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FINDING THE RELATIVE TRAJECTORY OF THE CARGO IN THE BODY OF A VEHICLE WHICH IS GOING DOWN OR UP ON A CURVED SECTION OF THE ROAD

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Abstract. *With a certain assumption, the load, which during relative movement along the bottom of the body makes translational movement, can be taken as a material point. In this case, finding the relative movement of the load in the body when the car moves along the road with a spatial axis is reduced to finding the relative trajectory of the material point.*

In known works, it is proposed to use the accompanying trihedron of the trajectory of the transfer motion and Frenet's formula to find the absolute acceleration vector of a point. The independent variable is not time, as in traditional problems, but the arc coordinate of the transfer trajectory, since only under this condition can Frenet's formulas be applied. The vector of absolute acceleration is obtained in projections onto the vertices of the moving accompanying trihedron, without looking for individual components (transfer, relative, and Coriolis acceleration), as is done in traditional approaches.

The purpose of the study is to investigate the relative movement of the load, taken as a material point, along the bottom of the body of a truck moving along a curvilinear section of the road with a spatial axis during its ascent or descent.

The spatial curve is characterized by two parameters that depend on the kinematics of the accompanying Frenet trihedron. These parameters are the curvature k and the twist σ of the curve. Their values at any point of the curve will be determined if the dependencies k=k(s) and σ=σ(s) are known, where s is the arc coordinate (the arc length of the curve). It is assumed that such a spatial curve is the axis of a curved section of the road that goes uphill or downhill.

The application of the accompanying trihedron of the curve and Frenet's formulas made it possible to describe the relative motion of the cargo in the body of a car moving along a road having a spatial axis with constant and variable speeds. The regularities of the relative movement of the cargo when the car is moving on the road, the axis of which is a helical line with constant curvature and twist, have been found. It was found that there is no significant difference between the value of the relative speed of the cargo when the car moves at a constant speed down or uphill at the initial stage. When braking on a descent, a significant relative movement of the cargo is observed, which increases with an increase in the braking acceleration, and when climbing, on the contrary, the relative movement decreases with an increase in the braking acceleration. The regularity of the *relative movement of the cargo when the car moves along the road with variable curvature and torsion of the axis line depends on the type of these dependencies as a function of the length of the arc of the axis line.*

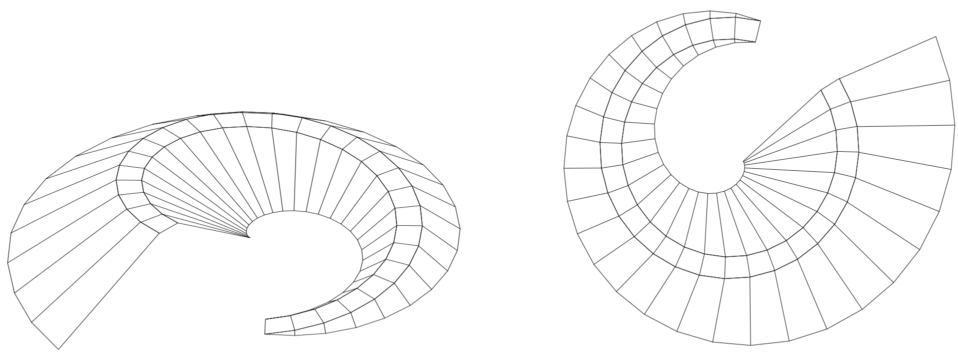
Key words: ***accompanying Frenet trihedron, the bottom of the car body, the trajectory of the load, the speed of the load, the length of the arc of the axis line***

Topicality. With a certain assumption, the cargo, which during the relative movement along the bottom of the body performs translational movement, can be taken as a material point. In this case, finding the relative movement of the cargo in the body when the car moves along the road with a spatial axis is reduced to finding the relative trajectory of the material point. This task is not simple, since the absolute acceleration of a point is the geometric sum of the transfer, relative and Coriolis acceleration, the direction and module of each of which can be variable depending on the trajectory, path and speed of the car.

Analysis of recent research and publications. In the paper [1], the relative motion of a particle on horizontal discs with and without blades rotating around a vertical axis was investigated. In the paper [2], the relative movement of a particle along the inner surface of a cylinder and other surfaces whose axis of rotation is arbitrarily oriented in space is considered. The movement of a material point along the inner surface of a cone with a vertical axis of rotation is considered in the work [3]. All these works are characterized by the fact that the trajectory of a point rigidly fixed on a moving surface is a circle, that is, the trajectory of the transfer movement is a flat curve of constant curvature. In work [4], the theoretical foundations of the relative motion of a point are considered, if the relative trajectory is a flat curve of variable curvature, and in work [5] – a spatial curve given by curvature and torsion. In the mentioned papers, it is proposed to use the accompanying trihedron of the transfer motion trajectory and Frenet's formula to find the absolute acceleration vector of a point. The independent variable is not time, as in traditional problems, but the arc coordinate of the transfer trajectory, since only under this condition can Frenet's formulas be applied. We obtain the vector of absolute acceleration in the projections onto the vertices of the moving accompanying trihedron, without looking for separate components (transfer, relative, and Coriolis acceleration), as is done in traditional approaches.

The purpose of the study is investigate the relative movement of the cargo, taken as a material point, along the bottom of the body of a truck moving along a curvilinear section of the road with a spatial axis during its ascent or descent.

