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MATHEMATICAL MODELING OF THE MATERIAL SEPARATION PROCESS ON STATIONARY SCREW SURFACES

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Abstract. *The motion of material particles on gravitational surfaces is used in special devices for their separation by physical and mechanical properties. For this purpose stationary screw surfaces of a constant step are applied.*

The purpose of the study is to investigate helical surfaces with different design parameters in order to improve their separation ability through mathematical and geometric modeling of the process without making surface models.

The problem of constructing the trajectory of a material particle on the surface under the action of its own weight is preceded by the problem of finding the trajectory on an inclined plane.

Modern software products make it possible not only to find the trajectory of the particle, but also to show it on the surface and even make an animation, which essentially replaces high-speed camera recording. This approach makes it possible to study the kinematic parameters of motion on different helical surfaces without full-scale samples of these surfaces, which significantly reduces the cost of finding the desired surfaces.

The trajectories of the particle on the surface of the helical conoid and the deployable helicoid are obtained. It is established that when moving on the surface of a helical conoid, the particle in the presence of friction first accelerates, and then stops at a considerable distance from its axis. To prevent this, its need to take a limited compartment of the conoid both in height and on its periphery. When a particle moves on the surface of a helicoid, its velocity becomes constant over time, and the trajectory after that will be a helical line.

The developed approaches make it possible to study the process of material separation not only after stabilization of motion, but also during the transition process, as it became possible to visualize it. This will allow you to choose the surface compartment of the optimal size, which will provide the desired productivity of separation due to the dispersion of particles with different coefficients of friction on its surface.

Key words: ***material particle, trajectory, material separation, helical conoid, helicoid***

Introduction. The motion of material particles on gravitational surfaces, that is the motion of particles on surfaces under the action of its own weight, is used in special devices for their separation by physical and mechanical properties. For this purpose stationary screw surfaces of a constant step are applied. In the mining industry, screw separators with different shapes of axial cross-section (gutter) are used for ore beneficiation [1]. Grain separation is carried out on linear helical surfaces, which are compartments of the oblique helicoid. Seeds with different coefficients of friction move after stabilization of the process at constant speeds along the helical lines at different distances from the axis of the helicoid. Despite the fact that such separators are passive working bodies and do not require energy costs to drive them, they also have disadvantages. This is a relatively low productivity and low separation ability (small difference in the trajectories of particles with different physical and mechanical properties) [2].

Analysis of recent research and publications. The calculation of the relationship between the kinematic parameters of motion, the coefficient of friction and the design parameters of the separator was carried out in [3, 4]. In [4] the calculations for the separator in the case when its surface is a deployable helicoid are considered and made. The transition process to stabilize the motion of a particle on such a surface is considered in detail in work [5].

Purpose. Investigate helical surfaces with different design parameters in order to improve their separation ability by mathematical and geometric modeling of the process without making surface models.

Materials and methods. Solving the problem of constructing the trajectory of a material particle on the surface under the action of its own weight is preceded by the problem of finding the trajectory on an inclined plane. If a material particle with a certain initial velocity v_o and a certain angle of inclination to the horizon falls on an inclined plane, it will move along a certain curve (in the absence of friction and air resistance, the trajectory will be a parabola). The centrifugal force caused by the curvature of the trajectory k, always acts along the main normal of the curve in the opposite direction of its direction and is determined from the expression mv^2k, where m – is the mass of the particle, v – its velocity. Since the trajectory on the plane is a flat curve, the vector of action of the centrifugal force is in the plane of the curve. The vector of this force is included in the basic equation of the dynamics of a point $m\bar{a} = \bar{F}$, where m – is the mass of the point (particle), $\bar{a}$ - the acceleration that gives it the equivalent of the forces

