

## **MATHEMATICAL MODELING OF ASYNCHRONOUS ELECTRIC DRIVE WITH PHASE-IMPULSE CONTROL**

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**Abstract.** *The process of forming a mathematical model of an asynchronous electric drive with phase-impulse control is considered.*

*The article presents the peculiarities of making of mathematical models of electromechanical energy converters and laws of physics on which they are based.*

*A critical analysis of scientific sources on the development, research and operation of control devices for adjustable asynchronous electric drive is made. It is emphasized that the emergence of a new modern element base encourages the search for new engineering solutions in this area.*

*It is noted that the definition of the priority method of speed control of an asynchronous electric motor as a basic element of asynchronous electric drive is a multifactorial and quite complex task.*

*The following are selected as research methods: fundamental provisions of the theory of electromechanical energy conversion, methods of mathematical analysis of elements of valve-electromechanical systems, mathematical modeling, numerical methods of solving differential equations.*

*For mathematical modeling of an asynchronous electric drive with phase-impulse control, a system of real stator coordinates is chosen, which allows to operate with real stator phase currents, both in unconverted natural coordinates, and at the same time to get rid of periodic coefficients in expressions for inductances and mutual inductance of flux linkage.*

*Taking to the account that the transformations in making of the mathematical model refer only to the rotor circuits of the motor, and the stator currents remain unchanged, it was pointed out the possibility of its matrix interpretation, considering stator phase circuit in addition to the rotor.*

*The expediency of adaptation of the built model of asynchronous electric drive with phase-impulse control to practical calculations in MathCad is indicated.*

**Key words:** *mathematical model, asynchronous electric drive, phase-impulse control*

**Introduction.** Mathematical modeling is one of the important methods of modern scientific research of physical properties of real objects, based on the formal uniqueness of the mathematical description of the modelled object and the model. At the same time, the study of the process is reduced to the analysis of its mathematical description. Mathematical modeling, as a method of scientific knowledge, depends on a number of factors - social, economic, technical, on the level of development of related sciences and on specific physical ideas about matter and the universe as a whole.

The mathematical description of the electromechanical energy converter (generalized electrical machine) is based on the known laws:

- the equation of equilibrium of voltage and EMF on the windings of the fixed and moving parts is based on the second law of Kirchhoff;
- Ampere's law in turn connects the flux linkage of the windings with currents flowing through the windings of an electric machine;
- the third law is Newton's second law - the law of equation of moments on the axle of the machine.

**Literature review and problem statement.** The long-term efforts of scientists and engineers have provided significant advancement of theoretical research in the field of asynchronous electric drives (AEDs), including regulated ones [4; 6; 9], and have also been embodied in many patents.

In several works, considering these issues, there is a classification of known methods for controlling the rotational speed of alternating current electric motors, mainly asynchronous electric motors with squirrel-cage rotor [9; 11].

The choice of a specific method of regulation is determined by many factors, the main of which are the requirements of the technology of a particular production, where it is planned to use a regulated AED and, of course, its reliability and efficiency.

The leading position in regulation is occupied by semiconductor control devices, which include a power switching device based on thyristors, triacs, transistors and a control unit that ensures its operation according to a certain algorithm.

A lot of studies have been devoted to the issues of development, research and operation of control devices for controlled asynchronous electric drives [9; 12; 13; 14]. Many of the proposed devices are protected by patents, and a number of them have found practical application [15 - 17]. It is remarkable that researches and the search for new engineering solutions in this area are going ahead, motivated by the advent of a modern elemental base.

It can be stated that the determination of the priority method of regulating the rotation speed of an asynchronous electric motor, as the basic element of the AED, is a multifactorial and rather complicated task.

**Object of study** - asynchronous electric fan drive.

**Subject of study** - mathematical model of an asynchronous electric drive with phase-impulse control.

**Research methods:** fundamental principles of the theory of electromechanical energy conversion, methods of mathematical analysis of elements of valve-electromechanical systems, methods of mathematical modeling, numerical methods for solving differential equations.

**Mathematical modeling of asynchronous electric fan drive with phase-impulse control.** For mathematical modeling of an asynchronous electric drive with phase-impulse control, a system of real stator coordinates was chosen, which has a number of known advantages [1,2,3,5].

Such a system allows operating with real currents of the stator phases, as well as in non-transformed natural coordinates, while simultaneously providing the opportunity to get rid of periodic coefficients in expressions for inductances and mutual inductances of flux linkages.

In the case of a transition to an idealized two-phase machine, which allows us to achieve the mentioned simplification of the system of differential equations describing the electromagnetic transient processes occurring in it, the stator phase currents are calculated in two stages - first for an idealized machine, and then using variable replacement formulas for a real machine.

When compiling a system of differential equations describing an asynchronous electric fan drive with phase-impulse control in various operating modes, a number of necessary transformations were performed.

