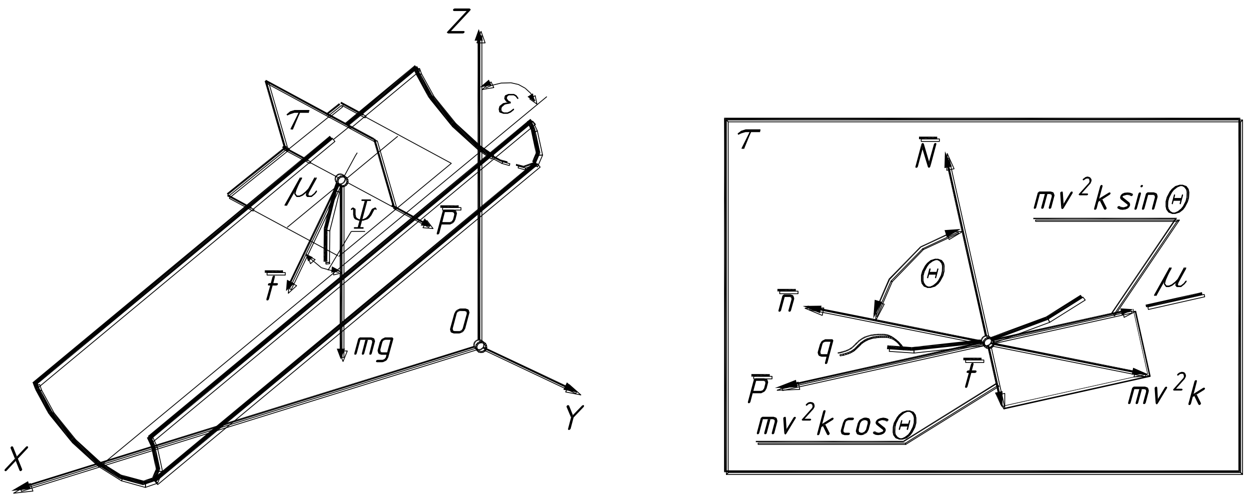
where R is the radius of the cylinder base; and u – the angle of rotation and the length of the cylinder's generator – the coordinates of a point on its surface – are variable parameters.

At $\varepsilon=0$, equation (1) of an inclined cylinder (formulas on the right) becomes the equation of a vertical cylinder (formulas on the left).

Research results and their discussion. Consider the motion of a material particle along an inclined cylinder under the action of its own weight. Suppose that the particle moves along a certain curve on the surface (trajectory). At a certain point on the trajectory, draw a plane μ tangent to the surface (Fig. 1, *a*). In the vicinity of the point where the particle is currently located, we can consider its motion along the surface as along the tangent plane μ. Therefore, we need to compose differential equations of motion, which in vector form will be written as one equation:

$$m\overline{w} = \overline{F}, \tag{2}$$

where *m* is the mass of the particle; $\overline{W}$ - particle acceleration; $\overline{F}$ - the resultant of the applied forces, which will be the weight of the particle mg ($g = 9.81\ m/s^2$) and the friction force.



a *b*

Fig. 1. Graphic illustrations for compiling differential equations of motion of a material particle along the inner surface of an inclined cylinder:
a – the tangent plane μ to the cylinder and the normal plane τ drawn to the trajectory at a certain point of it; *b* – decomposition of the acting forces in the normal plane τ trajectories

The vector equation (2) can be written in projections on the axis of the fixed system *OXYZ*, or on the axis of the moving system, which is somehow connected with the trajectory of motion. For the moving system, we will take the so-called Darboux trihedron of the trajectory, onto which we will project equation (2). It is formed as follows. In the tangent plane μ (touching the cylinder along a straight line), which is one of its three faces, we will draw a unit coordinate $\bar{t}$ tangent to the trajectory. The second coordinate $\bar{P}$, which is also in the plane μ, will be drawn perpendicular to $\bar{t}$. The plane τ passing through the coordinate $\bar{P}$ and perpendicular to $\bar{t}$, is the second face of the trihedron and is called the normal plane (Fig. 1, *a*). The third orthogonal $\bar{N}$ (normal to the surface) and the third face of the trihedron in Fig. 1, *a* are not shown.