$\overline{F}$. applied to the point. If we take the surface, the vector of centrifugal force must be decomposed into two mutually perpendicular components: one component $mv^2k\cdot\sin\varepsilon$ acts in the plane tangent to the surface perpendicular to the direction of motion, the other $mv^2k\cdot\cos\varepsilon$ - along the normal to the surface, increasing or decreasing the pressure on surface, where ε - is the angle between the normal to the surface and the main normal of the trajectory.The expressions $k\cos\varepsilon = k_н$ and $k\sin\varepsilon = k_г$ in differential geometry are called respectively normal and geodetic curvature of the curve on the surface. Normal curvature is determined by the coefficients of the first and second quadratic forms, and geodetic - by the coefficients of only the first quadratic shape of the surface. If we write the main equation of the dynamics of a point $m\overline{a} = \overline{F}$ in the projections on the orts of the accompanying Darboux triangular trajectory, it will be reduced to a system of differential equations (detailed derivation of these equations is shown in [5]):

$$\begin{cases} v\dfrac{dv}{ds} = g\cos\psi - f(g\cos\omega + v^2k_н); \\ v^2k_г = g\cos\varphi, \end{cases} \tag{1}$$

where f – coefficient of friction; s – is the length of the trajectory arc; $g=9{,}81$ *м/с*^{2} – acceleration of free fall; ψ, φ, ω - the angles between the particle weight vector and each of the orthographs of the triangle.

System (1) does not include the mass of the particle m, because the equations were reduced to it (this is possible in the absence of other applied forces than the force of gravity mg, acting on the particle). System (1) describes the motion of a material point on a gravitational surface in the general case, where the angles ψ, φ, ω, velocity v, geodesic $k_г$ and normal $k_н$ curvature of the trajectory are functions of its arc s or other parameter that defines the curve on the surface. If the surface is given by the parametric equations $X=X(\alpha, u)$; $Y=Y(\alpha, u)$; $Z=Z(\alpha, u)$, where α and u – are independent variables, then to solve system (1) means to find such dependence between variables α and u, that at each point of a curve which is formed on a surface at the found dependence, were performed conditions of this system.

If the system (1) is used for a specific surface, it will be nonlinear and numerical methods must be used to integrate it. Modern software products make it possible not only to find the trajectory of the particle, but also to show it on the surface and even make an animation, which essentially replaces high-speed video recording. This approach makes it possible to study the kinematic parameters of motion on different helical surfaces without full-scale samples of these surfaces, which significantly reduces the cost of finding the desired surfaces.

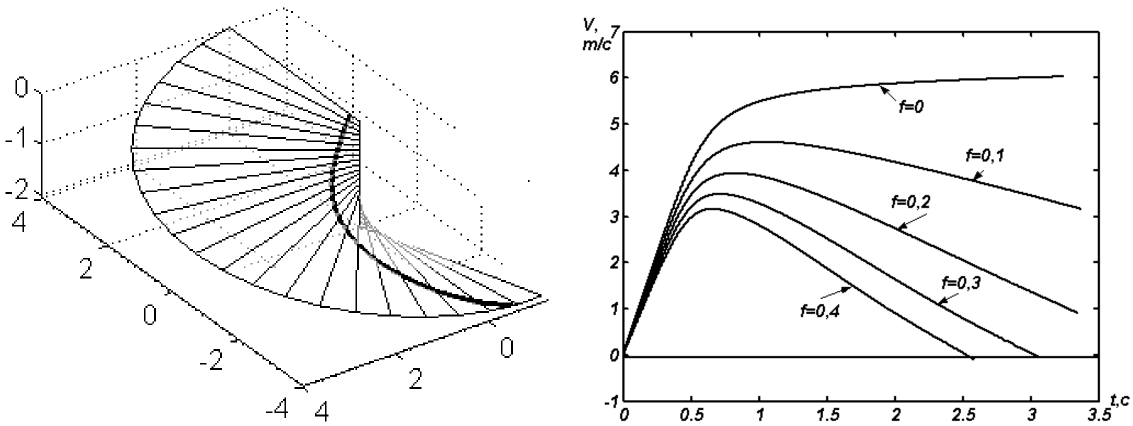
Results and discussion. Take for example some helical surfaces.

