UDC 631.312 DOI 10.31548/energiya1(65).2023.106

Construction of a linear surface according to the calculated trajectory of the movement of a material particle along it

S. F. Pylypaka, Doctor of Technical Sciences, Professor

A.V. Nesvidomin, Candidate of Technical Sciences, Docent

National University of Life and Environmental Sciences of Ukraine

E-mail: a.nesvidomin@gmail.com

Abstract. *Currently, studies of the movement of agricultural materials on gravity surfaces have been carried out. These works also indicate the possibility of solving the inverse problem - the construction of such a surface that would ensure the given trajectory of the particle's movement. In the article, we will consider the construction of a linear surface, which ensures the movement of a particle along a slant line. The property of such lines is a constant angle between the tangent line drawn to the curve at any point and the horizontal plane (the angle of rise of the curve), as well as the parallelism of the main normal of the curve to the horizontal plane*

The purpose of the research is to construct a linear surface according to the calculated trajectory of the movement of a material particle along it.

The construction of a linear surface according to a given trajectory of the movement of a material particle under the action of the force of its own weight is considered.

A system of equations was obtained that describes the movement of a material particle along a linear gravitational surface.

The considered example is the construction of a linear surface, which, with a known coefficient of friction, would ensure the movement of a particle along a helical line given by the angle of elevation and a constant curvature.

An example of the construction of a linear surface is also given, which, with a known coefficient of friction, would ensure the accelerated movement of a particle along the surface with a constant angle.

The results of the conducted research make it possible to construct a linear surface according to a given trajectory of the movement of a material particle under the action of its own weight.

Key words*: linear surface, motion trajectory, helical line*

Topicality. In many tasks during the construction of agricultural machines, the question of constructing a linear surface according to the calculated trajectory of the movement of a material particle along it arises.

Analysis of recent research and publications. Finding the trajectories of movement of material particles along gravitational surfaces is considered in works [1, 2]. Research concerns the movement of agricultural materials on working surfaces. These works also indicate the possibility of solving the inverse problem - constructing such a surface that would ensure a given trajectory of the particle's movement. In the article, we will consider the construction of a linear surface, which ensures the movement of a particle along a slant line. A property of such lines is a constant angle β between the tangent line drawn to the curve at any point and the horizontal plane (the angle of rise of the curve), as well as the parallelism of the main normal of the curve to the horizontal plane [3]. These two properties greatly facilitate surface design.

The purpose of the research is to construct a linear surface according to the calculated trajectory of the movement of a material particle along it.

Materials and methods of research. Let a material particle move along a spatial curved line. Let's attach to the particle the accompanying Frenet trihedron, whose ort $\bar{t}$ is tangent to the curve, and the ortho is the main normal $\bar{n}$ and binormals $\bar{b}$ are in the normal plane τ (Fig. 1, a). It is known from geometry that it is possible to choose such a direction of projection in which the normal plane τ is projected into a straight line. As can be seen from Fig. 1, b, which shows this case, the weight force *mg* (*m* is the mass of the particle, $g = 9.81\ m/s^2$), can be decomposed into two components: one component is directed along the center $\bar{t}$ (it causes the movement of the particle), the other is in the normal plane τ.

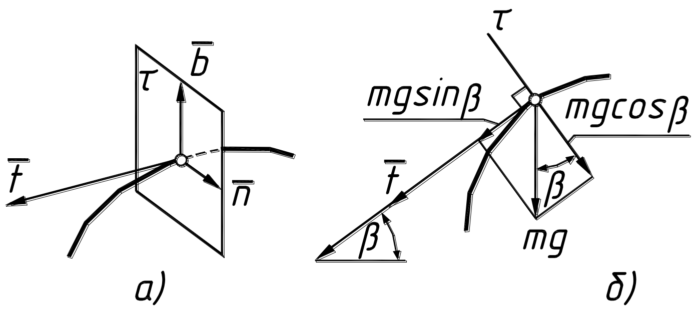

Fig. 1. Accompanying trihedron of the Frenet curve – the trajectory of the particle movement:
a - the normal plane τ is located arbitrarily in space;
b - the normal plane τ is projected into a straight line

Now we will choose the projection direction in such a way that the ort $\bar{t}$ is projected into a point. This case is shown in Fig. 2. As it was said earlier, the angle β= *const* , so the ortho of the main normal $\bar{n}$ at all points of the trajectory is parallel to the horizontal plane. Two forces act on a material particle in the normal plane: the component of the force of gravity $mg\cos\beta$ and centrifugal force mv^2k (v is the velocity of the particle, k is the curvature of the trajectory). Let's start constructing the surface. Let's draw a plane μ tangent to the future surface through the ort $\bar{t}$.

