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COMPUTER MODEL OFMOTION OF A PARTICLE ON A FIXED ROUGH SURFACES OF SECOND ORDER

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Abstract. *The aim - to develop Maple-model of a particle on all surfaces of 2nd order. Based on the proposed method of forming laws of motion of particles on rough surfaces in the interior of coordinates projected on orts of accompanying trihedron of trajectory was set maple-established model of a particle in all ruled rough surfaces 2nd order. Due developed computer tools has become possible to perform online research trajectory-kinematic characteristics of a particle on all surfaces of 2nd order, cylindrical surfaces orthogonal sections as transcendental curves of helical surfaces. This enabled us to analyze the movement of particles moving loose materials pipelines, supply of fertilizers to scattering disk separation grain inclined vibrational planes.*

Key words: ***material particle, rough surface, cylinder, hyperboloid of revolution, paraboloid of rotation***

Topicality. The movement of material particles on rough working surfaces of complex shape occurs when moving loose materials pipelines, supply of fertilizers to the scattering disk separation grain inclined vibrational planes. Understanding the patterns of movement of particles in random rough surface position in three dimensions allows purposefully to calculate structural and kinematic parameters of working bodies.

Analysis of recent research and publications. Analytical output law of motion of a particle on the rough surface is reduced to drawing up a system of differential equations of 2nd order desired dependency which is the trajectory of the particle, its velocity, acceleration, length of the path, the strength of the normal reaction, the move to its stop and other trajectory-kinematic characteristics . Sequence analytical output of differential equations and methods of solution is rather labor-intensive [1].

In contrast, computer simulation of a particle on the surface allows to remove bulky analytic transformation and provide a convenient interactive mode scientist for the necessary computational experiments on the analysis of a particle at different baseline throwing it on any rough surface. But the development of computer models of a particle on the surface needs to address a number of theoretical and practical nature which make up the relevance of research. First, is the development of general algorithm for automatic withdrawal of differential equations law of motion of particles on any surface that is randomly located in space; trajectory analysis, kinematic characteristics of a particle not only in time, but depending on the position of the particles on the surface and the direction of its movement on the surface; illustrate the results of research in the form of numerical data, graphics and motion simulations particles on the surface [2].

The purpose of research - development-Maple models of a particle on all surfaces of 2nd order.

Materials and methods of research. For arbitrary rotation position of the cylinder:

$$\boldsymbol{R}(u,v) = \boldsymbol{R}[a\cos(u),\ a\sin(u)\cos(\xi) - v\sin(\xi),\ a\sin(u)\sin(\xi) + v\cos(\xi)] \qquad (1)$$

developed three models of a particle, which allow to investigate its trajectory, according kinematic properties of time$t$, position$u$and the direction of moveα. Derived laws of motion of particles in the cylinder$t$are as follows:

– projected on orts$\boldsymbol{u}$ and $v$trihedron$\boldsymbol{OuvN}$:

$$\begin{cases} Ou := m\frac{d^2}{dt^2}u(t) = -\dfrac{mfa\frac{d}{dt}u(t)\left(a\left(\frac{d}{dt}u(t)\right)^2 - g\sin(u(t))\sin(\xi)\right)}{\sqrt{a^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2}} - mg\cos(u(t))\sin(\xi) \\ Ov := m\frac{d^2}{dt^2}v(t) = -\dfrac{mf\frac{d}{dt}v(t)\left(a\left(\frac{d}{dt}u(t)\right)^2 - g\sin(u(t))\sin(\xi)\right)}{\sqrt{a^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2}} - mg\cos(\xi) \end{cases}, \qquad (2)$$

– projected on orts$\boldsymbol{T}$ and$\boldsymbol{P}$trihedron$\boldsymbol{OTPN}$:

$$\begin{cases} OT := \dfrac{m\left(a^2\frac{d}{dt}u(t)\frac{d^2}{dt^2}u(t) + \frac{d}{dt}v(t)\frac{d^2}{dt^2}v(t)\right)}{\sqrt{a^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2}} = \dfrac{-mg\left(a\cos(u(t))\frac{d}{dt}u(t)\sin(\xi) + \frac{d}{dt}v(t)\cos(\xi)\right)}{\sqrt{a^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2}} - \\ \qquad mf\left(a\left(\frac{d}{dt}u(t)\right)^2 - g\sin(u(t))\sin(\xi)\right) \\ OP := -ma\left(\frac{d}{dt}u(t)\frac{d^2}{dt^2}v(t) - \frac{d}{dt}v(t)\frac{d^2}{dt^2}u(t)\right) = mg\left(a\frac{d}{dt}u(t)\cos(\xi) - \frac{d}{dt}v(t)\cos(u(t))\sin(\xi)\right). \end{cases} \qquad (3)$$