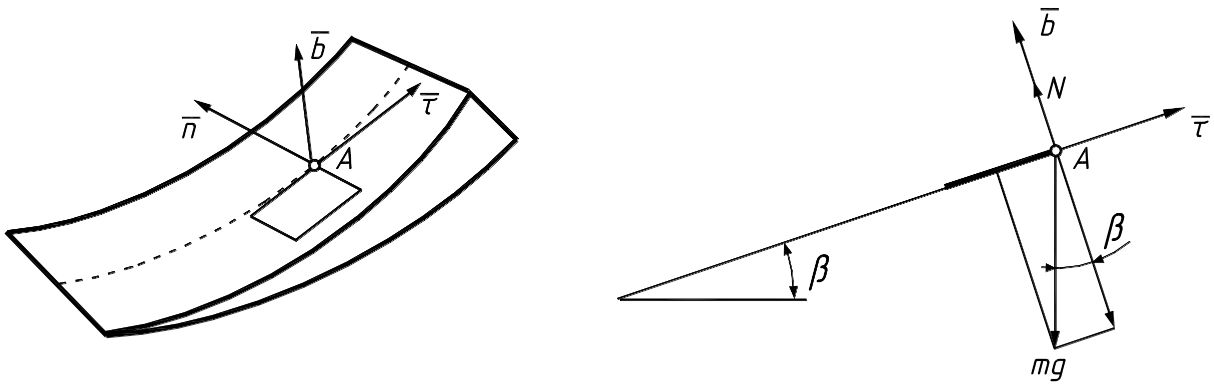
Materials and methods of research. The spatial curve is characterized by two parameters that depend on the kinematics of the accompanying Frenet trihedron. These parameters are the curvature k and the twist σ of the curve. Their values at any point of the curve will be determined if the dependencies $k = k(s)$ and $\sigma=\sigma(s)$ *are known*, where s is the arc coordinate (length of the arc of the curve). We will assume that such a spatial curve is the axis of a curvilinear section of the road going uphill or downhill (Fig. 1). Let's build a corresponding trihedron at point A of the road axis so that the point of the tangent $\bar{\tau}$ is directed in the direction of the car, and the point of the main normal $\bar{n}$ is towards the center the curvature of the curve, ort binormal $\bar{b}$ - up perpendicular to both previous orts. All three unit angles are mutually perpendicular. Let's assume that the rectangle built on the orts extended in the opposite direction $\bar{\tau}$ is $\bar{n}$ the bottom of the truck body (Fig. 2, *a*).



a *b*

Fig. 1. A road section with a curvilinear spatial axis in the form of a spiral line on an earth embankment:
a – axonometric image; *b* – top view

If the Frenet trihedron (car) moves along the axis of the road at a speed v_A, and in its body at this time a material point moves, described in the general case by three equations:



a *b*

Fig. 2. To determine the relative movement of a point in the body of a truck:

a – diagram of the location of the car body relative to the accompanying trihedron of the road axis;

b – diagram of decomposition of the weight of a material point (the main normal $\bar{n}$ is projected into a point, and the bottom of the body is projected into a line segment)

$$\rho_\tau = \rho_\tau(s); \qquad \rho_n = \rho_n(s); \qquad \rho_b = \rho_b(s), \tag{1}$$

where ρ_τ, ρ_n і ρ_b – the projections of a material point in the body of the car onto the orthos of the tangent $\bar{\tau}$, the main normal $\bar{n}$ and the binormal $\bar{b}$, respectively, then the absolute acceleration w of its movement will be determined from the expressions [5]:

$$\begin{aligned} w_\tau &= v_A v'_A(1 - k\rho_n + \rho'_\tau) + v_A^2\left[\rho''_\tau - k'\rho_n - k(k\rho_\tau - \sigma\rho_b + 2\rho'_n)\right]; \\ w_n &= v_A v'_A(k\rho_\tau - \sigma\rho_b + \rho'_n) + v_A^2\left[\rho''_n - k'\rho_\tau - \sigma'\rho_b + k(1 - k\rho_n + 2\rho'_\tau) - \sigma(\sigma\rho_n + 2\rho'_b)\right]; \\ w_n &= v_A v'_A(\sigma\rho_n + \rho'_b) + v_A^2\left[\rho''_b + \sigma'\rho_n + \sigma(k\rho_\tau - \sigma\rho_b + 2\rho'_n)\right]. \end{aligned} \tag{2}$$

Research results and their discussion. In our case, the material point moves along the bottom of the body, therefore $\rho_b = \rho'_b = \rho''_b = 0$. Considering dependencies (1) to be unknown (except for the last one), we will formulate the differential equation of the absolute motion of the point in the form:

$$m\bar{w} = \bar{F}, \tag{3}$$

where m is the mass of the particle; $\bar{w}$ - absolute acceleration vector; $\bar{F}$ - uniform force vector.

Let's write equation (3) in projections onto the orthos of the moving Frenet trihedron. We will take the slope line as the spatial axis of the road, so the angle β, which is the ortho tangent to the horizontal plane, will be constant at all points of the curve (Fig. 2, *b*). Taking this into account, we write:

$$\begin{aligned} mw_\tau &= -F_{T\tau} - mg\sin\beta; \\ mw_n &= -F_{Tn}; \\ mw_b &= -mg\cos\beta + N, \end{aligned} \tag{4}$$

where $F_{T\tau}$ and F_{Tn} - the components of the friction forces in the projections on the corresponding points;

N is the reaction force of the body bottom; $g = 9.81\ m/s^2$.