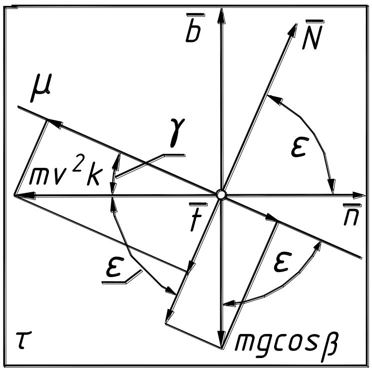

Fig. 2. Decomposition of the acting forces in the normal plane of the accompanying trihedron of the trajectory

In Fig. 2, it is projected in a straight line. Since the constructed surface is linear, the rectilinear surface will be the result of the intersection of the normal plane τ and the tangent plane μ. In order for the particle to move exactly along the specified line, it is necessary that at each point of the trajectory the projections of the force of gravity and the centrifugal force on the tangent plane μ are balanced. Based on Fig. 2, we can write:

$$mv^2k\sin\varepsilon = mg\cos\beta\cos\varepsilon, \tag{1}$$

where ε is the angle between the main normal $\bar{n}$ of the trajectory and the normal $\bar{N}$ to the surface. We project onto the ortho $\bar{t}$ the forces that give the particle acceleration $\frac{dv}{dt}=\frac{dv}{ds}\frac{ds}{dt}=v\frac{dv}{ds}$ (t *is* time, s is the length of the arc of the trajectory):

$$mv\frac{dv}{ds} = mg\sin\beta - fR, \tag{2}$$

where f is the coefficient of friction; R is the pressure exerted by the particle on the surface. After projecting the centrifugal force and the weight force on the normal to the surface, we obtain the pressure force R (see Fig. 2):

$$R = mg\cos\beta\sin\varepsilon + mv^2k\cos\varepsilon. \tag{3}$$

By substituting (3) into (2) and reducing the resulting equation, as well as equation (1) by the mass m , we obtain a system of equations that describes the movement of a material particle along a linear gravitational surface:

$$\begin{cases} v\dfrac{dv}{ds} = g\sin\beta - f(g\cos\beta\sin\varepsilon + v^2 k\cos\varepsilon); \\ v^2 k\sin\varepsilon = g\cos\beta\cos\varepsilon. \end{cases} \tag{4}$$

System (4) includes four unknown quantities: dependencies $v = v(s)$; $\varepsilon = \varepsilon(s)$; $k = k(s)$ and the angle β (the coefficient f is assumed to be known). Therefore, some values must be set, and the rest must be found from system (4). If we project the surface according to a predetermined trajectory of the movement of the particle, this means that the trajectory itself must be set by the angle β and its curvature $k = k(s)$, and the dependencies $v = v(s)$ and $\varepsilon = \varepsilon(s)$ to find from the solution of system (4). Let's consider examples.

Results of the studies and their discussion. *Example 1.* Construct a linear surface that, with a known coefficient of friction f, would ensure the movement of a particle along a helical line given by the angle of elevation β and a constant curvature $(k = const)$.

From the second equation of system (4), we write:

$$v^2 = \frac{g}{k}\cos\beta\,\mathrm{ctg}\,\varepsilon. \tag{5}$$

We differentiate equation (5) with respect to the parameter s and write it in the form:

$$v\frac{dv}{ds} = -\frac{g\cos\beta}{2k\sin^2\varepsilon}\frac{d\varepsilon}{ds}. \tag{6}$$

Substituting (5) and (6) into the first equation of system (4), after simplifications we obtain the differential equation:

$$-\frac{\cos\beta}{2k\sin^2\varepsilon}\frac{d\varepsilon}{ds} = \sin\beta - f\frac{\cos\beta}{\sin\varepsilon}. \tag{7}$$