Results and discussion. Some results of numerical experiments on the analysis of the trajectories$\boldsymbol{r}(t)$of particle and its speed$V(t)$depending on the radius of the cylinder a, its angle of inclination$\xi - 90°$, initial velocity$V_o$and direction$\alpha_o$of throwing particles, its initial position$u_o, v_o$and the coefficient of friction$f$shown in Fig. 1.Thus, the shape of the trajectories $\mathbf{r}(t)$of the particles on the vertical cylinder essentially depends on the angle$\alpha_o = 0°, 45°, 90°, 120°$throwing them over time t all the particles gain speed$V(t)$gravity (Fig. 1 ,and). For horizontal cylinder $\xi = 90°$to the motion of a particle to its full stop significantly depends on its initial positionя$[u_o, v_o]$on the surface, which affects the possible lead particles from the surface (Fig. 1b). Thus, the initial particle position$u_o = 0.9\pi$and throwing angle$\alpha_o = 60°, 90°$tookawayfrom horizontal surface of the cylinder at the time$t \approx 0.24c$ -will free fall. At this point the normal reaction force$F_N(t)$s zero, and graphics speed$V(t)$shows the velocity of separation from the surface. Particles with an angle throwing$\alpha_o = 0°, 30°$will not took of from the surface of horizontal cylinder. The nature of a particle on the inner surface of the cylinder includes advanced features like the vertical cylinder and horizontal, to which substantially affects the angleξdeviation from its vertical position [1]. For example, particles thrown on old$\xi = 80°$cylinder angle with the same throwing$\alpha_o = 60°$, initial velocity$V_o = 4$ м/сand friction$f = 0{,}3;\ 0{,}45$stop on its surface, and particles of friction$f = 0{,}01;\ 0{,}15$after zigzag reduce their speed to a certain size it will begin to type (Fig. 1, B). For horizontal and inclined cylinders probable movement of particles along its outer surface, where centrifugal force$F_C = m\,V^2\,k$are not pin it to the surface [2], but rather took it off to the top of the cylinder (Fig. 1, d) .

For vertical cylinder there is a correspondence between the provisions of the particles on the surface and parameters: u – the distance on a straight line generators; α - direction of movement (the angle between the tangent$\boldsymbol{\tau}(\alpha)$and rectilinear generators). We derive the following laws of motion of particles in rough inner surface of the vertical cylinder projection on orts***T*** and***P***trihedron***OTPN***according to function parameter$u$:

$$\begin{cases} OT := mV(u)\frac{d}{du}V(u) = -mg\frac{d}{du}v(u) - \frac{mfaV(u)^2}{a^2+\left(\frac{d}{du}v(u)\right)^2}, \\ OP := mV(u)^2 = -\frac{mg\left(a^2+\left(\frac{d}{du}v(u)\right)^2\right)}{\frac{d^2}{du^2}v(u)} \end{cases}, \qquad (4)$$

and according to function parameterα:

$$\begin{cases} OT := mV(u)\frac{d}{du}V(u) = -amg\,ctg(\alpha(u)) - fV(u)^2 \sin(\alpha(u)), \\ OP := mV(u)^2 = \frac{amg}{\frac{d}{du}\alpha(u)} \end{cases} \quad (5)$$

