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RESEARCH OF THE MOVEMENT OF A MATERIAL PARTICLE ON THE INTERNAL SURFACE OF A VERTICAL CYLINDER THAT PERFORMS PLANETARY MOTION

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Abstract. *Planetary motion of a cylinder is understood as its motion when it is in two rotational motions at the same time: it rotates around its own vertical axis with a constant angular velocity, and the cylinder axis itself rotates with a constant angular velocity around a vertical fixed axis. The movement of the particle will be complex and will consist of its relative movement along the inner surface of the cylinder and the translational movement of the cylinder itself. Such a drive scheme is used in cylindrical sieves for sorting seeds of agricultural crops.*

Problems on the complex motion of a particle can be successfully solved using the trihedron and Frenet's formulas.

The purpose of the study is to establish the complex movement of a material particle along the inner surface of a cylindrical sieve using a trihedron and Frenet's formulas at the same and different angular velocities of transfer and relative rotation of the sieve.

A characteristic feature of the application of the trihedron and Frenet formulas is that the independent variable in them is not time t, as is generally accepted in problems of kinematics and dynamics of a point, but the length of the arc s of the directional curve (in our case, a circle of radius R), so the relationship was established the connection between rotational movements through this parameter.

The system of differential equations is integrated by numerical methods. An exact analytical solution was found in the case when the motion of the particle stabilizes and its speed becomes constant. The obtained results were visualized.

Some regularities of the relative and absolute motion of a particle in a cylindrical sieve were established when the angular velocity of rotation of the cylinder around its own axis is zero and is not equal to zero.

In the first case, it was found that the particle on the surface of the cylinder occupies a position at which it is as far as possible from the axis of rotation of the cylinder around a vertical line and then it moves down the plane of the cylinder uniformly accelerated, uniformly or uniformly decelerated until it "sticks" depending on the value angular velocity.

In the second case, the particle behaves similarly: it remains at the maximum distance after the motion is stabilized. At the same time, it slides along the surface of the cylinder with a constant relative speed along a helical line. The direction of the rise of the *helical line changes to the opposite when the direction of the angular velocity of the cylinder changes. Such movement is possible in a certain range of angular velocities of the particle and the cylinder. As the angular velocity of the particle increases, there comes a moment when it cannot maintain the described state of sliding and begins to move along the surface of the cylinder with stops and is prone to "sticking". This state occurs sooner when the angular velocities of the particle and the cylinder have the same direction, and later when they are directed in opposite directions.*

Key words: ***accompanying Frenet trihedron, cylindrical sieve, particle motion trajectory, cylinder motion velocity, particle motion velocity***

Topicality. Planetary motion of a cylinder is understood as its motion when it is simultaneously in two rotational motions: it rotates around its own vertical axis with a constant angular velocity ω_r, and the cylinder axis itself rotates with a constant angular velocity ω around a vertical fixed axis. The movement of the particle will be complex and will consist of its relative movement along the inner surface of the cylinder and the translational movement of the cylinder itself. This drive scheme is used in cylindrical sieves for sorting seeds of agricultural crops [1].

Analysis of recent research and publications. The theory of the complex movement of a material particle along the rotating surfaces of the working bodies of agricultural machines is considered in fundamental works [1, 2]. In the work [1], the kinematics of a material particle along the inner surface of a cylindrical sieve is described in detail and the relative trajectory of movement on its sweep is constructed. At the same time, it was meant that the angular velocities of both rotational movements have the same direction. In work [3] it is shown that the problems of complex motion of a particle can be successfully solved using the trihedron and Frenet's formulas.

The purpose of research is investigate the complex movement of a material particle along the inner surface of a cylindrical sieve using a trihedron and Frenet's formulas at the same and different angular velocities of translational and relative rotation of the sieve.

Materials and methods of research. Since the axis of the moving cylinder rotates around the fixed axis $\tilde{O}z$, the center of the upper base of the cylinder (point A) describes a circle of radius $\tilde{O}A = R = 1/k$, where k is the curvature of this circle (Fig. 1, a). Let's take this circle as the trajectory of the transferable movement of the cylinder and construct the corresponding Frenet trihedron of this circle at point *A*. Then any point of the cylinder can be written in the projections on the vertices of this trihedron:

$$\rho_\tau = r\cos\psi; \qquad \rho_n = r\sin\psi; \qquad \rho_b = u, \tag{1}$$

where ρ_τ, ρ_n і ρ_b – the projections of the point of the cylinder on the orthos of the tangent $\bar{\tau}$, the main normal $\bar{n}$ and the binormal, $\bar{b}$ respectively;

ψ *is* the angle of rotation and u is the distance along the generating line of the cylinder – variable independent parameters of the cylinder.