When compiling the differential equations of electric equilibrium, the assumptions are made [1,5,8], which simplify the model with an allowable and insignificant decrease in the accuracy of the results of calculating the quasi-steady-state mode of an electric machine.

The system of equations of electrical equilibrium of the rotor winding is made up for a decelerated electric motor that is magnetically-invariant to a real rotating fan motor. The legitimacy of such a replacement is confirmed by the observance of one of the most important conditions of the electromechanical energy conversion - the mutual immobility of the magnetomotive forces of the rotating space [1,2,5,10].

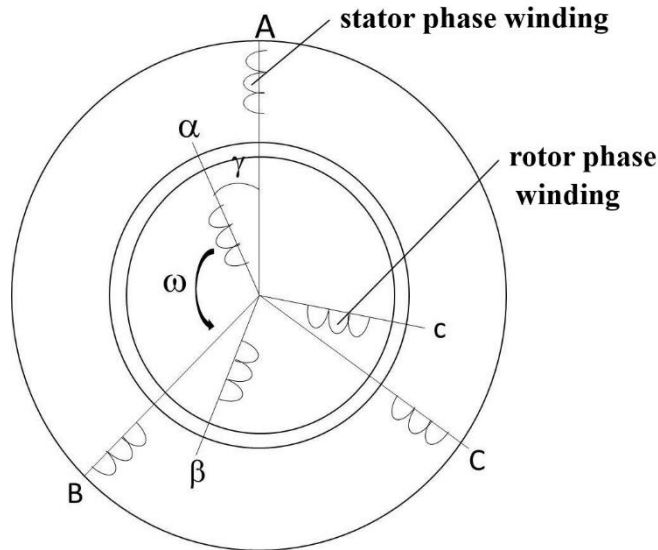
The system of equations of electrical equilibrium for the phase circuits of the stator windings and the rotor of the electric motor, recorded through currents and flux linkages, has the following form:

$$\begin{aligned} U_A &= i_A \cdot r_s + \frac{d\Psi_A}{dt}; 0 = i_a \cdot r_r + \frac{d\Psi_a}{dt} \\ U_B &= i_B \cdot r_s + \frac{d\Psi_B}{dt}; 0 = i_b \cdot r_r + \frac{d\Psi_b}{dt} \\ U_C &= i_C \cdot r_s + \frac{d\Psi_C}{dt}; 0 = i_c \cdot r_r + \frac{d\Psi_c}{dt} \end{aligned} \quad (1)$$

A continuous change in time of the mutual arrangement of the stator windings and the rotor of the electric motor (Fig. 1, angle  $\gamma$ ) causes a change in the mutual inductance of the same phases of the stator and rotor

$$M_{Aa} = M_{\phi \max} \cos \gamma \quad (2)$$

where  $M_{\phi \max}$  - is the maximum value of the mutual inductance "stator phase - rotor phase", which occurs when their axes coincide.



**Figure 1. The relative position of the stator windings and the rotor of an asynchronous electric motor**

For a symmetric asynchronous machine, the relations are

$$L_s = l_{\sigma s} + M_{\phi} = L_1 + M_{\phi}, \quad (3)$$

$$L_r = l_{\sigma r} + M_{\phi} = L_2 + M_{\phi}, \quad (4)$$

$$M_{\phi s} = M_{\phi} \cos 2\pi/3 = -0,5M_{\phi}, \quad (5)$$

$$M_{\phi r} = M_{\phi} \cos 2\pi/3 = -0,5M_{\phi}, \quad (6)$$

where  $L_s, L_r$  - total stator and rotor phase inductances;  $l_{\sigma s}$  i  $l_{\sigma r}$  - stator and rotor phase inductance;  $M_{\phi s}, M_{\phi r}$  - the mutual inductance between the stator phase windings and between the rotor windings, respectively, for the converted machine [1,2,10].

The transition from a real machine with a rotating rotor to a stationary (decelerated) machine in the mathematical model is reflected primarily by replacing the real phase currents of the rotor  $i_a, i_b, i_c$  with the currents of the fixed rotor  $i'_a, i'_b, i'_c$ . In this case, the squirrel-cage rotor of the electric motor is considered as equivalent three-phase.