For a helical conoid given by parametric equations:

$$X = u\cos\alpha; \qquad Y = u\sin\alpha; \qquad Z = b\alpha, \tag{2}$$

where *b* – is the helical parameter (constant value), the system (1) takes the form [6]:

$$\begin{cases} v^2 \sin\beta\left(\operatorname{th}w + \dfrac{d\beta}{d\alpha}\right) = bg\cos\beta; \\ \dfrac{v\sin\beta}{b}\dfrac{dv}{d\alpha} = g\sin\beta - f\,(v^2\dfrac{\sin 2\beta}{b\operatorname{ch}w} + g\operatorname{sh}w), \end{cases} \quad \text{де} \quad w = \int \operatorname{ctg}\beta\, d\alpha. \tag{3}$$



a *b*

Fig.1. Graphic illustrations of the motion of a material particle on the surface of a helical conoid (helical parameter b = 0.6):

a - fthe trajectory of the particle from the beginning of motion to the moment of stopping;

b – the dependence of the change in the velocity of particles with different coefficients of friction

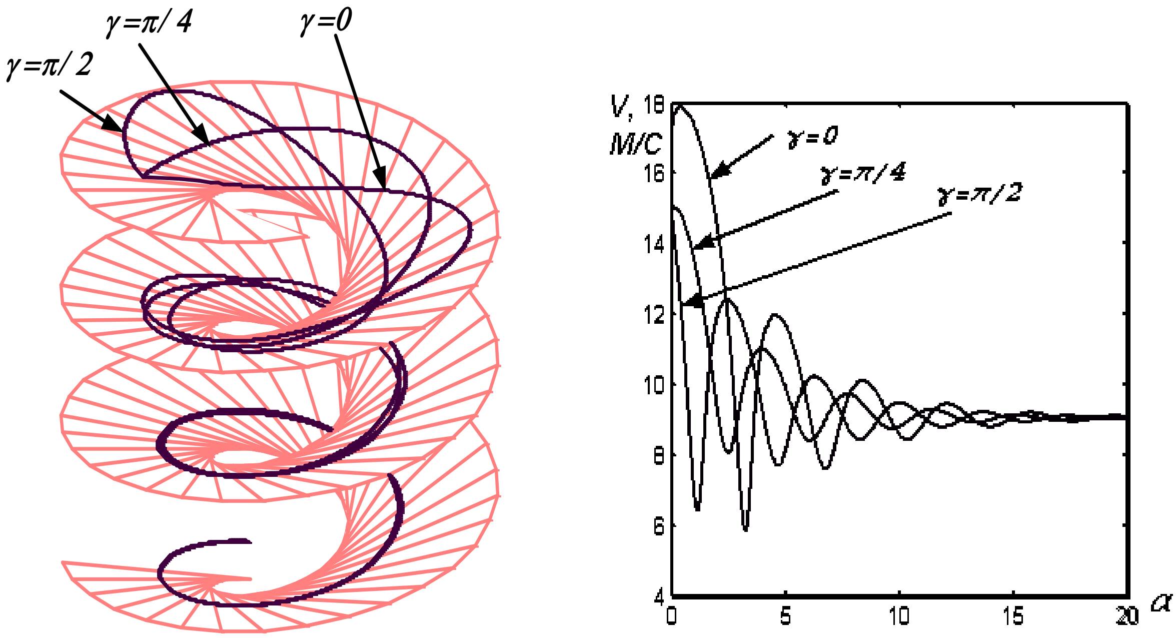
The solution of system (3) is two dependences: the velocity of the particle $v=v(\alpha)$ and the angle $\beta=\beta(\alpha)$ between the generators of the conoid and the trajectory. The integration of the system showed that the particle, starting its motion with an initial velocity close to zero, first accelerates, moves away from the conoid axis and stops (Fig. 1, *a*). This is explained by the fact that as you move away from the axis of the conoid, the angle of inclination of the trajectory to the horizontal plane decreases and there comes a time when the particle can no longer overcome the force of friction. The graph of the change in the velocity of the particle as a function of time (Fig. 1, *b*) shows that the particles with the highest coefficient of friction stop the fastest. In the absence of friction and air resistance, the particle will not stop: its velocity eventually approaches a constant value.