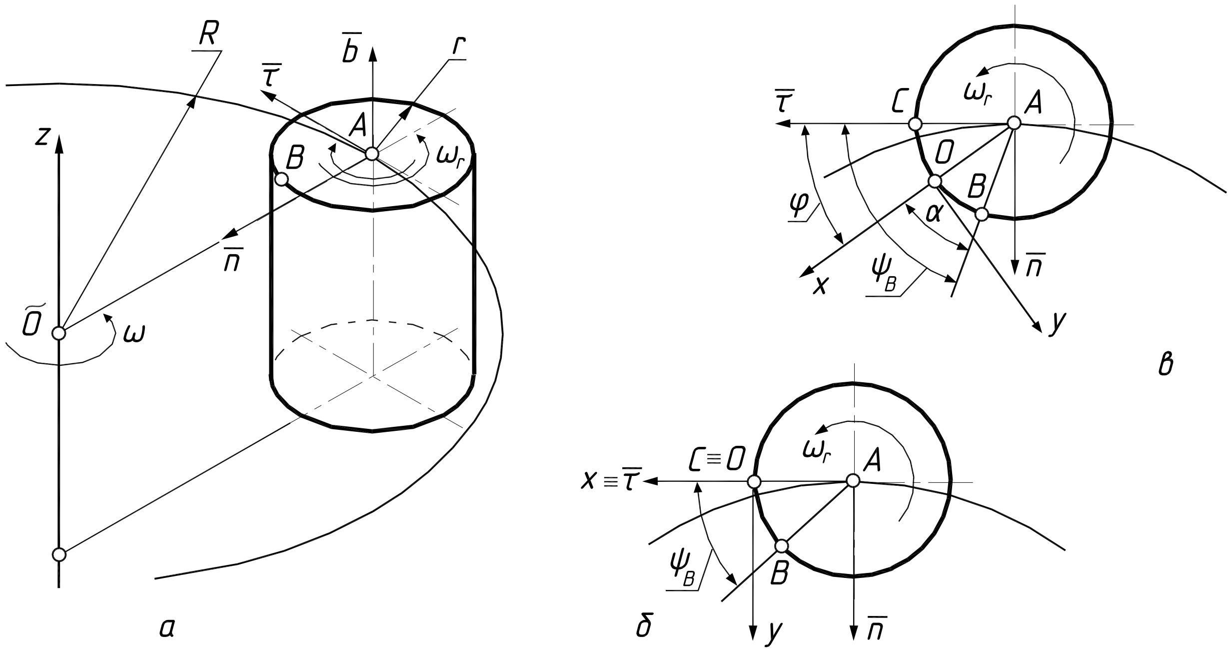


Fig. 1. To determine the relative motion of a particle along the inner surface of a cylinder performing planetary motion:
a - diagram of the planetary motion of the cylinder;
b - the initial moment of the particle's relative motion: the vertices of the companying trihedron and the axis *Oxyz* systems are parallel; the particle is located at the origin of the coordinates of the *Oxyz system*;
c - he particle in relative motion turned to the angle *α*; axes *Ox* and *Oy* have turned to an angle *φ*

A characteristic feature of the application of the trihedron and Frenet formulas is that the independent variable in them is not time *t*, as is generally accepted in problems of kinematics and dynamics of a point, but the length of the arc *s* of the directional curve (in our case, a circle of radius *R*), so we need to establish relationship between rotational movements through this parameter. Let the cylinder itself make *n* revolutions in one full revolution of the Frenet trihedron (point *A*, which is the origin of its coordinates). Then there will be a relationship between the angular velocities. $\omega_r = n\omega$. In the same time t, point A and point B on the cylinder will turn to the corners in their rotational movements:

$$\psi_A = \omega t; \qquad \psi_B = \omega_r t = n\omega t. \tag{2}$$

For the trajectory of the transfer movement (circle of radius R), the length of the trajectory will be determined from the expression:

$$s = R\omega t = \frac{\omega}{k}t, \text{where from } t = \frac{k}{\omega}s. \tag{3}$$

By substituting the expression for time t from (3) into the second expression (2), we obtain the dependence of the change in the angle ψ of point B on the position of the trihedron on the transfer trajectory:

$$\psi_B = nks = \frac{\omega_r}{\omega}ks. \tag{4}$$

Thus, at the current arc coordinate s, point A in the translational motion will turn to an angle, $\psi_A = ks$, and point B in relative motion will turn to an angle $\psi_B = \frac{\omega_r}{\omega}ks$ (Fig. 1, b).

If there is a particle on the inner surface of the cylinder, it can either slide on its surface or "stick". In the case of sliding, the particle will lag behind point B by an angle α (Fig. 1c). When it is located at point C, you can write:

$$\varphi = \psi_B - \alpha = \frac{\omega_r}{\omega}ks - \alpha. \tag{5}$$

By substituting the angle φ from (5) into equation (1), ψ_B, we obtain instead the projection of the particle onto the vertices of the accompanying trihedron:

$$\rho_\tau = r\cos\left(\frac{\omega_r}{\omega}ks - \alpha\right); \qquad \rho_n = r\sin\left(\frac{\omega_r}{\omega}ks - \alpha\right); \qquad \rho_b = u. \tag{6}$$

If the equations of motion of a particle in the system of the accompanying trihedron are known, then its absolute acceleration can be found using known formulas in the projections onto the vertices of this trihedron [3]:

$$\begin{aligned} w_\tau &= v^2\left[\rho_\tau'' - k\left(k\rho_\tau + 2\rho_n'\right)\right]; \\ w_n &= v^2\left[\rho_n'' + k\left(1 - k\rho_n + 2\rho_\tau'\right)\right]; \\ w_b &= v^2\rho_b'', \end{aligned} \tag{7}$$

where v is the speed of movement of the top of the trihedron in a circle of radius $R = 1/k$.

The first and second derivatives of equations (6) will be:

$$\begin{aligned}
&\rho'_\tau = -r\left(\frac{\omega_r}{\omega}k - \alpha'\right)\sin\left(\frac{\omega_r}{\omega}ks - \alpha\right);\\
&\rho'_n = r\left(\frac{\omega_r}{\omega}k - \alpha'\right)\cos\left(\frac{\omega_r}{\omega}ks - \alpha\right);\\
&\rho'_b = u';\\
&\rho''_\tau = -r\left[-\alpha''\sin\left(\frac{\omega_r}{\omega}ks - \alpha\right) + \left(\frac{\omega_r}{\omega}k - \alpha'\right)^2\cos\left(\frac{\omega_r}{\omega}ks - \alpha\right)\right];\\
&\rho''_n = r\left[-\alpha''\cos\left(\frac{\omega_r}{\omega}ks - \alpha\right) - \left(\frac{\omega_r}{\omega}k - \alpha'\right)^2\sin\left(\frac{\omega_r}{\omega}ks - \alpha\right)\right];\\
&\rho''_b = u''.
\end{aligned} \tag{8}$$