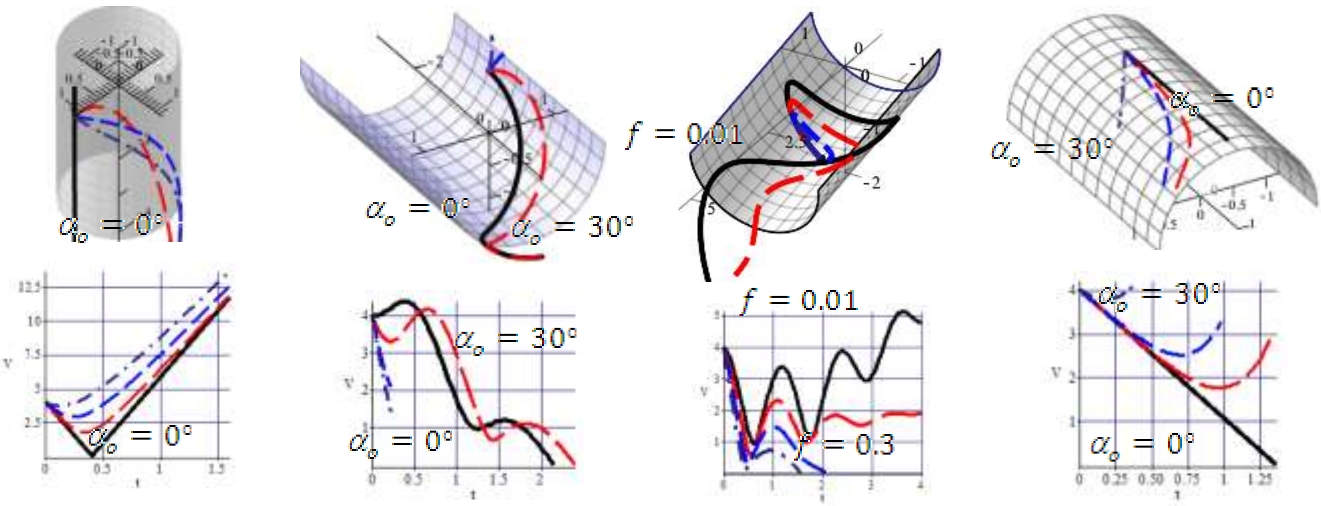
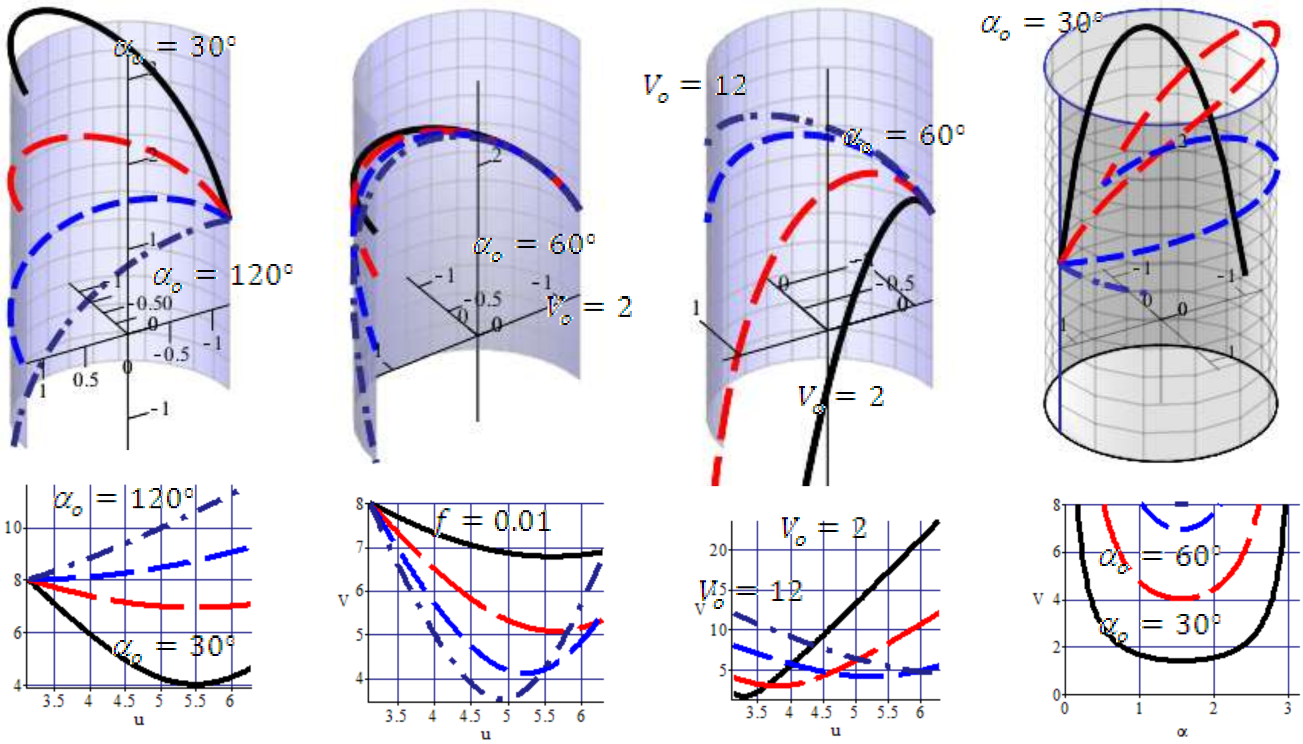


Fig. 1. Trajectories *r (t)* and graphics speed *V (t)* particles on the rough surface of the cylinder rotation to its various provisions in space.