From the last equation (4), we find the pressure force *N*:

$$N = m(w_b + g\cos\beta). \tag{5}$$

If the coefficient of friction *f* is known, then the force of friction is equal to $F_T = fN$. It is directed in the direction opposite to the direction of relative motion, that is, its direction coincides with the tangent to the relative trajectory. Taking this into account, the components of the friction forces on the orths $\bar{\tau}$ will $\bar{n}$ be written:

$$F_{T\tau} = -\frac{fN\rho'_\tau}{\sqrt{\rho'^2_\tau + \rho'^2_n}}; \qquad F_{Tn} = -\frac{fN\rho'_n}{\sqrt{\rho'^2_\tau + \rho'^2_n}}. \tag{6}$$

Substituting (5) into (6) and (6) into (4), we get:

$$\begin{aligned} mw_\tau &= -\frac{fm(w_b + g\cos\beta)\rho'_\tau}{\sqrt{\rho'^2_\tau + \rho'^2_n}} - mg\sin\beta; \\ mw_n &= -\frac{fm(w_b + g\cos\beta)\rho'_n}{\sqrt{\rho'^2_\tau + \rho'^2_n}}. \end{aligned} \tag{7}$$

We reduce both equations (7) by the mass *m* and substitute the absolute acceleration expressions from (2), taking into account that $\rho_b = \rho'_b = \rho''_b = 0$. After the transformations, we get:

$$\begin{aligned} \rho''_\tau &= k'\rho_n + k(k\rho_\tau + 2\rho'_n) - \frac{f\rho'_\tau}{\sqrt{\rho'^2_\tau + \rho'^2_n}}\left[\frac{v'_A}{v_A}\sigma\rho_n + \sigma'\rho_n + \sigma(k\rho_\tau + 2\rho'_n) + \frac{g}{v_A^2}\cos\beta\right] - \\ &\quad - \frac{g}{v_A^2}\sin\beta - \frac{v'_A}{v_A}(1 - k\rho_n + \rho'_\tau); \\ \rho''_n &= k'\rho_\tau - k(1 - k\rho_n + 2\rho'_\tau) + \sigma^2\rho_n - \\ &\quad - \frac{f\rho'_n}{\sqrt{\rho'^2_\tau + \rho'^2_n}}\left[\frac{v'_A}{v_A}\sigma\rho_n + \sigma'\rho_n + \sigma(k\rho_\tau + 2\rho'_n) + \frac{g}{v_A^2}\cos\beta\right] - \frac{v'_A}{v_A}(k\rho_\tau + \rho'_n). \end{aligned} \tag{8}$$

The system of differential equations (8) includes two unknown functions: $\rho_\tau = \rho_\tau(s)$ and $\rho_n = \rho_n(s)$. Differential characteristics the axes of the road *k = k (s)* and *σ=σ(s),* as well as the speed v_A of the movement of the trihedron (car) must be specified.

Let's consider examples. Let the axis of the road be a helical line with an elevation angle *β*, which is described by parametric equations:

$$x = a\cos\left(\frac{\cos\beta}{a}s\right); \quad y = a\sin\left(\frac{\cos\beta}{a}s\right); \quad z = s\sin\beta, \tag{9}$$

where *a* is a constant value equal to the radius of the circle – the horizontal projection of the helical line (road axis).

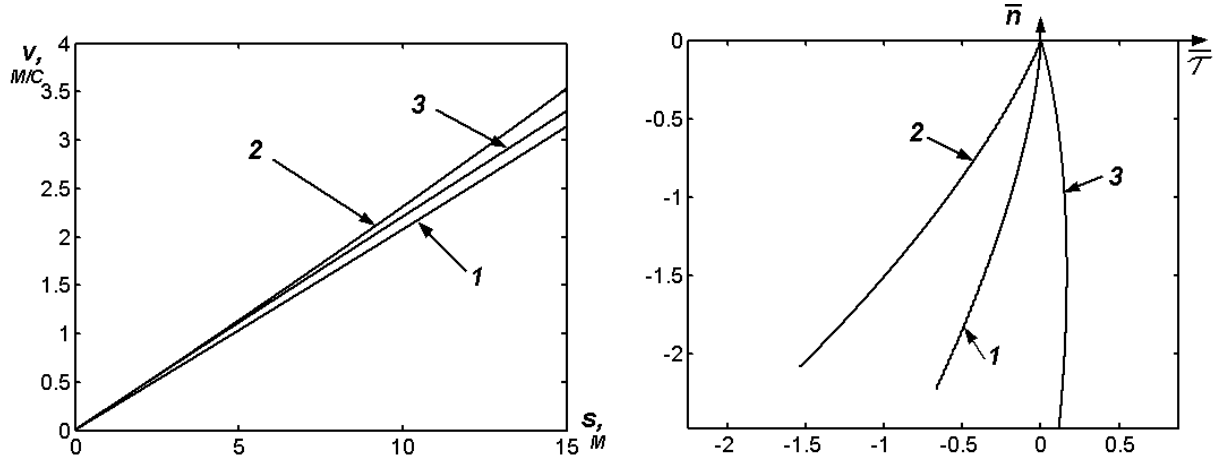
The curvature and twist of the helical line (9) are also constant values, which are found by the well-known formulas:

$$k = \frac{\cos^2\beta}{a}; \qquad \sigma = \frac{\sin\beta\cos\beta}{a}. \tag{10}$$