By substituting (8) into (7), we obtain the projections of the absolute acceleration vector of the particle on the axis of the trihedron:

$$\begin{aligned}
&w_\tau = v^2\left[r\alpha''\sin\left(\frac{\omega_r}{\omega}ks - \alpha\right) - r\left(k + \frac{\omega_r}{\omega}k - \alpha'\right)^2\cos\left(\frac{\omega_r}{\omega}ks - \alpha\right)\right];\\
&w_n = v^2\left[-r\alpha''\cos\left(\frac{\omega_r}{\omega}ks - \alpha\right) - r\left(k + \frac{\omega_r}{\omega}k - \alpha'\right)^2\sin\left(\frac{\omega_r}{\omega}ks - \alpha\right) + k\right];\\
&w_b = v^2u''.
\end{aligned} \tag{9}$$

To compile the differential equation of motion of a particle in the form $m\overline{w} = \overline{F}$, where is $\overline{w}$ – the vector of absolute acceleration, $\overline{F}$ – is the vector of applied forces, it is convenient to project these vectors onto the axis of the moving system $Oxyz$. This is explained by the fact that the relative motion of the particle occurs in a plane tangential to the cylinder. Such a plane is the plane Oyz of the moving system $Oxyz$. The component of the absolute acceleration along the *Ox axis* will be normal to the surface of the cylinder, so the pressure on the surface will depend on it.

The projections of the absolute acceleration vector on the axis of the *Oxyz system* can be found using the well-known formulas:

$$\begin{aligned}
&w_x = w_\tau\cos\varphi + w_n\sin\varphi;\\
&w_y = -w_\tau\sin\varphi + w_n\cos\varphi,
\end{aligned} \tag{10}$$

where φ is the angle between the axes Ox , Oy and the axes $\bar{\tau}$, $\bar{n}$.

Substituting (9) and (5) into (10), we find the projections of the absolute acceleration on the axis of the moving system *Oxyz*:

$$\begin{aligned} w_x &= v^2\left[-r\left(\frac{\omega_r}{\omega}k - \alpha' + k\right)^2 + k\sin\left(\frac{\omega_r}{\omega}ks - \alpha\right)\right]; \\ w_y &= v^2\left[-r\alpha'' + k\cos\left(\frac{\omega_r}{\omega}ks - \alpha\right)\right]; \\ w_z &= v^2 u''. \end{aligned} \qquad (11)$$

The projections of the absolute acceleration on the axis Oz and ort $\bar{b}$ are the same, since $Oz \parallel \bar{b}$.

Let's find the projections of the applied forces.

The weight of the particle *mg*, where g = 9.81 m/s^2, in the projections on the axis of the *Oxyz system* , will be written:

$$\{0; \quad 0; \quad -mg\}. \qquad (12)$$

Since the friction force is directed tangentially to the trajectory of the particle's relative motion in the opposite direction, we will find the projections of the tangent vector. They are determined by the first derivatives (8) under the condition that the angle $\varphi = \frac{\omega_r}{\omega}ks$ and its derivative $\varphi' = \frac{\omega_r}{\omega}k$ are equal to zero (this determines the sliding of the particle along the cylinder), so the vector projections will be written:

$$\{-r\alpha'\sin\alpha; \quad -r\alpha'\cos\alpha; \quad u'\}. \qquad (13)$$

Let's move from projections in the trihedron system to projections in the *Oxyz system*. To do this, we apply the transition formulas (10), taking in them the angle ($-\alpha$) *instead of the angle* φ due to the above-mentioned reasons. After performing the transition and bringing the vector to unit, its projection in the *Oxyz system* will be written:

$$\left\{0; \quad -\frac{r\alpha'}{\sqrt{r^2\alpha'^2 + u'^2}}; \quad \frac{u'}{\sqrt{r^2\alpha'^2 + u'^2}}\right\}. \qquad (14)$$

As can be seen from (14), this vector is located in the *Oyz plane*, which is tangent to the cylinder at the point *0* where the material particle is located (Fig. 1, c).

Knowing the absolute acceleration vector (11) of the particle, the applied forces and the direction of its movement in the *Oxyz system*, you can make a differential equation in projections on the axis of the *Oxyz system*:

$$\begin{aligned} mw_x &= -N; \\ mw_y &= -(fN)_y; \\ mw_z &= -(fN)_z - mg, \end{aligned} \tag{15}$$

where $(fN)_y$ і $(fN)_z$ are the components of the friction forces on the *Oy* and *Oz axes* , respectively, and *N* is the reaction force of the cylinder surface on the particle, *f* is the friction coefficient. From the first equation (15), we find $N = - mW_x$. Taking into account the direction cosines (14), the components of friction forces on the axis *Oy* and *Oz* will be written:

$$\begin{aligned} (fN)_y &= \frac{fmr\alpha'}{\sqrt{r^2\alpha'^2 + u'^2}} W_x; \\ (fN)_z &= -\frac{fmu'}{\sqrt{r^2\alpha'^2 + u'^2}} W_x. \end{aligned} \tag{16}$$

By substituting (16) into (15), we obtain a system of two equations:

$$\begin{aligned} mW_y &= -\frac{fmr\alpha'}{\sqrt{r^2\alpha'^2 + u'^2}} W_x; \\ mW_z &= \frac{fmu'}{\sqrt{r^2\alpha'^2 + u'^2}} W_x - mg. \end{aligned} \tag{17}$$

We reduce both equations (17) by the mass of the particle *m* , substitute expressions (11) into them, and after reductions and transformations we obtain (bearing in mind that $v = \frac{\omega}{k}$):

$$\begin{aligned} \alpha'' &= \frac{k}{r}\cos\left(\frac{\omega_r}{\omega}ks - \alpha\right) + \\ &+ \frac{f\alpha'}{\sqrt{r^2\alpha'^2 + u'^2}}\left[k\sin\left(\frac{\omega_r}{\omega}ks - \alpha\right) - r\left(\frac{\omega_r}{\omega}k - \alpha' + k\right)^2\right]; \\ u'' &= -\frac{gk^2}{\omega^2} + \frac{fu'}{\sqrt{r^2\alpha'^2 + u'^2}}\left[k\sin\left(\frac{\omega_r}{\omega}ks - \alpha\right) - r\left(\frac{\omega_r}{\omega}k - \alpha' + k\right)^2\right]. \end{aligned} \tag{18}$$

In the case when the surface of the cylinder is completely smooth (*f =0*), equations (18) are greatly simplified. In particular, the second equation becomes independent of the first and after integration acquires the form: $u = \frac{gk^2}{2\omega^2}s^2$. After substituting the expression s from (3) into it, we obtain: $u = \frac{gt^2}{2}$. Therefore, in the absence of friction and resistance of the medium, a particle along the wall of a cylinder in the vertical direction moves downward according to the law of free fall, and in performs oscillations in the transverse direction, which are described by the differential equation $\frac{d^2\alpha}{ds^2} = \frac{k}{r}\cos\left(\frac{\omega_r}{\omega}ks - \alpha\right)$.