Let us write the basic equation (2) of the dynamics of a point in the projection onto the ort $\bar{t}$. Since the acceleration *w* is the derivative of the velocity *v* with respect to time *t* (not to be confused with the orthogonal function $\bar{t}$), then it can be written in the transition from the variable *t* to the variable *s* – the length of the trajectory arc:

$$\frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v\frac{dv}{ds}; \qquad \frac{ds}{dt} = v. \tag{3}$$

The force that causes the particle to move and gives it acceleration is the component of the gravitational force, which in the projection onto the orthogonal plane $\bar{t}$ is written as $mg \cdot \cos \Psi$, where Ψ is the angle between the orthogonal plane $\bar{t}$ and the vector of the gravitational force *mg* (Fig. 1, *a*). Another force directed along the orthogonal plane $\bar{t}$ opposite to the direction of motion is the friction force fG, where *f* is the friction coefficient, *G* is the pressure exerted by the particle on the surface. Therefore, equation (2) in the projection onto the orthogonal plane $\bar{t}$ of the Darboux trihedron can be written as:

$$mv\frac{dv}{ds} = mg\cos\psi - fG. \tag{4}$$

The pressure force *G* is directed along the normal $\bar{N}$ to the surface and is the sum of two components: the gravity force in the projection onto the ort $\bar{N}$ ($mg \cdot \cos \omega$, where ω is the angle between the gravity vector and the normal to the surface $\bar{N}$) and the projection of the centrifugal force onto the same ort. The centrifugal force vector mv^2k, where *k* is the curvature of the trajectory at a given point, is directed along the main normal $\bar{n}$ trajectory in the opposite direction side of its direction. Since the main normal $\bar{n}$ trajectory and the normal $\bar{N}$ to the surface are in the normal plane τ, then we consider it without distortion (Fig. 1, *b*), that is, we choose the direction of view on it from the orthogonal plane $\bar{t}$; in this case, the orthogonal plane $\bar{t}$ is projected into a point, and the tangent plane μ is projected into a straight line. This same line will be tangent to the curve q - the line of section of the cylinder by the normal plane τ. Between the main normal $\bar{n}$ and the normal to $\bar{N}$ the cylinder surface there is an angle θ, which changes during the movement of the particle and depends on its position on the trajectory, i.e. is a function of the length of the arc of the trajectory s. Thus, the expression for the pressure force takes the form:

$$G = mg\cos\omega + mv^2 k\cos\theta. \tag{5}$$

Let us write the basic equation of the dynamics of the point (2) in the projection on the orth $\bar{P}$. The orth $\bar{P}$ is located in the plane τ perpendicular to the direction of motion and is the result of the intersection of this plane with the tangent plane μ. The component of the centrifugal force $mv^2k \cdot \sin\theta$ tries to displace the particle in the direction transverse to the trajectory up along the curve of the cross section q (Fig. 1,b). The particle is displaced until it is balanced by the component of the gravitational force $mg \cdot \cos\varphi$, where φ is the angle between the gravitational force vector and the orth $\bar{P}$. Therefore, the equation of motion in the projection on the orth $\bar{P}$ can be written:

$$mv^2 k\sin\theta = mg\cos\varphi. \tag{6}$$

Finally, $\bar{N}$ we essentially have the equation of motion of a particle in the projection onto the ort. This is the pressure force (5), which is balanced by the surface reaction (this equation is already included in equation (4), so the system will consist of two equations – in the projections onto the ort $\bar{t}$ and $\bar{P}$). It should be noted that the equality of expression (5) to zero indicates that at a given point of the trajectory there is no pressure on the surface and the particle is detached from the surface.

Let us combine equations (4) and (6) into a system, making some simplifications. First, the expressions $k\cos\theta = k_n$ and $k\sin\theta = k_r$ in differential geometry are called, respectively, the normal and geodesic curvatures of a curve on a surface [4]. Secondly, substituting expression (5) into equation (4) makes it possible to reduce it to the mass m. The same applies to equation (6). Taking into account the above, the system of differential equations of motion of a particle in projections onto the orth $\bar{t}$ and $\bar{P}$ the moving Darboux trihedron can be written (its derivation is considered in more detail in [5]):

$$\begin{cases} v\dfrac{dv}{ds} = g\cos\psi - f(g\cos\omega + v^2k_н); \\ v^2k_г = g\cos\varphi. \end{cases} \tag{7}$$

The angles ψ, φ, ω, velocity v, geodesic k_g and normal k_n curvature of the trajectory, which are included in the system (7), are functions of the arc s *of* the trajectory or another parameter that defines the curve on the surface. If the surface of an inclined cylinder is given by equations (1), in which α and u are independent variables, then solving the system (7) means finding such a dependence between the variables α and u, so that at each point of the curve that is formed on the surface with the found dependence, the conditions of the system (7) are fulfilled.

Let us find expressions for the angles ψ, φ, ω, of the geodesic k_g, normal k_n of the trajectory curvature and the arc differential ds, which are included in system (7). To find the specified quantities, it is necessary to have partial derivatives and expressions of the first and second quadratic forms of the surface of the inclined cylinder. The first, second and mixed derivatives of equations (1) on the right will be:

$$\begin{aligned} &X_\alpha = -R\cos\varepsilon\sin\alpha; && X_u = \sin\varepsilon; \\ &Y_\alpha = R\cos\alpha; && Y_u = 0; \\ &Z_\alpha = R\sin\varepsilon\cos\alpha; && Z_u = \cos\varepsilon; \\ &X_{\alpha\alpha} = -R\cos\varepsilon\cos\alpha; && X_{uu} = 0; \\ &Y_{\alpha\alpha} = -R\sin\alpha; && Y_{uu} = 0; \\ &Z_{\alpha\alpha} = R\sin\varepsilon\cos\alpha; && Z_{uu} = 0; \\ &X_{\alpha u} = 0; \quad Y_{\alpha u} = 0; && Z_{\alpha u} = 0. \end{aligned} \tag{8}$$