The direct conversion formulas linking both currents can be written based on the properties of the representing current vector [1,2,5]:

$$i'_a = 2/3[i_a \cos \gamma + i_b \cos(\gamma + \rho) + i_c \cos(\gamma - \rho)], \quad (7)$$

$$i'_b = 2/3[i_a \cos(\gamma - \rho) + i_b \cos \gamma + i_c \cos(\gamma + \rho)], \quad (8)$$

$$i'_c = 2/3[i_a \cos(\gamma + \rho) + i_b \cos(\gamma - \rho) - i_c \cos \gamma], \quad (9)$$

The reverse replacement of the considered rotor circuit variables is possible using the inverse transformation formulas

$$i_a = 2/3[i'_a \cos \gamma + i'_b \cos(\gamma - \rho) + i'_c \cos(\gamma + \rho)], \quad (10)$$

$$i_b = 2/3[i'_a \cos(\gamma + \rho) + i'_b \cos \gamma + i'_c \cos(\gamma - \rho)], \quad (11)$$

$$i_c = 2/3[i'_a \cos(\gamma - \rho) + i'_b \cos(\gamma + \rho) + i'_c \cos \gamma], \quad (12)$$

In expressions (7) – (12)  $\rho = 120^\circ$ . Obtaining the formulas of the direct and inverse transforms is easy to trace (Fig. 1) using particular provisions that reveal the properties of the representing vector [1, 2, 5, 8].

The projections of the spatial current vector (or magnetomotive force) of the m-phase in the general case of the machine on the axis of the phases are instantaneous values of the current (magnetomotive force) of the corresponding phases. The value of each magnetomotive force at a given constant value of the representing vector depends on the spatial position of the phases.

Representing vector of the rotor current of a three-phase asynchronous electric motor can be presented by the analytical expression:

$$\bar{i}_2 = 2/3(\bar{i}_a + \bar{i}_b + \bar{i}_c), \quad (13)$$

and through the currents of the converted (fixed) rotor

$$\bar{i}'_2 = 2/3(\bar{i}'_a + \bar{i}'_b + \bar{i}'_c), \quad (14)$$

In this case, it is imperative to meet the conditions of the invariance of the phases of the magnetomotive forces of the real and converted rotors created by the currents.

**Recording of direct transformation formulas in matrix form.** Considering that the transformations concern only rotor circuits of the electric motor (transition to the fixed axes of the stator phases), and the stator currents remain unchanged in accordance with [1], it is possible to write formulas (7 - 12) in matrix form, considering in addition to the rotor and stator phase circuits. Then, for the machine as a whole, the direct conversion formula is written as follows:

$$\bar{i}' = \Pi \cdot \bar{i} \quad (15)$$

and the inverse conversion

$$i = \Pi^{-1} \cdot \bar{i}' \quad (16)$$

In a somewhat more complete form, the conversion formulas (15) and (16) can be written as

$$\begin{bmatrix} i_A \\ i_B \\ i_C \\ i'_a \\ i'_b \\ i'_c \end{bmatrix} = \Pi \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_a \\ i_b \\ i_c \end{bmatrix} \quad (17)$$

$$\begin{bmatrix} i_A \\ i_B \\ i_C \\ i_a \\ i_b \\ i_c \end{bmatrix} = \Pi^{-1} \begin{bmatrix} i_A \\ i_B \\ i_C \\ i'_a \\ i'_b \\ i'_c \end{bmatrix} \quad (18)$$

The correspondence of expressions (17) and (18) to the previously written formulas for converting rotor currents (7) - (12) is obvious, in particular, if the former are written more detailed.

$$i'_a = (1/3 + 2/3 \cos \gamma) \cdot i_a + [1/3 + 2/3 \cos(\gamma + \rho)] \cdot i_b + [1/3 + 2/3 \cos(\gamma - \rho)] \cdot i_c \quad (19)$$

$$i'_b = [1/3 + 2/3 \cos(\gamma - \rho)] \cdot i_a + (1/3 + 2/3 \cos \gamma) \cdot i_b + [1/3 + 2/3 \cos(\gamma + \rho)] \cdot i_c \quad (20)$$

$$i'_c = [1/3 + 2/3 \cos(\gamma + \rho)] \cdot i_a + [1/3 + 2/3 \cos(\gamma - \rho)] \cdot i_b + (1/3 + 2/3 \cos \gamma) \cdot i_c \quad (21)$$

In these expressions, it is easy to distinguish the rotor zero sequence current

$$i_{or} = 1/3 (i_a + i_b + i_c). \quad (22)$$

For a rotor with no neutral wire,

$$i_{or} = 1/3 (i_a + i_b + i_c) = 0 \quad (23)$$

and then the expressions (19 - 21) become identical to the transformation formulas (10 - 12).