The trajectories $r(u)$and graphics speed $V(u)$of particles on the rough surface vertical cylinder based on: a) throwing angle $\alpha_o = 30°, 60°, 90°, 120°$at constant values of$f = 0{,}3$, $V_o = 8$ м/с і $a = 1{,}2$; b) the coefficient of friction$f = 0{,}01$ $0{,}15$; $0{,}3$; $0{,}45$at constant values$V_o = 8$ м/с і $\alpha_o = 60°$; c) the initial velocity$V_o = 2, 4, 8, 12$ м/с at constant values$f = 0.3$і $\alpha_o = 60°$, built in Fig. 2. We can see that particles thrown at an angle of$\alpha_o = 30°$ minimum speed is$V_{min} \approx 2.8$ м/сwhen the particle was at a distance$u \approx 4.9$rectilinear generators. Particles thrown straight generators perpendicular to the cylinder ($\alpha_o = 90°$), first reduce their speed to a certain size$V_{min} \approx 5.8$ м/с, and then gaining it. The higher the coefficient of friction *f*, the lower the value of the minimum speed of the particles and the faster it will fall. With decreasing initial velocity particles$V_o$shortened time to his downfall. Fig. 7 g built trajectory$r(\alpha)$and graphics speed$V(\alpha)$particles on the inner surface of the vertical cylinder on the interval$[\alpha_o;\ 180° - \alpha_o]$depending on the angle of throwing $\alpha_o = 30°, 60°, 90°, 120°$at constant values $f = 0.3$, $V_o = 8$ м/с i $a = 1.2$, which unlike previous cases graphics speed $V(\alpha)$ are symmetrical.

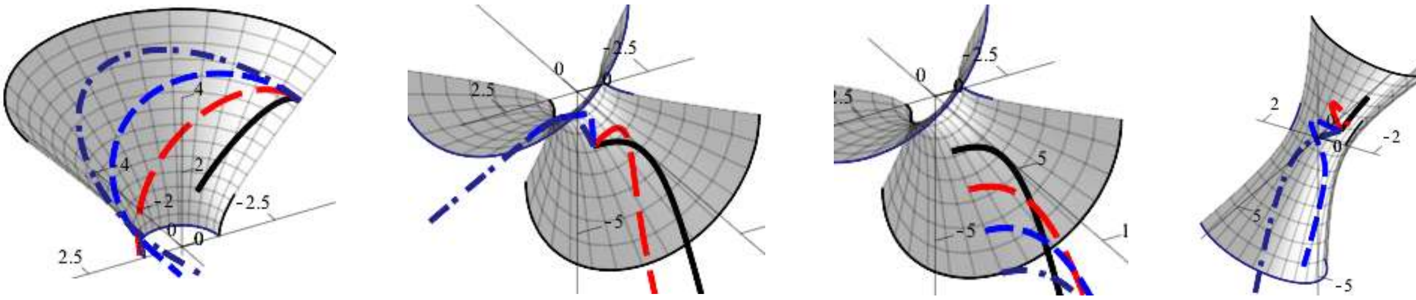


а) б) в) г)

Fig. 2. The trajectories of particle velocity and graphics on the inner surface of the vertical cylinder parameters:

а,б,в) u- position; г) α- direction

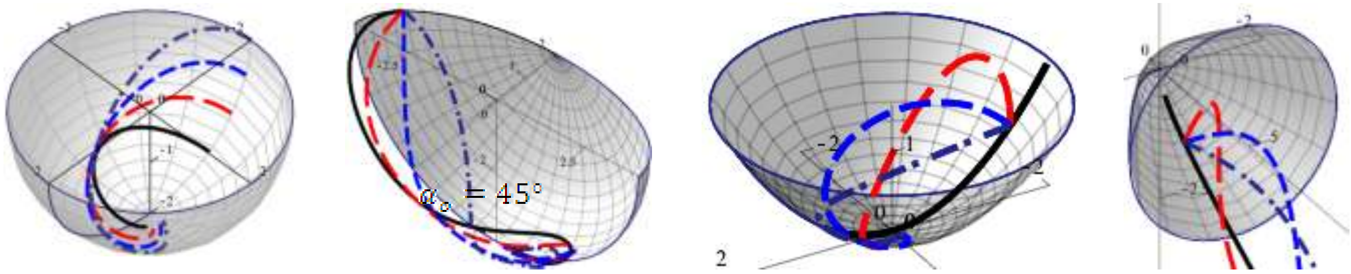
Based on the proposed method of forming laws of motion of particles on rough surfaces in the interior of coordinates projected on orty accompanying trihedron of trajectory was set maple-established model of a particle in all ruled rough surfaces 2nd order. In order to assess each of these surfaces to perform separation or movement of bulk material conducted experiments with complex computing trajectory analysis, kinematic properties of a particle at different initial conditions. In particular, the trajectory $r(t)$ on the vertical rotation hyperboloid odnoporozhnynnomu not cross his neck as detached particles (Fig. 3 a). Hyperboloid horizontal axis at certain ratios initial velocity V_o particle, its provisions $[u_o, v_o]$, throwing angle α_o and friction coefficient f, may be used for the separation of loose material into two fractions, separated by a neck (Fig. 3, b, in). Bulk material thrown on top of the slope of hyperboloid [3], will initially accumulate around the neck, which after passing trajectories of particles are closer to each other (Fig. 3, g).