Substituting (10) $k' = \sigma' = 0$ into (8), we obtain a system of two differential equations of the second order with respect to unknown functions $\rho_\tau = \rho_\tau(s)$ and $\rho_n = \rho_n(s)$. Its solution carried out by numerical methods with initial conditions according to which the position of the cargo before the start of the relative movement coincides with the origin of the coordinates of the trihedron, i.e. the cargo is located in front of the body in the left corner of the bottom (Fig. 2a). Initially, the integration was carried out at a constant speed of the car v_A = *10 m/s* and values of constants *a = 20 m; f=0.3.* The relative trajectory (trace of the cargo on the bottom of the body) was built according to the results of system integration (8), and the relative speed of the cargo according to the formula [5]:

$$v = v_A\sqrt{\rho_\tau'^2 + \rho_n'^2}. \tag{11}$$

In fig. 3, graphs of the relative speeds of the cargo are plotted, and in fig. 3, *b* – relative trajectories of movement along the bottom of the body for different angles *β* when moving the car on a section of the road center line *s = 15 m*.



a *b*

Fig. 3. Graphs of relative speeds and cargo trajectories (1 – $\beta=0^0$ (horizontal road); 2- $\beta=10^0$ (lift); 3 - $\beta=-10^0$ (descent):

a – graphs of relative speeds; *b* – relative trajectories

From the obtained results (Fig. 3), we conclude that there is no significant difference between the value of the relative speeds of the cargo when the car is moving at a constant speed on a horizontal road, down or uphill at the initial stage (within *15 m* of the car's movement). If in the transverse direction to the opposite side in all three cases the cargo moved approximately *2 m* , then the movement along the direction of movement of the car is significantly different: a significant backward movement during ascent and a slight forward movement during descent.

Let's determine the maximum speed of the car at which the relative movement of the cargo is possible. Equating the components of relative motion and their derivatives to zero from the second equation of system (8) we obtain (at the same tim$-\rho'_n\Big/\sqrt{\rho'^2_\tau+\rho'^2_n}=1$, since possible relative movement is directed along the main normal in the direction opposite to its direction):

$$k=\frac{fg}{v_A^2}\cos\beta\,. \tag{12}$$

Substituting the curvature expression (10) into (12), we solve with respect to v_A:

$$v_A=\sqrt{\frac{afg}{\cos\beta}}\,. \tag{13}$$

At a vehicle speed lower than the limit value (13), the relative movement of the cargo is impossible. For our case (*a=20 m; f=0.3*) v_A *=7.67 m/s* for $\beta=0^0$ (horizontal road) and v_A *=7.73 m/s* for $\beta=\pm 10^0$ (ascent or descent). Thus, when climbing or descending, the critical speed should be greater than on a horizontal road. This is explained by the fact that the helical line (road axis) has a smaller curvature than its projection (circle).

The relative movement of the cargo is also possible in the direction of the $\bar{\tau}$ orthogonal tangent at variable speed of the vehicle. In this case, the relative movement of the cargo can begin even with the car moving in a straight line when it reaches a certain acceleration. To find its value, let's equate the components of the relative motion and their derivatives to zero in the first equation of system (8). Taking $-\rho'_\tau \Big/ \sqrt{\rho'^2_\tau + \rho'^2_n} = 1$, since the possible relative motion is directed along the tangent $\bar{\tau}$ in the direction opposite to its direction, we obtain:

$$v_A \frac{dv_A}{ds} = g(f\cos\beta - \sin\beta). \tag{14}$$

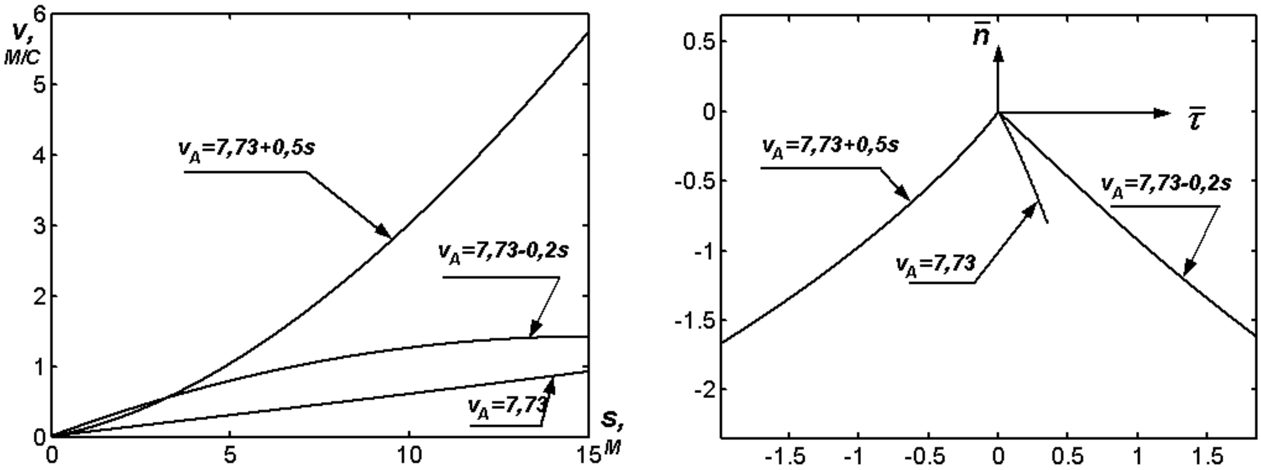
Let us show that the left part of expression (14) is the acceleration as a function of time:

$w_A = \frac{dv_A}{dt} = \frac{dv_A}{ds}\frac{ds}{dt} = v_A \frac{dv_A}{ds}$. Thus, the maximum acceleration of the car, at which relative movement is possible on a straight section of the road, will be determined from the expression:

$$w_A = g(f\cos\beta - \sin\beta). \tag{15}$$

For example, for the received data w_A *=1.2 m/s*^{2} when the car moves uphill ($\beta=10^0$), w_A *=2.9 m/s*^{2} for $\beta=0^0$ (horizontal road) and w_A *= 4.6 m/s*^{2} during descent ($\beta=-10^0$).