The coefficients G, F, E of the first quadratic form will be:

$$G = X_\alpha^2 + Y_\alpha^2 + Z_\alpha^2 = R^2;$$

$$F = X_\alpha X_u + Y_\alpha Y_u + Z_\alpha Z_u = 0; \quad E = X_u^2 + Y_u^2 + Z_u^2 = 1. \tag{9}$$

The coefficients N, L, M of the second quadratic form will be:

$$N=\frac{1}{\sqrt{GE-F^2}}\begin{vmatrix}X_{\alpha\alpha} & Y_{\alpha\alpha} & Z_{\alpha\alpha}\\ X_u & Y_u & Z_u\\ X_\alpha & Y_\alpha & Z_\alpha\end{vmatrix}=R; \tag{10}$$

$$M=\frac{1}{\sqrt{GE-F^2}}\begin{vmatrix}X_{\alpha u} & Y_{\alpha u} & Z_{\alpha u}\\ X_u & Y_u & Z_u\\ X_\alpha & Y_\alpha & Z_\alpha\end{vmatrix}=0;\quad L=\frac{1}{\sqrt{GE-F^2}}\begin{vmatrix}X_{uu} & Y_{uu} & Z_{uu}\\ X_u & Y_u & Z_u\\ X_\alpha & Y_\alpha & Z_\alpha\end{vmatrix}=0.$$

Let's find the first and second quadratic forms of the inclined cylinder and their ratio - the normal curvature:

$$I=ds^2=Edu^2+2Fdud\alpha+Gd\alpha^2=du^2+R^2d\alpha^2; \tag{11}$$

$$II=Ldu^2+2Mdud\alpha+Nd\alpha^2=Rd\alpha^2; \tag{12}$$

$$k_u=\frac{II}{I}=\frac{Rd\alpha^2}{du^2+R^2d\alpha^2}. \tag{13}$$

In order for a line to be given on the surface of an inclined cylinder, it is necessary to establish a certain dependence between the variables u and α. Such a dependence can be established through another quantity - the angle β between the straight-line generator of the cylinder and the tangent to the trajectory. Since the grid of coordinate lines on the surface is orthogonal, *(F =0),* then the element of the arc of the trajectory ds can be considered as the hypotenuse of an elementary right-angled triangle (Fig. 2), the legs of which are the elements of the lengths of the coordinate lines. This can be understood from expression (11), if it is interpreted as the Pythagorean theorem. From the right-angled triangle (Fig. 2) we have:

$$\mathrm{tg}\beta=\frac{R\cdot d\alpha}{du}\quad \text{or}\quad d\alpha=\frac{\mathrm{tg}\beta}{R}du,\quad \text{where}\quad \alpha=\frac{1}{R}\int \mathrm{tg}\beta\, du. \tag{14}$$

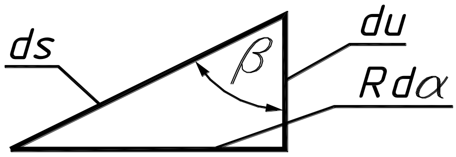


Fig. 2. To determine the angle β between the direction of particle motion and the rectilinear generating surface

Angle β in (14) we will consider as a function of the variable u: $\beta = \beta(u)$. Substituting $d\alpha$ from (14) into (11), we obtain the expressions for the differential of the arc and the normal curvature of the trajectory:

$$ds = \frac{du}{\cos\beta}; \qquad k_н = \frac{\sin^2\beta}{R}. \tag{15}$$

Let us find an expression for the geodesic curvature k_g of the trajectory, which is included in the system (7). The geodesic curvature of a line on a surface refers to the internal properties of the surface and can be determined through the coefficients (9) of the first quadratic form. For a ruled surface, which is the surface of an inclined cylinder, the geodesic curvature of a line can be found by the formula [4]:

$$k_г = \frac{\sqrt{G}}{\left(u'^2 + G\alpha'^2\right)^{3/2}}\left(u''\alpha' - \alpha''u' - \frac{1}{2}G_u\alpha'^3 - \frac{1}{2}\frac{G_\alpha}{G}u'\alpha'^2 - \frac{G_u}{G}u'^2\alpha'\right), \tag{16}$$

where G_α, G_u - partial derivatives of the coefficient G with respect to the corresponding parameter. Since we took the parameter *u as the independent variable*, and $\alpha = \alpha(u)$ according to (14), the derivatives included in expression (16) can be written:

$$G_u = 0; \quad G_\alpha = 0; \quad u' = 1; \quad u'' = 0; \quad \alpha' = \frac{\mathrm{tg}\beta}{R}; \quad \alpha'' = \frac{\beta'}{R\cos^2\beta}. \tag{17}$$