а) б) в) г)

Fig. 3. trajectories *r (t)* particles on the rough surface odnoporozhnynnoho rotation hyperboloid different position in space

Automatic execution of analytical transformations in the formation law of motion of a particle on the rough surface of the sphere, ellipsoid, paraboloid [4] and hyperboloid of rotation arbitrary position is so cumbersome that they illustrate is impractical - the user sets only the initial experimental conditions and selects appearance presenting the results (Fig. 4).



а) б) в) г)

Fig. 9. trajectories *r (t)* particles on rough surfaces:

a) scope; b) ellipsoid;b) paraboloid; c) hyperboloid of rotation

Conclusions

A set of computer models of automatic generation of a particle of the law as a system of two differential equations of 2nd order for any rough surface randomly located in space. It was developed through computer tools has become possible to perform online research trajectory-kinematic characteristics of a particle on all surfaces 2nd order, cylindrical surfaces with orthogonal sections as transcendental curves of helical surfaces.

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КОМП'ЮТЕРНІ МОДЕЛІ РУХУ ЧАСТИНКИ ПО НЕРУХОМИХ ШОРСТКИХ ПОВЕРХНЯХ 2-ГО ПОРЯДКУ

А. В. Несвідомін

Анотація. *Мета дослідження – розробка Maple-моделей руху частинки по всіх поверхнях 2-го порядку. На основі запропонованого методу формування законів руху частинки по шорстких поверхнях у внутрішніх їх координатах в проекціях на орти супровідних тригранників траєкторії було створено набір maple-моделей руху частинки по всіх лінійчатих шорстких поверхнях 2-го порядку. Завдяки розробленому комп'ютерному інструментарію стало можливим в інтерактивному режимі виконати дослідження траєкторно-кінематичних характеристик руху* *частинки по всіх поверхнях 2-го порядку, циліндричних поверхнях з ортогональними перерізами у вигляді трансцендентних кривих, гвинтових поверхнях. Це дало можливість аналізувати рух частинок при переміщенні сипкої сировини трубопроводами, подачі мінеральних добрив туконапрямниками до розсіювального диска, сепарування зернового вороху нахиленими коливальними площинами.*

Ключові слова: ***матеріальна частинка, шорстка поверхня, циліндр, однопорожнинний гіперболоїд обертання, параболоїд обертання***

КОМПЪЮТЕРНЫЕ МОДЕЛИ ДВИЖЕНИЯ ЧАСТИЦЫ ПО НЕПОДВИЖНЫМ ШЕРОХОВАТЫХ ПОВЕРХНОСТЯХ 2-ГО ПОРЯДКА

А. В. Несвидомин

Аннотация. *Цель исследования - разработка Maple-моделей движения частицы по всем поверхностям 2-го порядка. На основе предложенного метода формирования законов движения частицы по шероховатым поверхностям во внутренних их координатах в проекциях на орты сопроводительных трёхгранников траектории было создано набор Maple-моделей движения частицы по всем линейчатых шероховатых поверхностях 2-го порядка. Благодаря разработанному компьютерному инструментарию, стало возможным в интерактивном режиме выполнить исследования траекторно-кинематических характеристик движения частицы по всем поверхностям 2-го порядка, цилиндрических поверхностях с ортогональным сечением в виде трансцендентных кривых, винтовых поверхностях. Это дало возможность анализировать движение частиц при перемещении сыпучего сырья по трубопроводам, подачи минеральных удобрений туконаправителями к рассеивающему диску, сепарирования зернового вороха наклоненными колебательными плоскостями.*

Ключевые слова: ***материальная частица, шероховатая поверхность, цилиндр, однопустотелый гиперболоид вращения, параболоид вращения***