For a deployable helicoid given by parametric equations:

$$\begin{aligned} X &= R\cos\alpha + (R\alpha - u\cos\beta)\sin\alpha; \\ Y &= R\sin\alpha - (R\alpha - u\cos\beta)\cos\alpha; \\ Z &= u\sin\beta, \end{aligned} \tag{4}$$

where R – is the radius of the cylinder on which the helical line is located - the edge of the return surface (4), β - is the angle of elevation of the helical line,
the system (1) takes the form [5]:

$$\begin{cases} v' = \dfrac{g}{v}u'\sin\beta - f\left[\dfrac{g}{v}\cos\beta\sqrt{u'^2 + (R\alpha - u\cos\beta)^2} + \dfrac{v\sin\beta(R\alpha - u\cos\beta)}{\sqrt{u'^2 + (R\alpha - u\cos\beta)^2}}\right]; \\ u'' = \dfrac{g}{v^2}\sin\beta\left[u'^2 + (R\alpha - u\cos\beta)^2\right] + u'\dfrac{R - 2u'\cos\beta}{R\alpha - u\cos\beta} - (R\alpha - u\cos\beta)\cos\beta. \end{cases} \tag{5}$$



a *b*

Fig.2. The trajectories of the material particle and the corresponding graphs of velocities under the same initial conditions *(α_o=0, u_o=20 м, v_o=15 м/с, f=0,3)* and different directions at the beginning of the motion (γ - is the angle between the direction of the initial vector speed and rectilinear generating surface)

The solution of system (5) is the dependences $v=v(\alpha)$ and $u=u(\alpha)$. The first characterizes the change in velocity, and the second establishes the relationship between the independent variables of the surface (4) and thus sets the trajectory of the particle. In fig. 2, and shows the compartment of the surface of the deployable helicoid and the trajectory of the particles obtained by integrating the system (5). From fig. 2, and it is seen that after several revolutions of the particles, which began their movement in different directions on the surface, move further along a common trajectory, which is a helical line. Their speed also stabilizes and becomes constant after about three revolutions ($\alpha \approx 20$ *rad.* *a*ccording to Fig. 2, *b*). After stabilization of motion, , that is at $v=const$ and $\rho=const$, where ρ - is the distance from the axis of the helicoid to the particle on the surface, system (5) can be solved in elementary functions and determine the velocity v of the particle and the distance ρ depending on the design surface parameters R and β, and the coefficient of friction f [5].

Conclusions and prospects for further research. Modeling of the motion of a material particle on helical surfaces and its study by modern means of numerical integration and visualization have shown that for different surfaces the nature of the motion of the particle will also be different. When moving on the surface of the helical conoid, the particle in the presence of friction first accelerates, and then stops at a considerable distance from its axis. To prevent this, you need to take a limited compartment of the conoid both in height and on its periphery. When a particle moves on the surface of a deployable helicoid, its velocity becomes constant over time, and the trajectory after that will be a helical line.

Prospects for further research are to use the developed approaches to identify the possibility of separation of the material not only after the stabilization of the movement, but also during the transition process, as it became possible to visualize it. This will allow you to choose the surface compartment of the optimal size, which will provide the desired productivity of separation due to the dispersion of particles with different coefficients of friction on its surface. The developed approach will allow to model the motion of the particle on other helical surfaces (oblique closed helicoid, oblique open helicoid), as well as on helical surfaces with variable pitch without making models of these surfaces.

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ПРОЦЕСУ РОЗДІЛЕННЯ МАТЕРІАЛУ НА СТАЦІОНАРНИХ ГВИНТОВИХ ПОВЕРХНЯХ

С. Ф. Пилипака, А. В. Несвідомін

Анотація. *Рух матеріальних частинок по гравітаційних поверхнях використовується у спеціальних пристроях для їх сепарації за фізико-механічними властивостями. Для цього застосовуються стаціонарні гвинтові поверхні сталого кроку.*

Мета дослідження – дослідити гвинтові поверхні із різними конструктивними параметрами на предмет покращення їх роздільної здатності за допомогою математичного і геометричного моделювання процесу без виготовлення моделей поверхонь.