Now consider the uniformly accelerated and uniformly decelerated movements of a car on an uphill or downhill slope in the form of a dependence $v_A = v_{A0} + w_A s$ where v_{A0} is its initial speed. As the initial speed, we will take its limit value, found earlier, at which the relative movement of the cargo begins under the action of centrifugal force during ascent or descent.

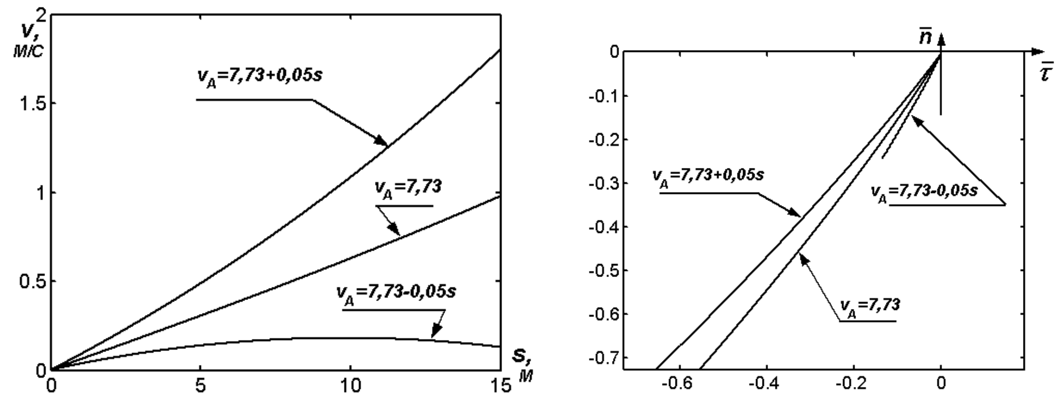


a *b*

Fig. 4. Graphs of relative velocities and trajectories of cargo during uniform, uniformly accelerated and uniformly decelerated downward movement of the car (*β=-10* ^{0}):

a – graphs of relative speeds; *b* – relative trajectories

In fig. 4 graphical dependencies are displayed when the car descends on a curved road. The smallest relative movement of the cargo was carried out at a constant speed of the car. When accelerating the car from this speed with an acceleration *of* $w_A=0.5\ m/s^2$ the cargo moves backwards along the path of the car, and when braking with an acceleration of $w_A=-0.2\ m/s^2$, it moves forward. The picture is somewhat different when lifting the car. In all driving modes, the cargo moves back along the vehicle's path (Fig. 5). Studies have shown that with sharp braking, the value of the relative movement of the cargo approaches zero.

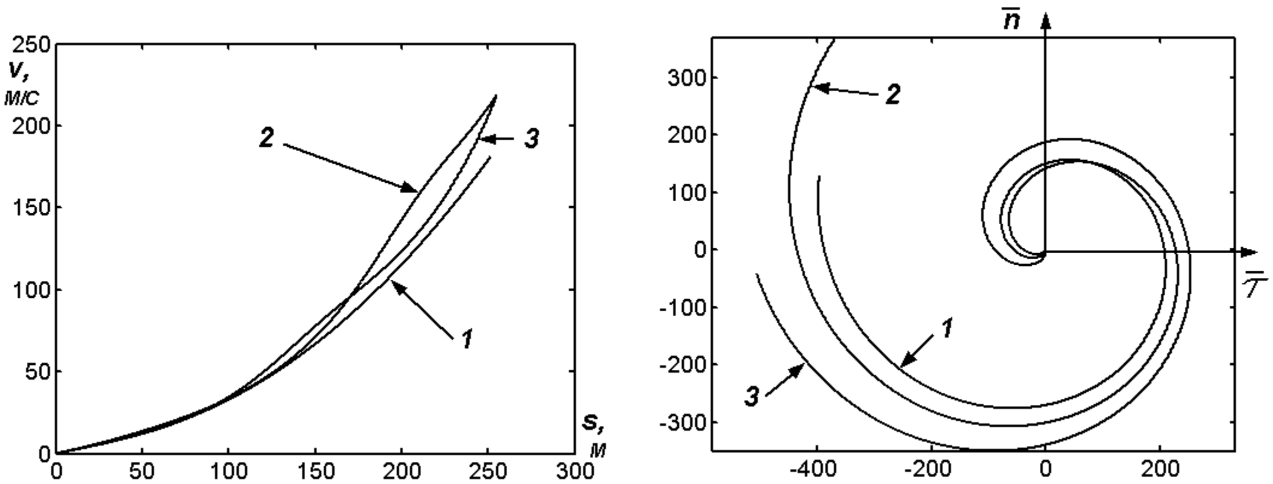


a *b*

Fig. 5. Graphs of relative velocities and trajectories of cargo with uniform, uniformly accelerated and uniformly decelerated movement of the car uphill (*β=10* ^{0}):