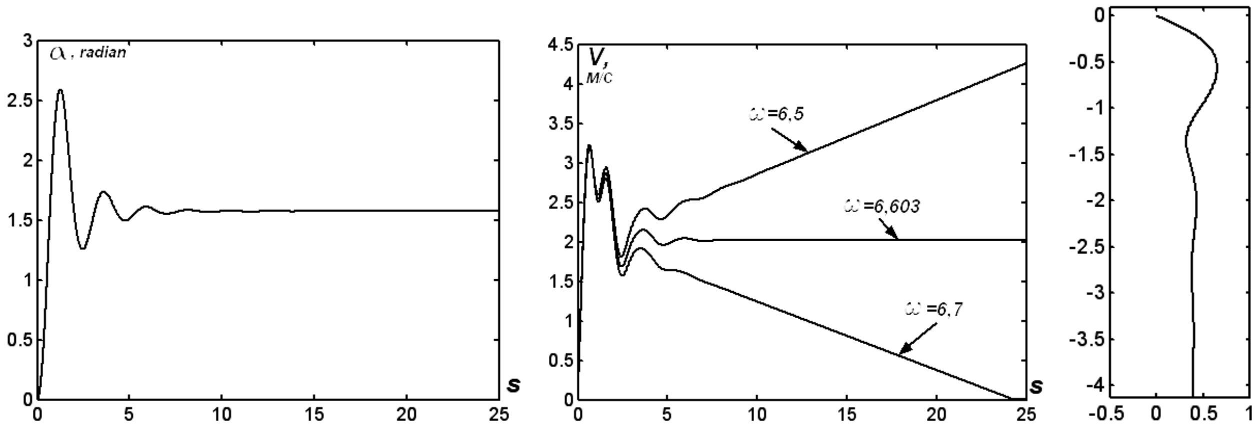
Research results and their discussion. Integration of equations (18) by numerical methods made it possible to find out some regularities of the relative and absolute motion of a particle in a cylindrical sieve.

1. Angular speed of rotation of the cylinder around its own axis $\omega_r = 0$.

In this case, the cylinder does not rotate around its axis, and its core rotates around the *Oz axis* with an angular velocity ω. The relative movement of a particle on the inner surface of the cylinder can be divided into two directions: down parallel to the axis and oscillations in the transverse direction. The total relative speed can be determined by the formula [3]:

$$v_r = \frac{\omega}{k}\sqrt{r^2\alpha'^2 + u'^2}. \tag{19}$$

When a particle hits the surface of the cylinder at point *0* (Fig. 1, b), it begins to move along the surface of the cylinder, trying to take a position in the radial direction under the action of centrifugal force, that is, it must return to 90 ^{0}. Calculations showed that the particle turns to a slightly larger angle, makes certain oscillations, and then its movement stabilizes at the 90 $^{0\ \mathrm{mark}}$ (Fig. 2a). Studies have shown that the relative velocity v_r (19) depends on the angular velocity ω . At a small angular velocity ω, the particle moves with uniform acceleration after stabilization (at ω = 6.5 *rad/s* in Fig. 2b). With an increase in angular speed, it can move uniformly decelerated up to a complete stop ("sticking"). In Fig. 2b, the graph for ω =6.7 *rad/s corresponds to this case* . It is possible to analytically find such an angular velocity, when the particle will move with a constant relative velocity after stabilization. In this case, the weight of the particle is balanced by the force of its friction on the surface of the cylinder. Since the friction force depends on the pressure on the particle due to the action of the centrifugal force, the equilibrium equation can be written in the form: $fm\omega^2(R+r)=mg$. From here we find the angular velocity ω at given R, r and f (in Fig. 2b, for this case corresponds to ω =6.603 *rad/s*). Oscillations in the transverse direction for these three modes of motion practically coincide (shown in Fig. 2a). In fig. 2, c shows the trajectory of the movement of the particle on the sweep of the cylinder, obtained by deposition in the horizontal direction of movement of the particle $r\alpha$, and in the vertical direction - downward movement u . These trajectories practically coincide.



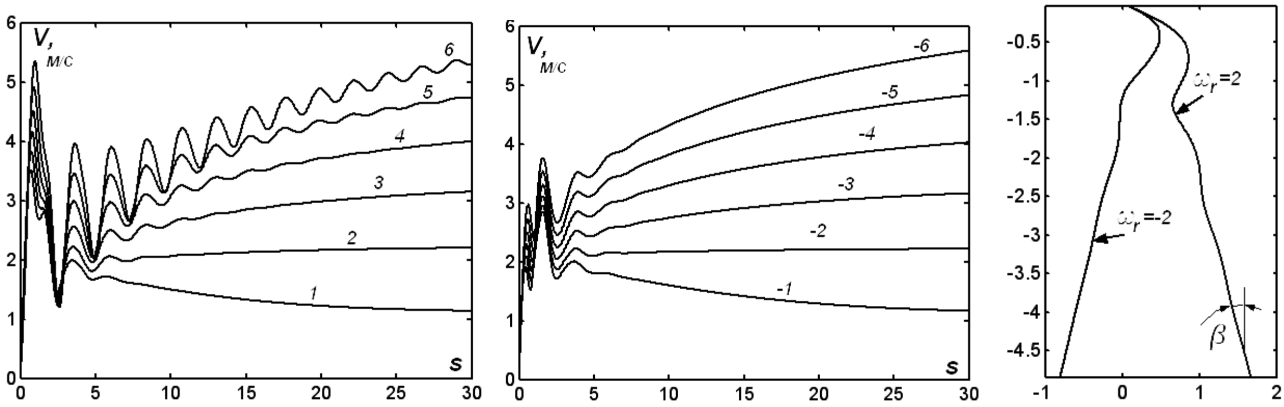
a *b* *c*

Fig. 2. Modes of motion of the particle at $\omega_r = 0$; *k* = 0.2; *r* = 0.25; *f* = 0.3; *ω* = 6.5; 6,603; 6,7:

a – graph of particle oscillations on the surface of the cylinder;
b – graphs of changes in the relative velocity of the particle;
c – trajectory of movement on the sweep of the cylinder

2. Angular speed of rotation of the cylinder around its own axis $\omega_r \neq 0$.

Before that, it was shown that $\omega_r = 0$ a particle can "stick" at a certain angular velocity ω. For example, for k = 2; r = 0.25; f = 0.3 and ω= 6.7, the relative velocity of the particle uniformly decreased to zero (Fig. 2b), i.e., the particle "stuck". Under the same conditions, we will provide the angular velocity ω_r with which the cylinder rotates around its own axis. Even at a low speed of rotation of the cylinder ($\omega_r = \pm 1$ *rad/s*), the particle no longer sticks. Its movement stabilizes over time and the relative speed becomes constant.



a *b* *c*

Fig. 3. Modes of particle motion at $\omega_r \neq 0$; *k* = 2; *r* = 0.25; *f* = 0.3:
a – graphs of changes in the relative velocity of the particle at positive values *of* ω_r (the value is indicated by a number);
b – graphs of changes in the relative velocity of the particle at negative values *of* ω_r (the direction of rotation of the cylinder in the transfer and relative movements is opposite);
c – trajectories of movement on the sweep of the cylinder at $\omega_r = \pm 2$ *rad/s*

ω_r increases, the relative velocity of the particle also increases. At the same time, and for the same directions of angular velocities ω and ω_r (Fig. 3a) and for the opposite ones (Fig. 3b), the relative speed of movement becomes constant over time. For example, for $\omega_r = \pm 2$ *rad/s*, it is approximately equal to 2.25 *m/ c*. Numerical integration of equations (18) showed that after the stabilization of the motion (when the relative velocity of the particle becomes constant), the component of the velocity down the generator and the component of sliding perpendicular to the generator are also constant. This means that the particle is in two uniform motions (rotational and translational), so it moves along a helical line. In fig. 3, c shows the trajectories of the movement of the particle on the sweep of the cylinder for $\omega_r = 2$ *rad/s* and $\omega_r = -2$ *rad/s* . Helical lines, which turn into straight lines on the scan, form angles *β with the vertical generator of the steel cylinder*.

Knowing that after the stabilization of the particle motion, the functions $\alpha=\alpha(s)$ and $u = u(s)$ are linear, one can find an analytical solution to the system of differential equations (18). Without resorting to derivation, we present the finished result:

$$\alpha = \frac{\omega_r}{\omega} ks + \varphi; \quad u = -\frac{\omega_r rgk^2}{\omega^3 \cos\varphi} s,$$

$$\text{де } \varphi = \arcsin \frac{-kr\omega^2 f^2 + \sqrt{\left(1+f^2\right)\left(\omega^4 + k^2 g^2\right) - \omega^4 f^2 k^2 r^2}}{\omega^2 \left(1+f^2\right)}. \tag{20}$$

For example, for values k =2; r = 0.25; f = 0.3; ω = 6.7; ω_r = 2 we obtain:

$$\alpha = 0{,}597s + 1{,}47; \qquad u = -0{,}648s. \tag{21}$$

Equations (21) describe the movement of a particle with given structural and kinematic parameters after stabilization of the movement with a constant relative speed. Knowing the derivatives of dependences (21), $\alpha' = 0{,}597$ і $u' = -0{,}648$ we can find the exact value of the relative velocity of the particle according to formula (19): v_r = 2.23 *m/s* . You can also find the angle β at which the particles cross the rectilinear generators of the cylinder: $\beta = \text{arctg}\left(r\alpha'/u'\right) \approx 13^0$. Since α' і u' they are determined by differentiating expressions (20) with respect to the variable s , it is possible to find generalized formulas for determining the relative speed v_r and the angle β :

$$\beta = \text{arctg} \frac{\omega^2 \cos\varphi}{gk}; \tag{22}$$

$$v_r = \frac{r\omega_r}{\omega \cos\varphi} \sqrt{\omega^2 \cos^2 \varphi + g^2 k^2}. \tag{23}$$

Since φ it does not depend on ω_r, it can be concluded that the value of the angle β (22) does not depend on the angular velocity ω_r, but only on the design parameters, the angular velocity ω and the friction coefficient f . In fig. 4 shows graphs of changes in the angle β for different angular velocities ω *as the friction coefficient* f increases. It can be seen from them that the angle β increases as the angular velocity ω *increases and the friction coefficient* f increases.

It can be seen from formula (23) that the relative velocity of the particle v_r is linearly dependent on the angular velocity ω_r with other parameters unchanged. Graphs of the dependence of the relative velocity v_r of the particle on the friction coefficient f for different angular velocities ω_r at ω =6.7 *rad/s* are shown in Fig. 5.