The differential equation (7) must be integrated by numerical methods, but under the condition that the surface is perfectly smooth $(f = 0)$, the variables are separated and after integration we obtain the dependence $\varepsilon = \varepsilon(s)$ in the final form:

$$\mathrm{ctg}\,\varepsilon = 2\mathrm{tg}\beta\cdot ks \quad \text{or} \quad \varepsilon = \mathrm{arcctg}(2\mathrm{tg}\beta\cdot ks). \tag{8}$$

By substituting (8) into (5), we obtain the dependence of the change in the speed of particle movement $v = v(s)$:

$$v^2 = 2g\sin\beta \cdot s = 2gH, \tag{9}$$

where $H = \sin\beta \cdot s$ is the height to which the particle descends during movement. The obtained result corresponds to Galileo's theorem, according to which the speed of a particle when moving along a perfectly smooth surface does not depend on the shape of the trajectory, but on the height *H.* From dependence (8), it can be concluded that at the initial moment of motion *(s = 0),* the angle, $\varepsilon = 90^o$, i.e., the rectilinear generating surface, is parallel to the horizontal plane. Then the angle ε decreases, approaching zero, which means that the surface is gradually approaching the surface of the binormals of the helical line.

For a real surface $f \neq 0$. In this case, as shown in work [2], with the help of numerical integration, the particle is first accelerated, and then its motion stabilizes and the trajectory is a line close to a helical one. It is obvious that the increase in speed occurs up to a certain limit, until the component of the force of gravity is balanced by the force of friction. In the future, the speed will be constant and the left side of the first equation of system (4) will be equal to zero. Taking into account the mentioned solution of system (4) gives the result:

$$\sin\varepsilon = f\mathrm{ctg}\beta; \qquad v^2 = \frac{g\sin\beta}{fk}\sqrt{1 - f^2\mathrm{ctg}^2\beta}. \tag{10}$$

The next stage is the design of the surface. The parametric equations of the slope line given by the elevation angle β and curvature $k = k(s)$ have the form [3]:

$$x = \cos\beta\int\cos\left(\frac{1}{\cos\beta}\int k ds\right)ds; \quad y = \cos\beta\int\sin\left(\frac{1}{\cos\beta}\int k ds\right)ds; \quad z = s\sin\beta. \tag{11}$$

Since $k = const$, the integration of equations (11) leads to the parametric equations of the helical line:

$$x = \frac{\cos^2\beta}{k}\sin\frac{ks}{\cos\beta}; \qquad y = -\frac{\cos^2\beta}{k}\cos\frac{ks}{\cos\beta}; \quad z = s\sin\beta. \tag{12}$$

The helical line (12) is located on a cylinder of radius $r = \frac{\cos^2\beta}{k}$ and has a step. $h = \frac{\pi}{k}\sin 2\beta.$ To construct a helical surface, it is necessary to draw a straight line at an angle $\gamma = 90^0 - \arcsin(f\operatorname{ctg}\beta)$ (Fig. 2) to the main normal, which is parallel to the horizontal plane, in the normal plane of each point of the helical line. Omitting the derivation, we present the parametric equations of the described surface:

$$\begin{aligned} X &= \frac{\cos^2\beta}{k}\sin\frac{ks}{\cos\beta} - u\left(f\operatorname{ctg}\beta\sin\frac{ks}{\cos\beta} + \sqrt{1 - f^2\operatorname{ctg}^2\beta}\sin\beta\cos\frac{ks}{\cos\beta}\right); \\ Y &= \frac{\cos^2\beta}{k}\cos\frac{ks}{\cos\beta} - u\left(f\operatorname{ctg}\beta\cos\frac{ks}{\cos\beta} - \sqrt{1 - f^2\operatorname{ctg}^2\beta}\sin\beta\sin\frac{ks}{\cos\beta}\right); \\ Z &= -\left(s\sin\beta + u\sqrt{1 - f^2\operatorname{ctg}^2\beta}\cos\beta\right), \end{aligned} \qquad (13)$$

where s , u are variable parameters of the surface, respectively, the length of the arc of the trajectory and the rectilinear generator. In Fig. 3, a helical linear surface is constructed according to equations (13). The trajectory of the particle is shown by a double line. Initial data: $k = 0.1$; $\beta = 20^0$; $f = 0.3$. Calculated values according to formulas (10): $\varepsilon = 55.5^0$; $v = 7.96$ *m/s*. The trajectory of the movement is a spiral line on a cylinder with a radius of $r = 8.8$ *m* and a pitch of $h = 20.2$ *m*. The constructed surface differs from the surface considered in the work [2, p. 327] in that the rectilinear generators are tangential to the axis of the spiral line while in the specified work they cross the axis.