Substituting (17) and (9) into (16) gives the expression for the geodesic curvature of the trajectory:

$$k_г = -\beta'\cos\beta. \tag{18}$$

We have to find expressions for the angles ψ, φ, ω, included in system (7). Let us write the parametric equations of the line on the surface of the cylinder, taking into account the dependence $\alpha = \alpha(u)$ (14):

$$\begin{aligned} x &= R\cos\varepsilon\cos\left(\frac{1}{R}\int \mathrm{tg}\beta\cdot du\right) + u\sin\varepsilon; \\ y &= R\sin\left(\frac{1}{R}\int \mathrm{tg}\beta\cdot du\right); \\ z &= -R\sin\varepsilon\cos\left(\frac{1}{R}\int \mathrm{tg}\beta\cdot du\right) + u\cos\varepsilon. \end{aligned} \tag{19}$$

To find the angle ψ between the orthogonal tangent $\bar{t}$ and the vector of the force of gravity mg , we first find the direction of the vector $\bar{t}$. To do this, we differentiate equation (19) with respect to the independent variable u:

$$\begin{aligned} x' &= -\cos\varepsilon \operatorname{tg}\beta \sin\left(\frac{1}{R}\int \operatorname{tg}\beta \cdot du\right) + \sin\varepsilon; \\ y' &= \operatorname{tg}\beta \cos\left(\frac{1}{R}\int \operatorname{tg}\beta \cdot du\right); \\ z' &= \sin\varepsilon \operatorname{tg}\beta \sin\left(\frac{1}{R}\int \operatorname{tg}\beta \cdot du\right) + \cos\varepsilon . \end{aligned} \tag{20}$$

Since the vector of the weight force mg is parallel to the OZ *axis* of the fixed coordinate system, the angle ψ is determined by the formula:

$$\cos\psi = \frac{z'}{\sqrt{x'^2 + y'^2 + z'^2}} = \sin\varepsilon \sin\beta \sin\left(\frac{1}{R}\int \operatorname{tg}\beta \cdot du\right) + \cos\varepsilon \cos\beta. \tag{21}$$

The angle ω is the angle between the vector of the force of gravity mg and the normal to the surface $\bar{N}$. The normal to the surface is found from the cross product of the vectors tangent to the coordinate lines:

$$\bar{N} = \begin{vmatrix} X & Y & Z \\ X_\alpha & Y_\alpha & Z_\alpha \\ X_u & Y_u & Z_u \end{vmatrix}, \qquad \begin{aligned} N_x &= Y_\alpha Z_u - Y_u Z_\alpha; \\ N_y &= -X_\alpha Z_u + X_u Z_\alpha; \\ N_z &= X_\alpha Y_u - X_u Y_\alpha . \end{aligned} \tag{22}$$

Substituting the partial derivatives from (8) into (22), we obtain the coordinates of the normal vector to the surface of the inclined cylinder:

$$N_x = R\cos\varepsilon\cos\alpha; \qquad N_y = R\sin\alpha; \qquad N_z = -R\sin\varepsilon\cos\alpha. \tag{23}$$

Let us find an expression for the angle ω, taking into account the dependence $\alpha = \alpha(u)$ from (14):

$$\cos\omega = \frac{N_z}{\sqrt{N_x^2 + N_y^2 + N_z^2}} = -\sin\varepsilon \cos\left(\frac{1}{R}\int \operatorname{tg}\beta \cdot du\right). \tag{24}$$

Finally, to find the expression for the angle φ, you need to know the coordinates of the vector $\bar{P}$. Since it is perpendicular to the two vectors $\bar{N}$ and $\bar{t}$, its coordinates will be determined from the cross product of the specified vectors:

$$\overline{P}=\begin{vmatrix} X & Y & Z \\ N_x & N_y & N_z \\ x' & y' & z' \end{vmatrix}, \qquad \begin{aligned} P_x &= z'N_y - y'N_z; \\ P_y &= -z'N_x + x'N_z; \\ P_z &= y'N_x - x'N_y. \end{aligned} \tag{25}$$

After substituting the corresponding expressions (20) and (23) into (25), we find the projections of the vector $\overline{P}$ on the axis of the fixed coordinate system:

$$\begin{aligned} P_x &= R\sin\varepsilon\,\mathrm{tg}\beta + R\cos\varepsilon\sin\left(\frac{1}{R}\int \mathrm{tg}\beta\cdot du\right); \\ P_y &= -R\cos\left(\frac{1}{R}\int \mathrm{tg}\beta\cdot du\right); \\ P_z &= R\cos\varepsilon\,\mathrm{tg}\beta - R\sin\varepsilon\sin\left(\frac{1}{R}\int \mathrm{tg}\beta\cdot du\right). \end{aligned} \tag{26}$$