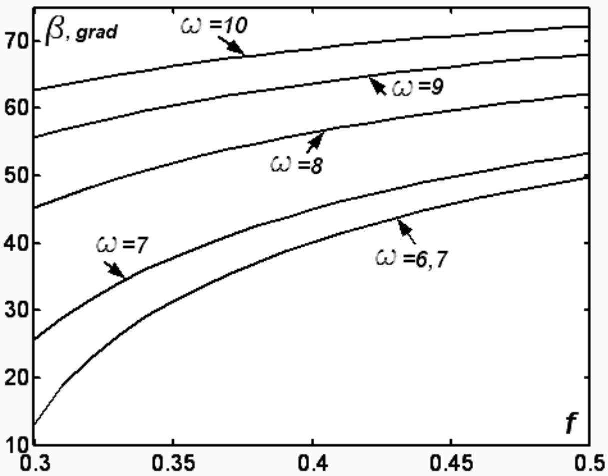


Fig. 4. Graphs of dependences of the angle *β* on the coefficient of friction *f* at different angular velocities *ω*

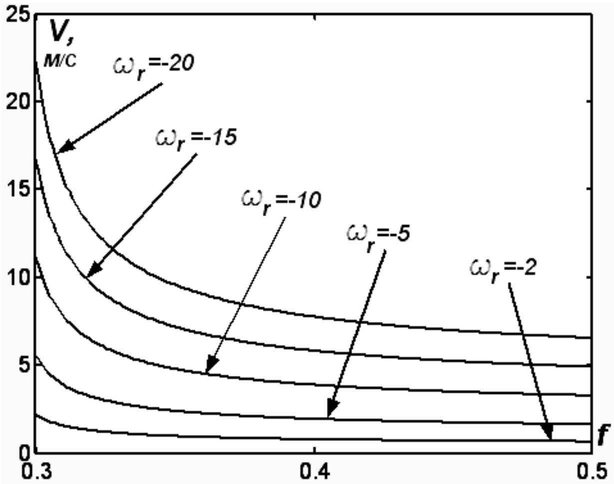


Fig. 5. Graphs of the dependence of the relative velocity v_r of the particle on the friction coefficient *f* at different angular velocities ω_r

Based on the dependences (20), it is possible to find the angle β that characterizes the helical line on the cylinder, and the relative speed v_r at which the particle moves along this line for any angular velocities ω_r. At the same time, for the determined angular velocity ω, given design parameters and friction coefficient *f, we find the angle* β using formula (22). When the angular velocity $\omega_{r\ increases,}$ the angle β does not change, and the speed of movement of the particle v_r increases according to (23) in direct proportion to ω_r, i.e. the trajectory of the movement of the particle remains the same helical line along which the particle moves faster with an increase in the angular speed ω_r.

Numerical integration of equations (18) shows that the exact analytical solution (20), formulas (22), (23) reflect the behavior of the particle only in a rather narrow range of angular velocities ω and ω_r. For example, when the angular velocity $\omega_{r\ increases}$ to 5 and 6 *rad/s*, fluctuations in the relative velocity increase (Fig. 3a).

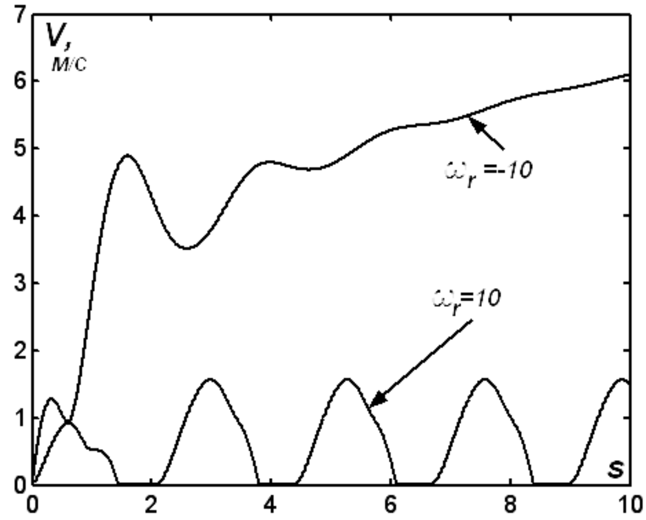
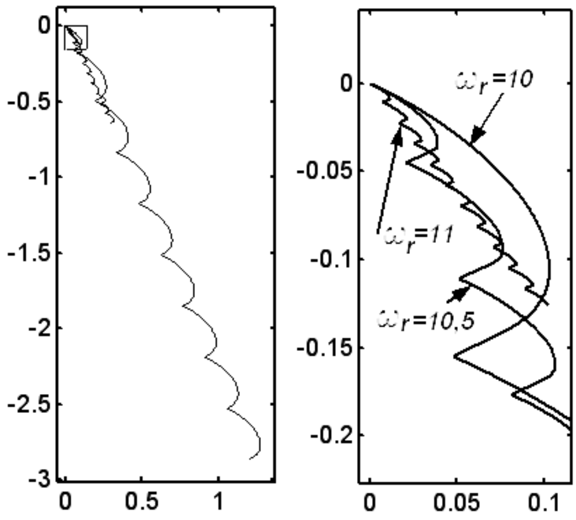


Fig. 6. Graphs of changes in the relative velocity v_r of a particle during rotation of the cylinder in opposite directions (ω_r = ±10 *rad/s*) at ω= 6.7 *rad/s*