The expressions for the flux linkages of the phases of the stator and rotor of a three-phase asynchronous electric motor, taking into account generally accepted assumptions [1,5, 8, 9], have the form

$$\Psi_A = L_{\Phi S} \cdot i_A + M_{\Phi S} \cdot i_B + M_{\Phi S} \cdot i_C + M_{\Phi} \cdot \cos \gamma \cdot i'_a + M_{\Phi} \cdot \cos(\gamma + \rho) \cdot i'_b +$$

$$+M_{\Phi} \cdot \cos(\gamma - \rho) \cdot i_c, \quad (24)$$

$$\Psi_B = L_{\Phi S} \cdot i_B + M_{\Phi S} \cdot i_A + M_{\Phi S} \cdot i_c + M_{\Phi} \cdot \cos(\gamma - \rho) \cdot i_a + M_{\Phi} \cdot \cos \gamma \cdot i_B + \\ + M_{\Phi} \cdot \cos(\gamma + \rho) \cdot i_c, \quad (25)$$

$$\Psi_C = L_{\Phi S} \cdot i_c + M_{\Phi S} \cdot i_A + M_{\Phi S} \cdot i_B + M_{\Phi} \cdot \cos(\gamma + \rho) \cdot i_a + \\ + M_{\Phi} \cdot \cos(\gamma - \rho) \cdot i_B + M_{\Phi} \cdot \cos \gamma \cdot i_c \quad (26)$$

$$\Psi_a = L_{\Phi r} \cdot i_a + M_{\Phi r} \cdot i_b + M_{\Phi r} \cdot i_c + M_{\Phi} \cdot \cos \gamma \cdot i_A + M_{\Phi} \cdot \cos(\gamma + \rho) \cdot i_B + \\ + M_{\Phi} \cdot \cos(\gamma + \rho) \cdot i_c, \quad (27)$$

$$\Psi_b = L_{\Phi r} \cdot i_b + M_{\Phi r} \cdot i_a + M_{\Phi r} \cdot i_c - M_{\Phi} \cdot \cos(\gamma - \rho) \cdot i_A + M_{\Phi} \cdot \cos \gamma \cdot i_B + \\ + M_{\Phi} \cdot \cos(\gamma - \rho) \cdot i_c, \quad (28)$$

$$\Psi_c = L_{\Phi r} \cdot i_c + M_{\Phi r} \cdot i_a + M_{\Phi r} \cdot i_b + M_{\Phi} \cdot \cos(\gamma - \rho) \cdot i_A + M_{\Phi} \cdot \cos(\gamma - \rho) \cdot \\ \times i_B + M_{\Phi} \cdot \cos \gamma \cdot i_c, \quad (29)$$

Considering that for a four-wire power supply system of the stator of the electric motor, the current in the neutral wire is

$$i_{oS} = i_A + i_B + i_C,$$

as well as previously performed rotor current transformations, expression (10) - (12) can be written as following:

$$\Psi_A = (L_{\Phi S} - M_{\Phi S}) \cdot i_A + M_{\Phi S} \cdot i_{oS} + \frac{3}{2} M_{\Phi S} \cdot i'_a = \\ = L_S \cdot i_A + M_{\Phi S} \cdot i_{oS} + M \cdot i'_a, \quad (30)$$

$$\Psi_B = (L_{\Phi S} - M_{\Phi S}) \cdot i_B + M_{\Phi S} \cdot i_{oS} + \frac{3}{2} M_{\Phi S} \cdot i'_b = \\ = L_S \cdot i_B + M_{\Phi S} \cdot i_{oS} + M \cdot i'_b, \quad (31)$$

$$\Psi_C = (L_{\Phi S} - M_{\Phi S}) \cdot i_C + M_{\Phi S} \cdot i_{oS} + \frac{3}{2} M_{\Phi S} \cdot i'_c = \\ = L_S \cdot i_C + M_{\Phi S} \cdot i_{oS} + M \cdot i'_c. \quad (32)$$

For rotor phase circuits

$$\Psi_a = (L_{\Phi r} - M_{\Phi r}) \cdot i_a + \frac{3}{2} M_{\Phi} \cdot i'_A = L_r \cdot i_a + M \cdot i'_A, \quad (33)$$

$$\Psi_b = (L_{\Phi r} - M_{\Phi r}) \cdot i_b + \frac{3}{2} M_{\Phi} \cdot i'_B = L_r \cdot i_b + M \cdot i'_B, \quad (34)$$

$$\Psi_c = (L_{\Phi r} - M_{\Phi r}) \cdot i_c + \frac{3}{2} M_{\Phi} \cdot i'_C = L_r \cdot i_c + M \cdot i'_C. \quad (35)$$



Here  $i'_A, i'_B, i'_C$  — are projections of the stator current representing vector on the rotor phase axis.