Using the projections of vector $\overline{P}$ (26), we find an expression for the cosine of the angle φ:

$$\cos\varphi = \frac{P_z}{\sqrt{P_x^2 + P_y^2 + P_z^2}} = \cos\varepsilon\sin\beta - \sin\varepsilon\cos\beta\sin\left(\frac{1}{R}\int \mathrm{tg}\beta\cdot du\right). \tag{27}$$

Now we have all the expressions included in system (7) in functions of one variable u. Substitute into (7) the expression ds and k_n from (15), k_g from (18) and the expressions for the cosines of the angles from (21), (24) and (27):

$$\begin{cases} vv'\cos\beta = g\left[\sin\varepsilon\sin\beta\sin\left(\dfrac{1}{R}\displaystyle\int \mathrm{tg}\beta\cdot du\right) + \cos\varepsilon\cos\beta\right] - \\ \qquad - f\left[v^2\dfrac{\sin^2\beta}{R} - g\sin\varepsilon\cos\left(\dfrac{1}{R}\displaystyle\int \mathrm{tg}\beta\cdot du\right)\right]; \\ -v^2\beta'\cos\beta = g\left[\cos\varepsilon\sin\beta - \sin\varepsilon\cos\beta\sin\left(\dfrac{1}{R}\displaystyle\int \mathrm{tg}\beta\cdot du\right)\right]. \end{cases} \tag{28}$$

Since system (28) is a system of integro-differential equations, we turn to a system of differential equations by differentiating the last expression (14) twice:

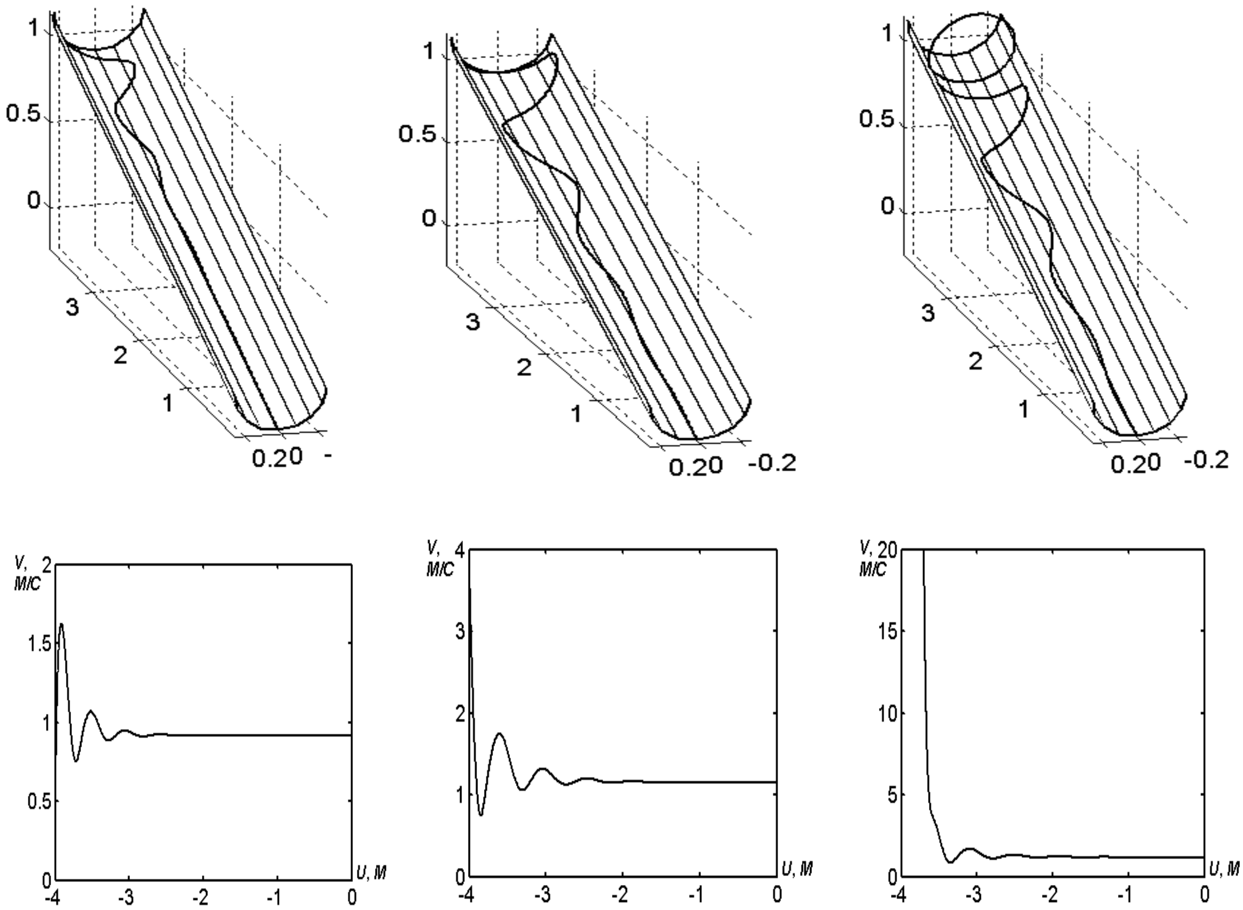
$$\begin{aligned} &\alpha' = \frac{\mathrm{tg}\beta}{R}; \qquad \alpha'' = \frac{\beta'}{R\cos^2\beta}, \qquad \text{where} \\ &\beta' = R\alpha''\cos^2\beta; \quad \cos\beta = \frac{1}{\sqrt{1+R^2\alpha'^2}}; \quad \sin\beta = \frac{R\alpha'}{\sqrt{1+R^2\alpha'^2}}. \end{aligned} \tag{29}$$