a – graphs of relative speeds; *b* – relative trajectories

We considered the relative motion of the cargo in the case when the car moves along the road, the axis line of which is a helical line. This is the only spatial curve of its kind with constant curvature and twist. This, in turn, leads to a certain regularity of the relative movement of the cargo. For example, if a car moves at a constant speed on the road, as if on a helical surface, making successive turns after turns, then the cargo will move in the body all the time, gaining speed at the same time (provided that the bottom of the body is not limited by the sides). In fig. 6 shows the graphical dependences for the relative motion of a cargo in the body of a car moving at a speed of *10 m/s* down, uphill or in a circle, making two complete turns. From fig. 6,b it follows that the longest relative trajectory is described by the cargo on the body of the car moving uphill. There is an explanation for this, because the component of the weight of the cargo *is mg* $\sin\beta$ (Fig. 2, b) acts on the cargo along the tangent $\bar{\tau}$ in the direction opposite to orthogonal, which favors relative motion.



a b

Fig. 6. Graphs of the relative speeds and trajectories of the cargo when the car is moving at a speed of v_A = *10 m/s* (1 – *β=0* ^{0} (horizontal road); 2- *β=10* ^{0} (lift); 3 - *β=-10* ^{0} (descent):

a – graphs of relative speeds; *b* – relative trajectories

Now consider the case when the axis of the road is a spatial slope curve with variable curvature and twist. If we take a chain line as the horizontal projection of such a curve, then the curve itself will be described by parametric equations:

$$x = a\cos\beta\,\mathrm{Arsh}\left(\frac{s}{\mathrm{a}}\right); \quad y = \cos\beta\sqrt{a^2 + s^2}\,; \quad z = s\sin\beta, \tag{16}$$

where *a* is a constant value; *β* is the angle of rise of the curve (also a constant value).

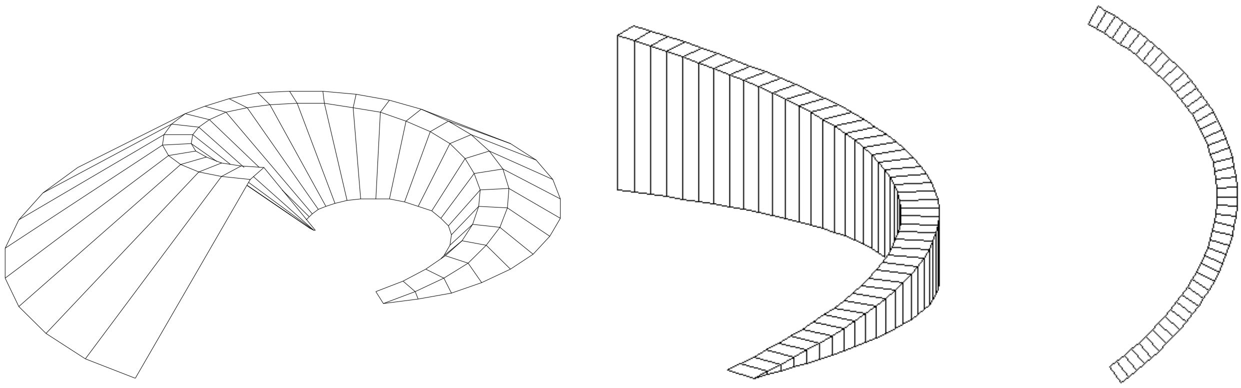


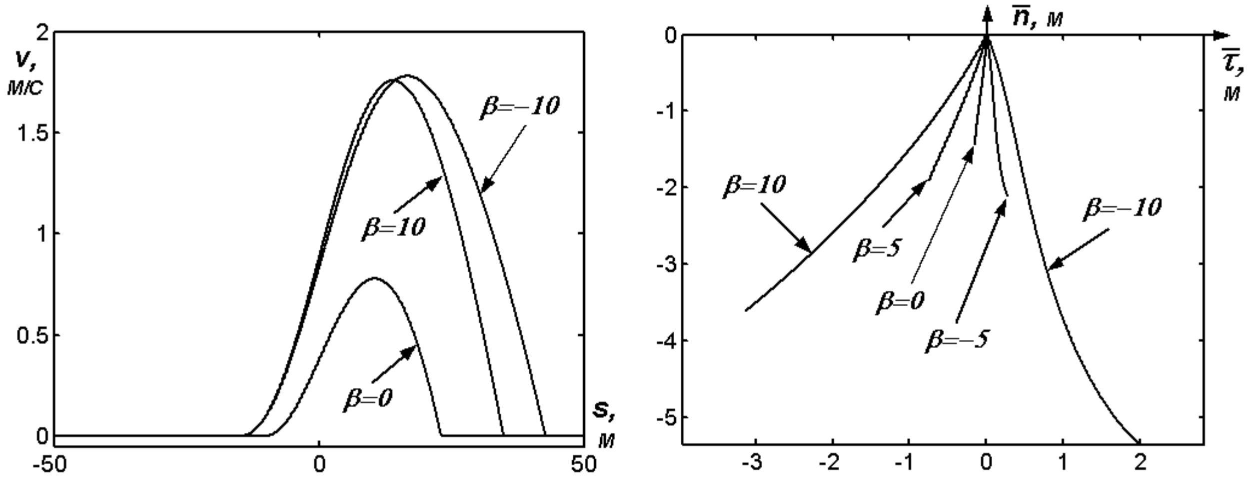
Fig. 7. Axonometric image and top view of a road in which the horizontal projection of the axis line is a chain line