Taking into account the expression (23), the direct transformation matrix  $\Pi$  is slightly modified, particularly, its part relating to the phase currents of the electric motor rotor

$$\Pi = \frac{2}{3} \begin{bmatrix} \cos\gamma & |\cos(\gamma + \rho)| & |\cos(\gamma - \rho)| \\ \frac{\cos(\gamma - \rho)}{|\cos(\gamma - \rho)|} & \cos\gamma & |\cos(\gamma + \rho)| \\ \frac{\cos(\gamma + \rho)}{|\cos(\gamma + \rho)|} & |\cos(\gamma - \rho)| & \cos\gamma \end{bmatrix}. \quad (36)$$

Using this matrix to perform a number of mathematical transformations related to expressions (33) - (35), we obtain, by analogy with the transformation formulas for rotor currents, similar formulas that reflect the relationship between the flux linkages of a fixed (decelerated) rotor and a rotating rotor of a real machine

$$\begin{bmatrix} \psi'_a \\ \psi'_b \\ \psi'_c \end{bmatrix} = \Pi \cdot \begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \end{bmatrix} = \frac{2}{3} \cdot \begin{bmatrix} \cos\gamma & |\cos(\gamma + \rho)| & |\cos(\gamma - \rho)| \\ \frac{\cos(\gamma - \rho)}{|\cos(\gamma - \rho)|} & \cos\gamma & |\cos(\gamma + \rho)| \\ \frac{\cos(\gamma + \rho)}{|\cos(\gamma + \rho)|} & |\cos(\gamma - \rho)| & \cos\gamma \end{bmatrix} \cdot \begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \end{bmatrix}. \quad (37)$$

After substituting the values of the flux linkages of the phases of the real rotor from expressions (33) - (35), we obtain the following expressions

$$\psi'_a = (L_{\phi r} - M_{\phi r}) \cdot i'_a + \frac{3}{2} M_{\phi} \cdot i_A = (L_{\phi r} - M_{\phi r}) \cdot i'_a + M \cdot i_A \quad (38)$$

$$\psi'_b = (L_{\phi r} - M_{\phi r}) \cdot i'_b + \frac{3}{2} M_{\phi} \cdot i_B = (L_{\phi r} - M_{\phi r}) \cdot i'_b + M \cdot i_B \quad (39)$$

$$\psi'_c = (L_{\phi r} - M_{\phi r}) \cdot i'_c + \frac{3}{2} M_{\phi} \cdot i_C = (L_{\phi r} - M_{\phi r}) \cdot i'_c + M \cdot i_C \quad (40)$$

Here, a number of intermediate transformations are omitted to reduce the presentation.

Considering that  $L_{\phi r} - M_{\phi r} = L_r$  we can bring the whole group of formulas for the flux linkages of the phases of the stator and rotor of the electric motor (expressions (30) - (32) and (38 - 40)) to the form

$$\Psi_A = L_S \cdot i_A + M_{\phi S} \cdot i_{oS} + M \cdot i'_a, \quad (41)$$

$$\Psi_B = L_S \cdot i_B + M_{\phi S} \cdot i_{oS} + M \cdot i'_b, \quad (42)$$

$$\Psi_C = L_S \cdot i_C + M_{\phi S} \cdot i_{oS} + M \cdot i'_c, \quad (43)$$

$$\psi'_a = L_r \cdot i'_a + M \cdot i'_a, \quad (44)$$

$$\psi'_b = L_r \cdot i'_b + M \cdot i'_b, \quad (45)$$

$$\Psi'_c = L_r \cdot i'_c + M \cdot i_c. \quad (46)$$

We perform the transformation of the equations of electrical equilibrium of the rotor of the electric motor (real and rotating) to the decelerated one, multiplying equation (1) by the transformation matrix column

$$\Pi_r = \begin{bmatrix} \frac{2}{3} \cos \gamma \\ \frac{2}{3} \cos(\gamma + \rho) \\ \frac{2}{3} \cos(\gamma - \rho) \end{bmatrix}. \quad (47)$$

The result of the multiplication is

$$O = \left[ \frac{2}{3} \cos \gamma \cdot \frac{d\Psi_a}{dt} + \cos(\gamma + \rho) \cdot \frac{d\Psi_b}{dt} + \cos(\gamma - \rho) \cdot \frac{d\Psi_c}{dt} \right] + i'_a \cdot r_r \quad (48)$$

We introduce additional transformations as following

$$\frac{d}{dt}(\Psi_a \cdot \cos \gamma) = -\Psi_a \cdot \frac{d\gamma}{dt} \cdot \sin \gamma + \cos \gamma \cdot \frac{d\Psi_a}{dt}, \quad (49)$$

$$\frac{d}{dt}[\Psi_b \cdot \cos(\gamma - \rho)] = \Psi_b \cdot \frac{d\gamma}{dt} \cdot \sin(\gamma + \rho) + \cos(\gamma + \rho) \cdot \frac{d\Psi_b}{dt}, \quad (50)$$

$$\frac{d}{dt}[\Psi_c \cdot \cos(\gamma - \rho)] = \Psi_c \cdot \frac{d\gamma}{dt} \sin(\gamma - \rho) + \cos(\gamma - \rho) \cdot \frac{d\Psi_c}{dt}. \quad (51)$$