Let us substitute expressions (29) and the last expression (14) into system (28) and reduce it to a form convenient for integration in the *MatLab environment* using the *Simulink dynamic systems modeling package:*

$$\begin{cases} v' = \dfrac{g}{v}\left(R\alpha' \sin\varepsilon \sin\alpha + \cos\varepsilon\right) - \\ \qquad\qquad - f\left(\dfrac{Rv\alpha'^2}{\sqrt{1+R^2\alpha'^2}} - \dfrac{g}{v}\sqrt{1+R^2\alpha'^2}\sin\varepsilon\cos\alpha\right); \\ \alpha'' = \dfrac{g(1+R^2\alpha'^2)}{Rv^2}(\sin\varepsilon\sin\alpha - R\alpha'\cos\varepsilon)\ . \end{cases} \tag{30}$$

Let us analyze the system of differential equations (30). If a material particle is forced to move along the bottom of an inclined cylinder down along the lower straight-line generator with some initial velocity v_o , then the nature of the motion will obviously depend on the angle of inclination of the cylinder axis ε. If the inclination of the cylinder to the horizon is less than the angle of friction, then it is obvious that the motion will be slowed down, and if it is greater, it will be accelerated. Let us check the system (30) for the case when the angle of inclination of the cylinder to the horizon is equal to the angular friction. Taking into account that is $\varepsilon$the angle between the vertical direction and the axis of the cylinder, we can write: f = ctg ε. The motion along the bottom of the cylinder along the straight-line generator corresponds to the angle value $\alpha=180^{0} - const$. Substituting these two quantities into system (30) from the second equation we obtain the identity, and the first gives: $v' = 0$, hence $v = const$. Thus, having given the particle an initial velocity along the lower generator of the cylinder down, it will continue to move with a constant velocity. The question arises: how will it move if the initial velocity is directed at a certain angle to the generator and the movement does not begin from the lower generator? To investigate such a movement, numerical integration of system (30) was applied using the *Simulink package*. The calculations were carried out at $R = 0.25\ m$ and the length of the generators *4 m*. The coefficient of friction was taken to be $f = 0.3$, from which the angle was determinedε : ε= arcctg $(0,3)=73,3^{0}$. For integration, three blocks were used – integrators, the input of each of which was given constant integrations: the initial velocity

v_o, the position of a point on the surface of the cylinder at the initial moment of movement (for all subsequent examples, the movement began from the extreme lateral generatrix at $\alpha_o = 90^0$) and the direction of movement at an angle β_o to the generatrix. The cylinder is constructed according to equations (1), and for clarity, only its lower part is shown in the form of a trough. In Fig. 3, the trajectories of the particle motion and the graphs of its velocity change under different initial conditions are plotted. In Fig. 3, *a*, the trajectory of the particle, the motion of which began with an initial velocity close to zero (the value of the angle β in this case, it is not of significant importance).