In fig. 7 shows a section of the road, the axis of which is a curve (16) at *a=25 m;* $\beta=10^{\,0}$. To use the system of differential equations (8), it is necessary to have the expressions of curvature and torsion and their derivatives. For curve (16), they will be written:

$$k = \frac{a\cos\beta}{a^2 + s^2}; \qquad \sigma = \frac{a\sin\beta}{a^2 + s^2};$$
$$k' = -\frac{2as\cos\beta}{\left(a^2 + s^2\right)^2}; \qquad \sigma' = -\frac{2as\sin\beta}{\left(a^2 + s^2\right)^2}. \tag{17}$$

It should be noted that substituting (17) into (8) and integrating the system makes it possible to find the relative movement of the cargo at different values of the angle *β*, including $\beta=0^{\,0}$, i.e. for a horizontal road. Such a case was considered in [5] when $v_A=10$ *m/* s The generalized system (8) gives the same results for the same initial conditions in the case of car movement on a horizontal road ($\beta=0^{\,0}$).

Let's consider the movement of a car uphill or downhill with constant and variable speeds. Studies have shown that at a constant speed of the car, the relative movement of the cargo begins on a certain section of the road and stops when the curvature of the axis decreases to a critical level. At the same time, the relative speed and, accordingly, the length is relative trajectories are larger when the car is moving at $\beta\neq0$ (i.e., when going downhill or going uphill). From fig. 8, and it can be seen that the relative movement of the cargo begins at the same point of the road, regardless of the ascent or descent, but continues longer when descending.

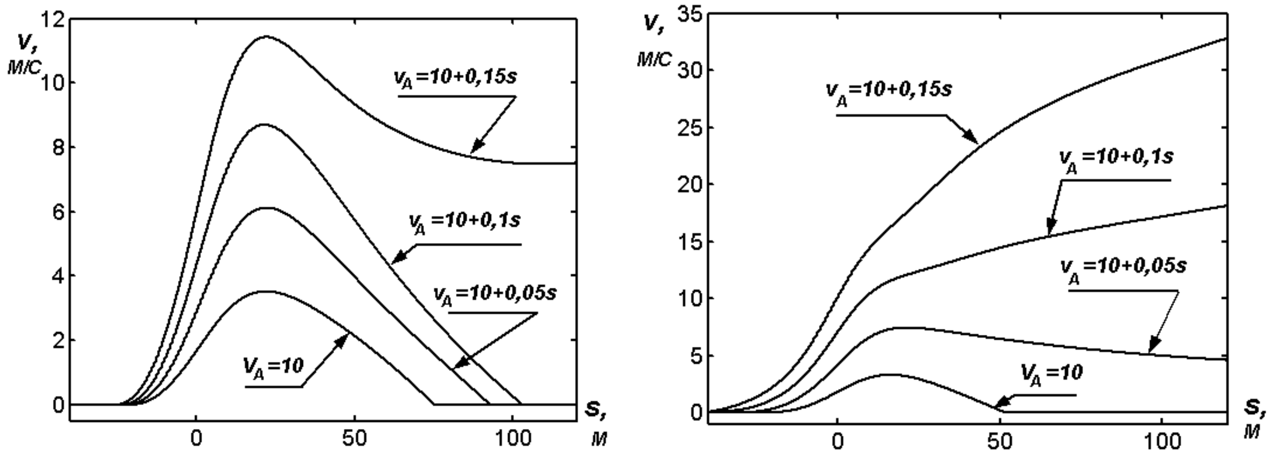


a *b*

Fig. 8. Graphs of relative velocities and trajectories of the cargo when the car is moving at a speed of v_A = *10 m/s* along the road, the axis of which has variable curvature and torsion:

a – graphs of relative speeds; *b* – relative trajectories

When the car accelerates, the relative movement of the cargo is significantly different when the car moves uphill and downhill (Fig. 9). If during the descent the cargo stops with an insufficient amount of acceleration of the car w_A, then during the ascent it sharply gains corresponding trajectories of the relative movement of the cargo
speed with the same modes of movement (for example, even with the minimum acceleration w_A=*0.05 m/s* ^{2} the cargo does not stop). However, if you start braking to lift the car at the initial speed v_A =*10 m/s*, then the cargo will not move relative to the body. Only at the initial speed v_A = *15 m/s* does it start moving and stops even before the car stops (Fig. 10). Studies have shown that with sharp braking when the car descends, it is possible for the cargo to stop relative to the body at the same time as the car stops.



a b

Fig. 9. Graphs of the relative speeds of the cargo at a constant speed and acceleration of the car (*a=25; f=0.3*):

a – descent (β=-10 ^{0}); *b* – rise (β=10 ^{0})