Summing up expressions (49) - (51) and rearranging, we obtain

$$\begin{aligned} & \cos \gamma \cdot \frac{d\Psi_a}{dt} + \cos(\gamma + \rho) \cdot \frac{d\Psi_b}{dt} - \cos(\gamma - \rho) \cdot \frac{d\Psi_c}{dt} = \\ & = \frac{d}{dt}[\Psi_a \cdot \cos \gamma + \Psi_b \cdot \cos(\gamma + \rho) + \Psi_c \cdot \cos(\gamma - \rho)] + \\ & + \omega[\Psi_a \cdot \sin \gamma + \Psi_b \cdot \sin(\gamma + \rho) + \Psi_c \cdot \sin(\gamma - \rho)]. \end{aligned} \quad (52)$$

Here  $\omega = \frac{d\gamma}{dt}$

An analysis of Fig. 1 allows us to write the following expressions:

$$\sin \gamma = \frac{1}{\sqrt{3}}[\cos(\gamma - \rho) - \cos(\gamma + \rho)], \quad (53)$$

$$\sin(\gamma + \rho) = \frac{1}{\sqrt{3}}[\cos(\gamma + \rho) - \cos \gamma], \quad (54)$$

$$\sin(\gamma - \rho) = \frac{1}{\sqrt{3}}[\cos \gamma - \cos(\gamma - \rho)]. \quad (55)$$

Then the second part of expression (52) is transformed to the following form (intermediate transformations are omitted)

$$\omega[\Psi_a \cdot \sin \gamma + \Psi_b \cdot \sin(\gamma + \rho) + \Psi_c \cdot \sin(\gamma - \rho)] = \frac{\omega}{\sqrt{3}}(\Psi'_b - \Psi'_c). \quad (56)$$

Expression (48) - the equation of electrical equilibrium of the stationary rotor phase circuit) can be written as

$$0 = \frac{d\Psi'_a}{dt} + \frac{\omega}{\sqrt{3}}(\Psi'_b - \Psi'_c) + i'_a \cdot r_r, \quad (57)$$

where  $\frac{\omega}{\sqrt{3}}(\Psi'_b - \Psi'_c)$  - is EMF of rotation of the phase of the rotor with changing the magnetic flux formed by the windings of phases b and c.

The system of equations for the circuits of all phases of the rotor will be as follows

$$\begin{aligned} 0 &= \frac{d\Psi'_a}{dt} + \frac{\omega}{\sqrt{3}}(\Psi'_b - \Psi'_c) + i'_a \cdot r_r, \\ 0 &= \frac{d\Psi'_b}{dt} + \frac{\omega}{\sqrt{3}}(\Psi'_c - \Psi'_a) + i'_b \cdot r_r, \\ 0 &= \frac{d\Psi'_c}{dt} + \frac{\omega}{\sqrt{3}}(\Psi'_a - \Psi'_b) + i'_c \cdot r_r. \end{aligned} \quad (58)$$

The general system of equations of electrical equilibrium describing the state of the asynchronous electric fan drive, taking into account the possible presence of upstream elements in the stator phase circuits ( $r_{\mathcal{L}A}, r_{\mathcal{L}B}, r_{\mathcal{L}C}, L_{\mathcal{L}A}, L_{\mathcal{L}B}, L_{\mathcal{L}C}$ ), has the following form

$$\begin{aligned} u_A &= i_a \cdot r_{\mathcal{L}A} + L_{\mathcal{L}A} \frac{di_A}{dt} + i_A \cdot r_S + \frac{d\Psi_a}{dt}, \\ u_B &= i_B \cdot r_{\mathcal{L}B} + L_{\mathcal{L}B} \frac{di_B}{dt} + i_B \cdot r_S + \frac{d\Psi_B}{dt}, \\ u_C &= i_C \cdot r_{\mathcal{L}C} + L_{\mathcal{L}C} \frac{di_C}{dt} + i_C \cdot r_S + \frac{d\Psi_C}{dt}, \\ 0 &= i'_a \cdot r_r + \frac{\omega}{\sqrt{3}}(\Psi'_b - \Psi'_c) + \frac{d\Psi'_a}{dt}, \\ 0 &= i'_b \cdot r_r + \frac{\omega}{\sqrt{3}}(\Psi'_c - \Psi'_a) + \frac{d\Psi'_b}{dt}, \\ 0 &= i'_c \cdot r_r + \frac{\omega}{\sqrt{3}}(\Psi'_a - \Psi'_b) + \frac{d\Psi'_c}{dt}, \end{aligned} \quad (59)$$