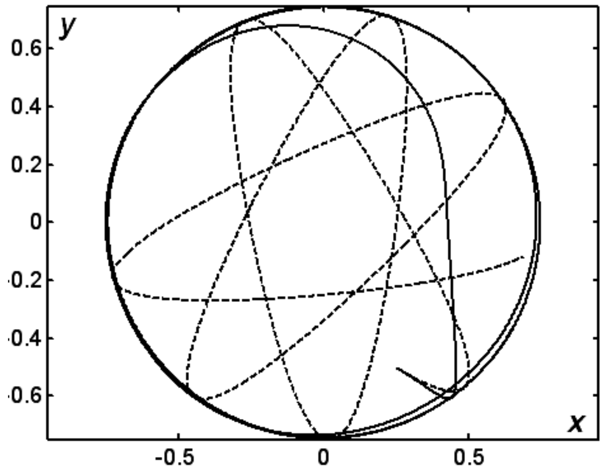


a *b*

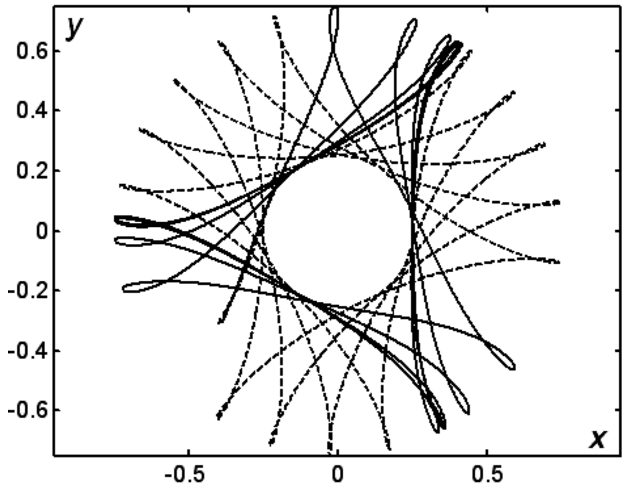
Fig. 7. Trajectories of the relative motion of a particle on the sweep of a cylinder at different angular velocities ω_r:

a – motion trajectories; *b* – enlarged fragment

With a further increase *in* ω_r to 10 *rad/s,* these oscillations become such that at a particular moment in time v_r = 0, i.e., the particle moves along the surface of the cylinder with stops (Fig. 6). Therefore, formulas (20), (22), (23) do not work in this case, but remain valid for $\omega_{r=-10.}$ With further growth *of* ω_r the relative velocity v_r of the particle drops sharply. Figure 7a shows the trajectories traveled by the particle on the sweep of the cylinder for the same period of time. If at ω_r = 10 *rad/s* the particle made 9 periodic oscillations with stops at the turning points and descended only almost 3 *m*, then at ω_r = 11 *rad/s* it made much smaller oscillations and descended only 0.13 *m* (Fig. 7, b). If you further increase ω_r, then the particle practically "sticks".



a



b

Fig. 8. Horizontal projections of the absolute trajectory of the particle movement at; k = 2; r = 0.25; f = 0.3; ω= 6.7:

a – ω_r =- 15 *rad/s*; *b* – ω_r =- 21 *rad/s*

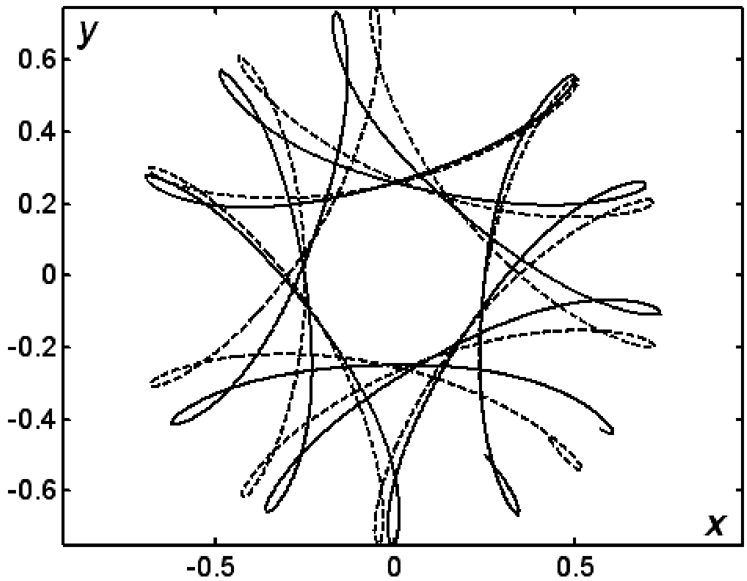


Fig. 9. Horizontal projection of particle motion at ω_r =- 21 *rad/s* and previous other parameters

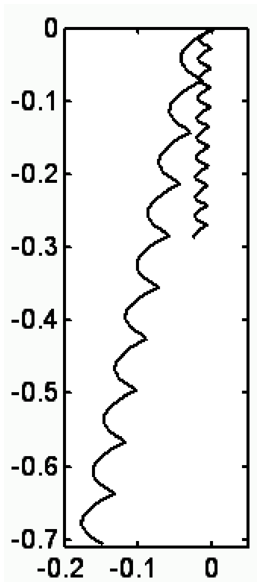


Fig. 10. The trajectory of the particle movement on the sweep of the cylinder at ω_r =- 22 *rad/s* and ω_r =- 23 *rad/s*

The situation is completely different when the rotational motions ω and ω_r have the opposite direction. At ω =6.7 – *const*, formulas (20), (22), (23) remain valid when ω_r

increases to -20 *rad/s*. At the same time, the relative speed increases to 22 *m/s* (Fig. 5, f = 0.3) at a constant angle β= 13 ^{0}. At ω_r=-21 *rad/s*, a phenomenon similar to resonance begins: the speed of the particle does not stabilize and grows indefinitely, the trajectory of its movement approaches the trajectory in the absence of friction. Studies have shown that in this case the pressure force has a sign-changing character, that is, the particle at some moments of time does not press against the cylinder wall, but breaks away from it, that is, its movement becomes uncertain. Equations (18) work in this case, because it is mathematically determined that the particle remains on the surface of the cylinder all the time and the direction of the pressure force does not matter (in the physical model, this can be explained by an example when the particle is between two coaxial cylinders with infinitely close radii).

ω_r were studied. The formulas for finding them are given in the work [4] :

$$x_B = r\cos(\varphi + ks) + \frac{1}{k}\sin(ks);$$
$$y_B = r\sin(\varphi + ks) - \frac{1}{k}\cos(ks), \qquad (24)$$

where the value of the angle φ is described by expression (5). If we take $\alpha = 0$ in expression (5), then equation (24) will describe the horizontal projection of the absolute trajectory of a particle that does not move along the cylinder ("stuck"). It turned out that a particle in the frequency range ω_r, when it is possible to stabilize its motion with the achievement of a constant relative speed v_r, moves along a cylindrical surface with the maximum possible radius $R + r$ (Fig. 8a, the trajectory is depicted by a solid line). The dashed line shows the trajectory of a point fixed firmly on the cylinder wall. At ω_r =- 21 *rad/s*, the horizontal projection of the absolute trajectory of the particle's motion significantly changes its shape in some places, approaching a straight line. (Fig. 8, b).