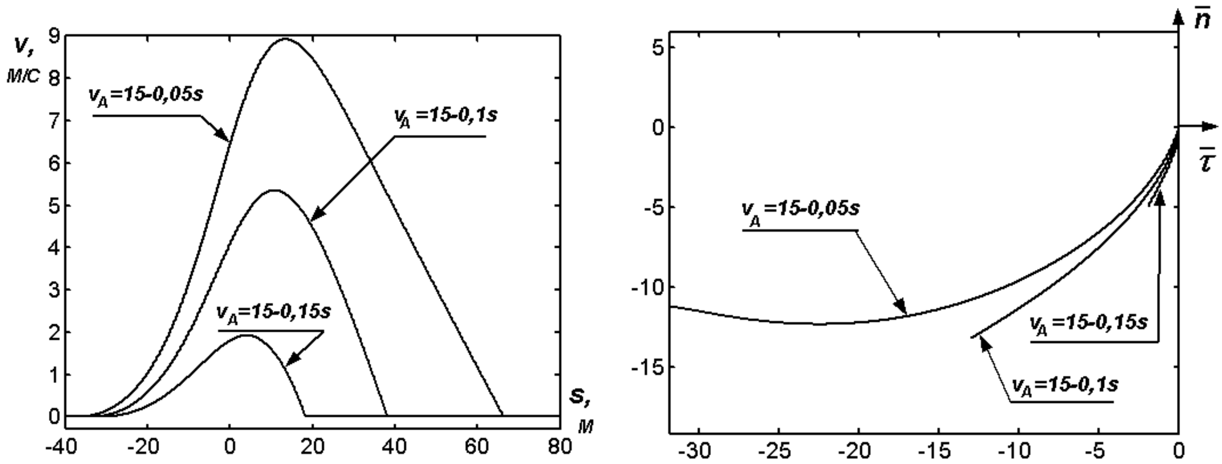


Fig. 10. Graphs of relative velocities during braking on a climb (β=10^0)

Conclusions and perspectives. The application of the accompanying trihedron of the curve and Frenet's formulas made it possible to describe the relative motion of the cargo in the body of a car moving along a road having a spatial axis with constant and variable speeds. The regularities of the relative movement of the cargo when the car is moving on the road, the axis of which is a helical line with constant curvature and twist, have been found. It was found that there is no significant difference between the value of the relative speed of the cargo when the car moves at a constant speed down or uphill at the initial stage. When braking on a descent, a significant relative movement of the cargo is observed, which increases with an increase in the braking acceleration, and when climbing, on the contrary, the relative movement decreases with an increase in the braking acceleration. The regularity of the relative movement of the cargo when the car moves along the road with variable curvature and torsion of the axis line depends on the type of these dependencies as a function of the length of the arc of the axis line.

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ЗНАХОДЖЕННЯ ВІДНОСНОЇ ТРАЄКТОРІЇ РУХУ ВАНТАЖУ В КУЗОВІ АВТОМОБІЛЯ, ЯКИЙ СПУСКАЄТЬСЯ АБО ПІДНІМАЄТЬСЯ КРИВОЛІНІЙНОЮ ДІЛЯНКОЮ ДОРОГИ

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Анотація. *З певним допущенням вантаж, який при відносному переміщенні по днищу кузова здійснює поступальний рух, можна прийняти за матеріальну точку. В* *такому випадку знаходження відносного переміщення вантажу в кузові при русі автомобіля дорогою із просторовою віссю зводиться до знаходження відносної траєкторії матеріальної точки.*

У відомих працях для знаходження векторе абсолютного прискорення точки запропоновано застосовувати супровідний тригранник траєкторії переносного руху та формули Френе. За незалежну змінну прийнято не час, як у традиційних задачах, а дугову координату переносної траєкторії, оскільки тільки за цієї умови можна застосувати формули Френе. Вектор абсолютного прискорення отримується в проекціях на орти рухомого супровідного тригранника, не розшукуючи окремих складових (переносного, відносного і коріолісового прискорень), як це робиться у традиційних підходах.

Мета дослідження - ***д****ослідити відносний рух вантажу, прийнятого за матеріальну точку, по днищу кузова вантажного автомобіля, який рухається криволінійною ділянкою дороги з просторовою віссю при його підйомі або спусканні вниз.*

Просторова крива характеризується двома параметрами, від яких залежить кінематика супровідного тригранника Френе. Такими параметрами є кривина k і скрут σ кривої. Їх значення в будь-якій точці кривої будуть визначені, якщо відомі залежності $k=k(s)$ і $\sigma=\sigma(s)$, де s – дугова координата (довжина дуги кривої). Прийнято , що такою просторовою кривою є вісь криволінійної ділянки дороги, яка йде на підйом або спуск.

Застосування супровідного тригранника кривої та формул Френе дало можливість описати відносний рух вантажу в кузові автомобіля, який рухається дорогою, яка має просторову вісь, із постійною та змінною швидкостями. Знайдено закономірності відносного руху вантажу при русі автомобіля дорогою, віссю якої є гвинтова лінія із постійними кривиною і скрутом. З'ясовано, що суттєвої різниці між величиною відносної швидкості вантажу при русі автомобіля із постійною швидкістю вниз або на підйом на початковому етапі немає. При гальмуванні на спуску спостерігається значне відносне переміщення вантажу, яке росте із збільшенням прискорення гальмування, а при підйомі – навпаки, відносне переміщення зменшується із збільшенням прискорення гальмування. Закономірність відносного переміщення вантажу при русі автомобіля дорогою із змінними кривиною і скрутом осьової лінії залежить від виду цих залежностей у функції довжини дуги осьової лінії.

Ключові слова: ***супровідний тригранник Френе, днище кузова автомобіля, траєкторія руху вантажу, швидкість вантажу, довжина дуги осьової лінії***