After substituting of the previously obtained expressions for flux linkages in the equations and supplementing with the equation of motion, we obtain the following system of equations:

$$\begin{aligned} u_A &= i_A \cdot r_{\mathcal{L}A} \frac{di_A}{dt} + i_A \cdot r_S + L_S \frac{di_A}{dt} + M \frac{di'_a}{dt} + M \frac{di_{OS}}{dt}, \\ u_B &= i_B \cdot r_{\mathcal{L}B} \frac{di_B}{dt} + i_B \cdot r_S + L_S \frac{di_B}{dt} + M \frac{di'_b}{dt} + M \frac{di_{OS}}{dt}, \end{aligned}$$

$$\begin{aligned}
 u_c &= i_c \cdot r_{\pi c} \frac{di_c}{dt} + i_c \cdot r_s + L_s \frac{di_c}{dt} + M \frac{di_c'}{dt} + M \frac{di_{os}}{dt}, \\
 u_{or} &= i_a' \cdot r_r + L_r \frac{di_a'}{dt} + \frac{\omega}{\sqrt{3}} [L_r(i_b' - i_c') + M(i_B - i_C)] + M \frac{di_A}{dt}, \\
 u_{or} &= i_b' \cdot r_r + L_r \frac{di_b'}{dt} + \frac{\omega}{\sqrt{3}} [L_r(i_c' - i_a') + M(i_C - i_A)] + M \frac{di_B}{dt}, \\
 u_{or} &= i_c' \cdot r_r + L_r \frac{di_c'}{dt} + \frac{\omega}{\sqrt{3}} [L_r(i_a' - i_b') + M(i_A - i_B)] + M \frac{di_C}{dt}, \\
 \frac{d\omega_r}{dt} &= \frac{p}{J} (M_{\mathfrak{E}} - M_C),
 \end{aligned} \tag{60}$$

where  $u_{or} = -M \frac{di_{os}}{dt}$ .

Auxiliary equation linking variables

$$i_A + i_B + i_C = i_{os}. \tag{61}$$

The expression for the electromagnetic moment developed by the electric motor is taken in the form [51]

$$M_{\mathfrak{E}} = \frac{p \cdot M}{\sqrt{3}} [i_a'(i_B - i_C) + i_b'(i_C - i_A) + i_c'(i_A - i_B)]. \tag{62}$$

**Conclusions.** The system of equations (60) is a mathematical model of an asynchronous electric drive, not related to intermediate transformations, and therefore more convenient for use. It seems appropriate to adapt the mathematical model for performing of practical calculations in Mathcad.

Note that the equations which describe the operation of an asynchronous electric drive with phase-impulse control allow their practical implementation in the form of blocks and subsystems of Matlab Simulink.

Based on the above, we can conclude that the developed model of asynchronous electric drive with phase-impulse control is quite adequate and can be used in industrial conditions to increase its efficiency and reliability.

### References

1. Kopylov, I. P. (2001). Mathematical modeling of electrical machines: textbook. manual for universities. Moscow: Publishing House "Vysshaya Shkola", 327.
2. Filz, R. V. (1979). Mathematical foundations of the theory of electromechanical converters. Kyiv: Nauk. Dumka, 208.
3. Sokolov, M. M., Petrov, L. P., Masandilov, L. B. et al. (1967). Electromagnetic transients in an asynchronous electric drive. Moscow: Energia, 200.

4. Heruntsev, P. E., Shklovskaya, Y. B. (1984). Modeling an asynchronous motor with a zero wire during thyristor control in the stator and rotor circuit. Electrical Engineering and Electrical Equipment: Compendium of scientific studies. Institute of Electrodynamics, Academy of Sciences of the Ukrainian SSR, Kyiv, 58-63.
5. Valts, O. M., Kagan, A. V., Ryabukha, V. I. (2010). Mathematical modeling in electromechanics: educational complex. St. Petersburg: Publishing House of SZTU, 176.
6. Shreiner, R. T. (2000). Mathematical modeling of AC electric drives with semiconductor frequency converters. Ekaterinburg. URO RAS, 254.
7. Ivanov-Smolensky, A. V. (2004). Electric machines: in 2 vols. T. 1. Moskow: Publishing House MEI, 652.
8. Bober, V. I. (2012). Mathematical model of an electric machine of alternating current. Electromechanical and energy saving systems. Edition 3/2012 (19), 474-479.
9. Donskoy, N. V. (2007). Regulated AC electric drives: monograph. Cheboksary: Publishing house of Chuvash. University, 204.
10. Burulko, L. K. (2014). B91 Mathematical modeling of electromechanical systems: training manual. Part 1. Mathematical modeling of AC electric power converters. Tomsk: Publishing House of Tomsk Polytechnic University, 104 p.
11. Methods for controlling the speed of asynchronous motors. [Electronic resource]. Access Mode: <https://msd.com.ua/chastotno-reguliruemyi-asinxronnyi-elektroprivod/sposoby-regulirovaniya-skorosti-asinxronnyx-dvigatelj/>.
12. Frolov, Yu., Shelyakin, V. (2016). Adjustable asynchronous electric drive. Tutorial. Lan. St. Petersburg, 464.
13. Braslavsky, I. Ya. , Ishmatov, Z. Sh., Polyakov, V. N. (2004). Energy-saving asynchronous electric drive. Moskow: ACADEMA, 202.
- 14.. Weinger, A. M. (2009). Adjustable AC drives. Synopsis of introductory lectures. Moscow, 102 p.
15. Station of automatic control of ventilation KLIMAT-T (control up to 100 fans). [Electronic resource]. Access mode: <http://www.evromash.ru/catalog/venti/vos/avtomatika/klimat/>.
16. Climate control station. [Electronic resource]. Access Mode: <http://agromash-ufa.ru/product/stantsiya-upravleniya-mikroklimatom>.
17. Control station for variable-frequency drive of pumping units SU-CHE. [Electronic resource]. Access Mode: [http://ukrsk.com.ua/su\\_jae.html](http://ukrsk.com.ua/su_jae.html).