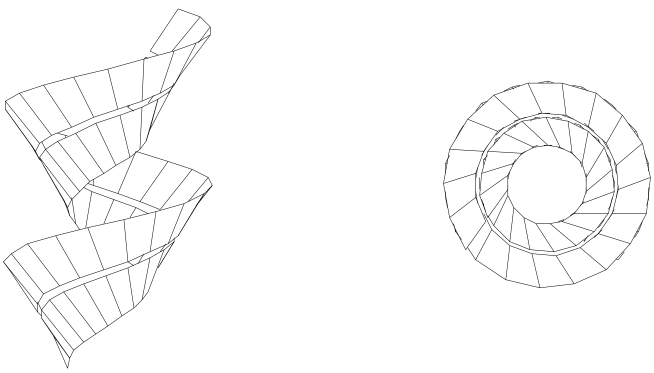

Fig. 3. Helical linear surface that ensures a constant speed of movement of a particle along it (on the left – frontal projection, on the right – horizontal)

*Example 2.*_Construct a linear surface that, with a known coefficient of friction *f*, would ensure the accelerated movement of a particle along the surface with a constant angle $\varepsilon(\varepsilon = const)$.

solution process will be the same as in the previous example. Differentiation of expression (5) taking into account that $\varepsilon = const$, $k = k(s)$ gives:

$$v\frac{dv}{ds} = -\frac{g\cos\beta\cos\varepsilon}{2k^2\sin\varepsilon}\cdot\frac{dk}{ds}. \quad (14)$$

After substituting (5) and (14) into the first equation of system (4), we obtain a differential equation that can be integrated:

$$\frac{dk}{k^2} = -\frac{2}{\cos\varepsilon}(\mathrm{tg}\beta\sin\varepsilon - f)ds \text{ where from } k = \frac{\cos\varepsilon}{2(\mathrm{tg}\beta\sin\varepsilon - f)s}. \quad (15)$$

Substitution of the expression of the curvature *k* from (15) into (5) gives the dependence of the change in the speed of particle movement:

$$v^2 = \frac{2g}{\sin\varepsilon}(\sin\beta\sin\varepsilon - f\cos\beta)s. \quad (16)$$

When $f = 0$, expression (16) turns into expression (9). System (4) is solved , because with given constants β, ε and *f* found the curvature of the trajectory in (15) and the velocity of the particle in (16). We look for the shape of the trajectory itself by integrating equations (11) while substituting the curvature expression from (15) into them:

$$\begin{aligned} x &= \frac{a\cos\beta}{1+a^2}\left[\frac{s}{a}\cos(a\ln s) + s\sin(a\ln s)\right]; \\ y &= \frac{a\cos\beta}{1+a^2}\left[\frac{s}{a}\sin(a\ln s) - s\cos(a\ln s)\right]; \\ z &= s\sin\beta, \quad \text{де} \quad a = \frac{\cos\varepsilon}{2(\sin\beta\sin\varepsilon - f\cos\beta)}. \end{aligned} \quad (17)$$

In Fig. 4, a spatial curve is constructed based on equations (17), which is the calculated trajectory of the movement of a material particle and, at the same time, a guide curve for the designed surface. The design of the surface itself is carried out in the same way as in the previous example. Output data: $\varepsilon = 60^0$; $\beta = 20^0$; $f = 0.3$. Calculated values according to formulas (15), (16): $k = \frac{16{,}4}{s}$; $v = 0{,}563\sqrt{s}$.