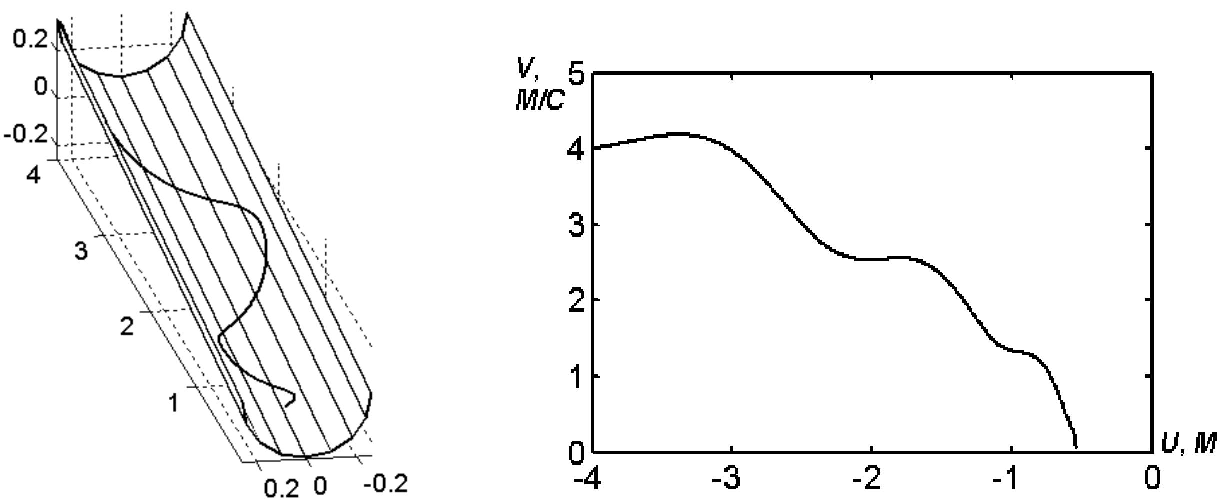


a *b* *c*

Fig. 3. Trajectory (top) and velocity graph (bottom) of a particle moving along the surface of a cylinder inclined to the horizon at an angle equal to the angular friction:
a – the initial velocity of the particle is close to zero, angle $\beta=45^0$; *b* – initial particle velocity $v_o = 4$ *m/s*, angle $\beta \approx 90^0$; *c* – initial particle velocity $v_o = 20$ *m/s*, angle $\beta \approx 90^0$

From Fig. 3, *a* it is seen that the particle first accelerates, then its motion becomes similar to the oscillatory one, which damps out and the rest of the way it moves along the lower generating cylinder with a constant speed close to *1 m/s*. When the initial speed increases, the particle also performs oscillatory motion (Fig. 3, *b*), but its stabilization occurs somewhat later to a value slightly greater than *1 m/s*. If the particle is given a significant initial speed at an angle β close to 90^{0} , it will make one full revolution, and then it will move, as in the previous cases (Fig. 3, *c*). Studies have shown that as the angle decreases, $\varepsilon$the nature of the particle's motion becomes more and more similar to its motion along the inner surface of a cylinder with a vertical axis [6], i.e. the number of full revolutions of the particle, other things being equal, increases. Conversely, as the angle increases, $\varepsilon$the number of complete revolutions decreases, the particle speed drops sharply, and even with a significant initial speed, it makes only 1-2 complete revolutions around the inner surface of the cylinder.

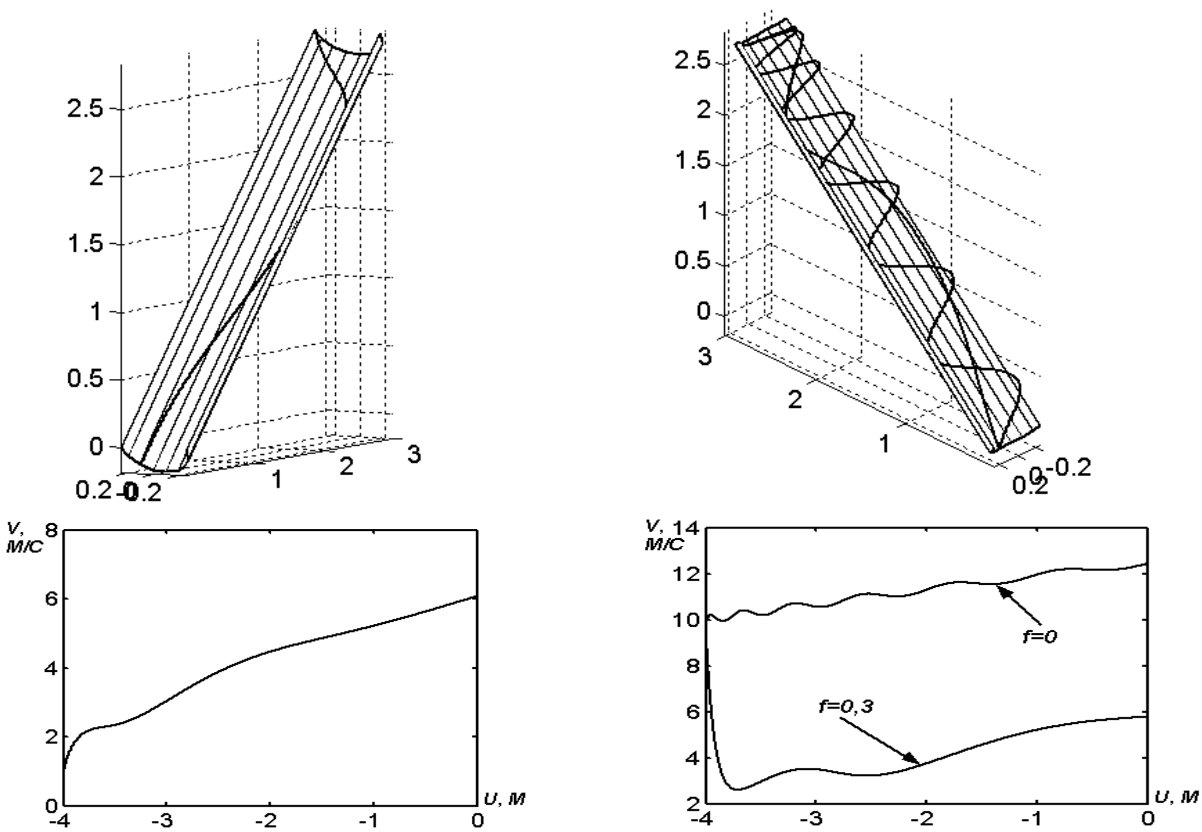
If the angle of inclination of the cylinder to the horizon is less than the angle of friction, the particle, having traveled a certain distance, will stop (Fig. 4).