## **МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ АСИНХРОННОГО ЕЛЕКТРОПРИВОДА З ПОФАЗНОІМПУЛЬСНИМ УПРАВЛІННЯМ**

***Н. Г. Батечко, М. Т. Лут, С. В. Шостак, О. Зінченко***

***Анотація.** Розглянуто процес формування математичної моделі асинхронного електропривода з пофазноімпульсним керуванням.*

*Наведено особливості побудови математичних моделей електромеханічних перетворювачів енергії та закони фізики, на яких вони ґрунтуються.*

*Проведено критичний аналіз наукових джерел щодо питань розроблення, дослідження та експлуатації приладів керування регульованим асинхронним електроприводом. Наголошено, що поява нової сучасної елементної бази спонукає до пошуку нових інженерних рішень у цій галузі.*

*Зауважено, що визначення пріоритетного способу регулювання частоти обертання асинхронного електродвигуна, як базового елемента асинхронного електропривода є багатофакторну та досить складну задачу.*

*В якості методів дослідження обрані фундаментальні положення теорії електромеханічного перетворення енергії, методи математичного аналізу елементів вентильно-електромеханічних систем, математичного моделювання, чисельні методи розв'язання диференціальних рівнянь.*

*Для математичного моделювання асинхронного електропривода з пофазноімпульсним керуванням обрано систему реальних координат статора, що дозволяє оперувати з реальними токами фаз статора, як і в неперетворених природних координатах, так і одночасно надати можливість позбутися від періодичних коефіцієнтів у виразах для індуктивностей та взаєміндуктивності потокозчеплень.*

*Враховуючи, що перетворення при побудові математичної моделі стосуються лише роторних кіл електродвигуна, а струми статора залишаються без змін, було вказано на можливість матричної її інтерпретації, розглядаючи додатково до ротора і кола фази статора.*

*Зазначено на доцільності адаптації побудованої моделі асинхронного електропривода з пофазноімпульсним керуванням до практичних розрахунків в MathCad.*

**Ключові слова:** *математична модель, асинхронний електропривод, пофазноімпульсне керування*

## **МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ АСИНХРОННОГО ЭЛЕКТРОПРИВОДА С ПОФАЗНОИМПУЛЬСНЫМ УПРАВЛЕНИЕМ**

**Н. Г. Батечко, Н. Т. Лут, С. В. Шостак, А. Зинченко**

**Аннотация.** *Рассмотрено процесс формирования математической модели асинхронного электропривода с пофазноимпульсным управлением.*

*Приведено особенности построения математических моделей электромеханических преобразователей энергии и законы физики, на которых они основываются.*

*Проведено критический анализ научных источников относительно вопросов разработки, исследования и эксплуатации приборов управления регулируемым асинхронным электроприводом. Акцентировано, что появление новой современной элементной базы приводит к поиску новых инженерных решений в этой отрасли.*

*В качестве методов исследования выбраны фундаментальные положения теории электромеханического преобразования энергии, методы математического анализа элементов вентильно-электромеханических систем, математического моделирования, численные методы решения дифференциальных уравнений.*

*Для математического моделирования асинхронного электропривода с пофазноимпульсным управлением выбрано систему реальных координат статора, что позволяет оперировать с реальными токами фаз статора как и в непреобразованных естественных координатах, так и одновременно предоставляя возможность избавиться от периодических коэффициентов в выражениях для индуктивностей и взаимоиндуктивностей в потокосцеплениях.*

*Учитывая, что преобразования при построении математической модели касаются только роторных цепей электродвигателя, а токи статора остаются без изменений, было указано на возможность матричной их интерпретации, рассматривая дополнительно к ротору и цепи фазы статора.*

*Указано на целесообразность адаптации построенной модели асинхронного электропривода с пофазноимпульсным управлением к практическим расчетам в MathCad.*

**Ключевые слова:** *математическая модель, асинхронный электропривод, пофазноимпульсное управление*