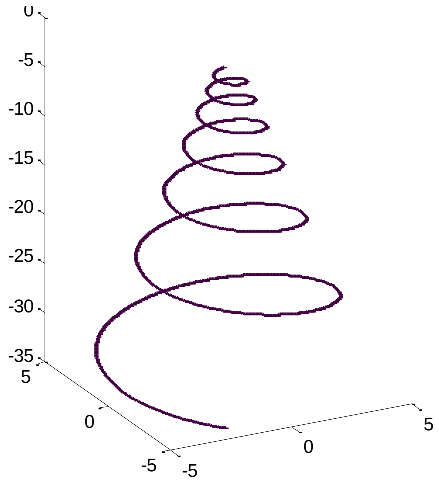

Fig. 4. Conical slope line, which is the trajectory of the movement of a material particle and at the same time the guide curve of a linear surface with a fixed angle ε slope of the generators

Conclusions. The designing of a ruled surface on a planned trajectory of motion of a mass point under operating of force of dead weight is reviewed. The concrete examples are adduced.

List of references

1. Василенко П. М. Теория движения частицы по шероховатым поверхностям сельскохозяйственных машин. К.: УАСХН, 1960. 283 с.

2. Заика П. М. Избранные задачи земледельческой механики. К.: Изд-во УСХА, 1992. 507 с.

3. Войтюк Д. Г., Пилипака С. Ф. Конструювання просторової кривої лінії із заданою кривиною, як траєкторії руху матеріальної точки. Збірник наукових праць Національного аграрного університету. Серія "Механізація сільськогосподарського виробництва". 2001. Т.10. К.: НАУ, 2001. С.74-78.

References

1. Vasilenko, P. M. (1960). Teoriya dvizheniya chastitsy po sherokhovatym poverkhnostyam sel′skokhozyaystvennykh mashin [Theory of movement of particles on rough surfaces of agricultural machines]. Kiyv: UASKHN, 283.

2. Zaika, P. M. (1992). Izbrannyye zadachi zemledel′cheskoy mekhaniki [Selected tasks of agricultural mechanics]. Kyiv: Izd-vo USKHA, 507.

3. Voitiuk, D. H., Pylypaka, S. F. (2001). Konstruiuvannia prostorovoi kryvoi linii iz zadanoiu kryvynoiu, yak traiektorii rukhu materialnoi tochky [Construction of a spatial curved line with a given curvature, as a trajectory of movement of a material point]. Zbirnyk naukovykh prats Natsionalnoho ahrarnoho universytetu. Seriia "Mekhanizatsiia silskohospodarskoho vyrobnytstva", 10, 74-78.

КОНСТРУЮВАННЯ ЛІНІЙЧАТОЇ ПОВЕРХНІ ЗА РОЗРАХУНКОВОЮ ТРАЄКТОРІЄЮ РУХУ МАТЕРІАЛЬНОЇ ЧАСТИНКИ ПО НІЙ

С. Ф. Пилипака, А. В. Несвідомін

Анотація. *Нині проведені дослідження руху сільськогосподарських матеріалів по гравітаційних поверхнях. У цих працях також вказано на можливість розв'язання оберненої задачі – конструювання такої поверхні, яка забезпечила б задану траєкторію руху частинки. У статті розглянемо конструювання лінійчатої поверхні, яка забезпечує рух частинки по лінії укосу. Властивістю таких ліній є сталий кут між дотичною прямою, проведеною до кривої в будь-якій точці, і горизонтальною площиною (кут підйому кривої), а також паралельність головної нормалі кривої до горизонтальної площини*

Мета дослідження - конструювання лінійчатої поверхні за розрахунковою траєкторією руху матеріальної частинки по ній.

Розглянуто конструювання лінійчатої поверхні за заданою траєкторією руху матеріальної частинки під дією сили власної ваги.

Одержано систему рівнянь, яка описує рух матеріальної частинки по лінійчатій гравітаційній поверхні.

Розглянутий приклад побудови лінійчатої поверхні, яка при відомому коефіцієнті тертя забезпечила б рух частики по гвинтовій лінії, заданій кутом підйому і сталою кривиною.

Також наведений приклад побудови лінійчатої поверхні, яка при відомому коефіцієнті тертя забезпечила б прискорений рух частинки по поверхні із постійним кутом .

Результати проведеного дослідження дають можливість конструювати лінійчату поверхню за заданою траєкторією руху матеріальної частинки під дією сили власної ваги.

Ключові слова: ***лінійчата поверхня, траєкторія руху, гвинтова лінія***