a *b*

Fig. 4. Graphic illustrations of the movement of a particle along the inner surface of a cylinder when it is tilted to the horizon at an angle smaller than the friction angle ($\varepsilon=85^{0}$, $\beta=0^{0}$, $v_{o}=4$ m/s, $f=0.3$):
a – trajectory of movement; *b* – velocity change graph

Finally, at an angle of inclination of the cylinder to the horizon greater than the friction angle, the particle will move unevenly, maintaining a tendency to increase speed and stabilize the motion without oscillations (Fig. 5). However, this does not apply to the motion in the absence of friction (Fig. 5, b), in which the oscillations will continue indefinitely with an increase in their period.



a *b*

Fig. 5. Trajectory (top) and velocity graph (bottom) of a particle moving along the surface of a cylinder inclined to the horizon at an angle greater than the friction angle (ε=45 ^{0}):

a – the initial velocity of the particle is close to zero, angle $\beta=45^0$, $f=0.3$; *b* – initial particle velocity $v_o=10$ *m/s,* angle $\beta\approx90^0$ and different coefficients of friction

Conclusions and perspectives. The motion of a material particle along the inner surface of an inclined cylinder under the action of its own weight in the presence of friction can be divided into three cases: *1)* the angle of inclination of the generating cylinder to the horizon is greater than the angle of friction; *2)* the angle of inclination of the generating cylinder to the horizon is equal to the angle of friction*; 3)* the angle of inclination of the generating cylinder to the horizon is less than the angle of friction. If we exclude the rectilinear motion along the lower generating cylinder, then all three cases are characterized by oscillatory motion, which over time acquires certain signs of stability, in particular, by decreasing the amplitude. In this case, in the first case, the particle velocity increases over time, in the second - it stabilizes and becomes constant, in the third - it decreases until the particle stops completely. In the absence of friction, in all three cases, the oscillations will continue indefinitely with an increase in their period.

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ДОСЛІДЖЕННЯ РУХУ МАТЕРІАЛЬНОЇ ЧАСТИНКИ ПО ВНУТРІШНІЙ ПОВЕРХНІ СТАЦІОНАРНОГО ПОХИЛОГО ЦИЛІНДРА

С. Ф. Пилипака, А. В. Несвідомін

Анотація. *Циліндричні поверхні як робочі органи сільськогосподарських машин мають досить широке застосування.*

Рух матеріальної частинки по циліндричних поверхнях розглянуто у працях академіків УААН П. М. Василенка і П. М. Заїки. Проте проінтегрувати диференціальні рівняння руху П. М. Василенку не вдалося і він розглядає наближені розв’язки. Подібну задачу розв’язав П.М.Заїка, розглянувши рух частинки по внутрішній поверхні похилого циліндра, що обертається навколо власної осі.

Мета дослідження – знайти кінематичні параметри руху матеріальної частинки по внутрішній поверхні стаціонарного похилого циліндра під дією сили власної ваги за різних початкових умов.

У статті досліджено рух матеріальної частинки по внутрішній поверхні похилого циліндра під дією сили власної ваги. Розглянуто випадки, коли частинка рухається прискорено, сповільнено і із постійною швидкістю. Систему диференціальних рівнянь розв’язано чисельними методами. Зроблено візуалізацію одержаних результатів.

Встановлено, що рух матеріальної частинки по внутрішній поверхні похилого циліндра під дією сили власної ваги за наявності тертя можна розділити на три випадки: 1) кут нахилу твірних циліндра до горизонту більший від кута тертя; 2) кут нахилу твірних циліндра до горизонту рівний кутові тертя; 3) кут нахилу твірних циліндра до горизонту менший від кута тертя. Якщо виключити прямолінійний рух по нижній твірній циліндра, то для всіх трьох випадків характерний коливальний рух, який з часом набуває певних ознак стабільності, зокрема, по зменшенню величини амплітуди. При цьому у першому випадку швидкість частинки з часом росте, у другому – стабілізується і стає постійною, у третьому – зменшується до повної зупинки частинки. За відсутності тертя у всіх трьох випадках коливання будуть продовжуватися нескінченно довго при зростанні їх періоду.

Ключові слова: ***матеріальна частинка, циліндр, рівняння руху частинки, сила власної ваги***