Of course, in this case, the centrifugal force, and therefore the pressure force, will be minimal. When the angular velocity increases to the value ω_r = -22 *rad/s*, the pressure increases and the particle begins to move with stops. The horizontal projection of the absolute trajectory of the particle is shown in Fig. 9 (solid line), and the trajectory on the sweep of the cylinder is shown in Fig. 10 (curve with a larger amplitude and oscillation period). With a further increase *in* ω_r, the amplitude and period of oscillations of the trajectory on the sweep decrease, and the horizontal projection of the absolute trajectory approaches the trajectory of a fixed point on the surface of the cylinder. This means that a further increase in the angular velocity ω_r leads to "sticking" of the particle. Therefore, upon reaching a certain value of the angular velocity ω_r at a constant angular velocity ω, *a sharp decrease in the relative velocity* $v_{r \text{ of the particle movement}}$ begins. It begins to slide along the cylinder with stops and is prone to "sticking", and this happens faster when the angular velocities ω and ω_r have the same direction and somewhat later when they are directed in opposite directions.

Conclusions and perspectives. The movement of a material particle along the inside of a vertical cylinder, the axis of which rotates around a fixed vertical line with an angular velocity ω, was studied for two cases: 1) the angular velocity of rotation ω_r of the cylinder around its own axis is zero; 2) the angular velocity ω_r of the rotation of the cylinder around its own axis is not equal to zero.

In the first case, it is found that the particle on the surface of the cylinder occupies the position at which it is the most distant from the axis of rotation of the cylinder around a vertical line, i.e. at a distance $R + r$ and further it moves down the plane of the cylinder uniformly accelerated, uniformly or uniformly decelerated up to "sticking" depending on the value of the angular velocity ω.

In the second case, when $\omega_r \neq 0$, the particle behaves similarly: it remains at the maximum distance $R + r$ after the motion is stabilized. At the same time, it slides along the surface of the cylinder with a constant relative speed v_r along a helical line. The direction of the rise of the helical line changes to the opposite when the direction of the angular velocity $\omega_{r \text{ changes}}$. Such movement is possible in a certain range of angular velocities ω and ω_r. As the angular velocity ω_r increases, the moment comes when the particle cannot maintain the described state of sliding, it begins to move along the surface of the cylinder with stops and is prone to "sticking". This state occurs sooner when the angular velocities ω and ω_r have the same direction, and later when they are directed in opposite directions.

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ДОСЛІДЖЕННЯ РУХУ МАТЕРІАЛЬНОЇ ЧАСТИНКИ ПО ВНУТРІШНІЙ ПОВЕРХНІ ВЕРТИКАЛЬНОГО ЦИЛІНДРА, ЯКИЙ ЗДІЙСНЮЄ ПЛАНЕТАРНИЙ РУХ

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Анотація. *Під планетарним рухом циліндра розуміють такий його рух, коли він одночасно перебуває в двох обертальних рухах: обертається навколо власної вертикальної осі із постійною кутовою швидкістю, і сама вісь циліндра обертається із постійною кутовою швидкістю ω навколо вертикальної нерухомої осі. Рух частинки буде складний і складатиметься із відносного її руху по внутрішній поверхні циліндра і переносного руху самого циліндра. Така схема приводу застосовується в циліндричних решетах для сортування насіння сільськогосподарських культур.*

Задачі на складний рух частинки можна успішно розв'язувати із застосуванням тригранника і формул Френе.

Мета дослідження - встановити складний рух матеріальної частинки по внутрішній поверхні циліндричного решета за допомогою тригранника і формул

Френе при однакових і різних за напрямом кутових швидкостях переносного і відносного обертання решета.

Характерною властивістю застосування триганника і формул Френе є те, що незалежною змінною в них служить не час t, як це загальноприйнято в задачах кінематики і динаміки точки, а довжина дуги s напрямної кривої (в нашому випадку – кола радіуса R), тому був встановлений взаємозв'язок між обертальними рухами через цей параметр.

Систему диференціальних рівнянь проінтегровано чисельними методами. Знайдено точний аналітичний розв'язок у випадку, коли рух частинки стабілізується і її швидкість стає постійною. Зроблено візуалізацію одержаних результатів.

Встановлено деякі закономірності відносного та абсолютного руху частинки у циліндричному решеті, коли кутова швидкість обертання циліндра навколо власної осі дорівнює нулю і не дорівнює нулю.

У першому випадку з'ясовано, що частинка на поверхні циліндра займає положення, за якого вона максимально віддалена від осі обертання циліндра навколо вертикальної прямої і далі вона рухається вниз по твірній циліндра рівноприскорено, рівномірно або ж рівносповільнено аж до „залипання" в залежності від величини кутової швидкості.

У другому випадку частинка поводить себе аналогічно: вона залишається на максимально віддаленій відстані після стабілізації руху. При цьому вона ковзає по поверхні циліндра із постійною відносною швидкістю по гвинтовій лінії. Напрям підйому гвинтової лінії міняється на протилежний при зміні напряму кутової швидкості циліндра. Такий рух можливий в певному діапазоні кутових швидкостей частинки і циліндра. При зростанні кутової швидкості частинки наступає момент, коли вона не може зберігати описаний стан ковзання і починає рухатися по поверхні циліндра із зупинками і схильна до „залипання". Такий стан наступає швидше, коли кутові швидкості частинки та циліндра мають однаковий напрям, і пізніше, коли вони спрямовані в протилежні сторони.

Ключові слова: *супровідний триганник Френе, циліндричне решето, траєкторія руху частинки, швидкість руху циліндра, швидкість руху частинки*