Розв’язанню задачі побудови траєкторії руху матеріальної частинки по поверхні під дією сили власної ваги передує задача знаходження траєкторії на похилій площині.

Сучасні програмні продукти дають можливість не тільки знаходити траєкторію руху частинки, а і показати її на поверхні і навіть зробити анімацію, яка по суті заміняє швидкісну зйомку. Такий підхід дає змогу досліджувати кінематичні параметри руху по різних гвинтових поверхнях без натурних зразків цих поверхонь, що значно здешевлює пошук потрібних поверхонь.

Отримані траєкторії руху частинки по поверхні гвинтового коноїда та розгортного гелікоїда. Встановлено, що при русі по поверхні гвинтового коноїда частинка за наявності тертя спочатку розганяється, а потім зупиняється на

значній відстані від його осі. Щоб запобігти цьому, потрібно брати обмежений відсік коноїда як по висоті, так і по його периферії. При русі частинки по поверхні розгортного гелікоїда її швидкість з часом стає постійною, а траєкторією після цього буде гвинтова лінія.

Розроблені підходи дають можливість вивчати процес сепарації матеріалу не тільки після стабілізації руху, а і під час перехідного процесу, оскільки стала можливою його візуалізація. Це дозволить підібрати відсік поверхні оптимального розміру, який забезпечить потрібну продуктивність сепарації за рахунок роззосередження частинок із різним коефіцієнтом тертя по його поверхні.

Ключові слова: ***матеріальна частинка, траєкторія руху, сепарація матеріалу, гвинтовий коноїд, розгортний гелікоїд***

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ПРОЦЕССОВ РАЗДЕЛЕНИЯ МАТЕРИАЛОВ НА СТАЦИОНАРНЫХ ВИНТОВЫХ ПОВЕРХНОСТЯХ

С. Ф. Пилипака, А. В. Несвидомин

Аннотация. *Движение материальных частиц по гравитационных поверхностях используется в специальных устройствах для их сепарации по физико-механическими свойствами. Для этого применяются стационарные винтовые поверхности постоянного шага.*

Цель исследования - исследовать винтовые поверхности с различными конструктивными параметрами на предмет улучшения их разрешающей способности с помощью математического и геометрического моделирования процесса без изготовления моделей поверхностей.

Решению задачи построения траектории движения материальной частицы по поверхности под действием силы собственного веса предшествует задача нахождения траектории на наклонной плоскости.

Современные программные продукты дают возможность не только находить траекторию движения частицы, а и показать ее на поверхности и даже сделать анимацию, которая по сути заменяет скоростную съемку. Такой подход позволяет исследовать кинематические параметры движения по разным винтовым поверхностях без натурных образцов этих поверхностей, что значительно удешевляет поиск нужных поверхностей.

Получены траектории движения частицы по поверхности винтового коноида и развертываемого геликоида. Установлено, что при движении по поверхности винтового коноида частица при наличии трения сначала разгоняется, а потом останавливается на значительном расстоянии от его оси. Чтобы предотвратить это, нужно брать ограниченный отсек коноида как по высоте, так и по его периферии. При движении частицы по поверхности развертываемого геликоида ее скорость со временем становится постоянной, а траекторией после этого будет винтовая линия.

Разработанные подходы дают возможность изучать процесс сепарации материала не только после стабилизации движения, а и во время переходного процесса, поскольку стала возможной его визуализация. Это позволит подобрать отсек поверхности оптимального размера, который обеспечит нужную *производительность сепарации за счет рассредоточения частиц с различным коэффициентом трения по его поверхности.*

Ключевые слова: ***материальная частица, траектория движения, сепарация материала, винтовой коноид, развертываемый геликоид***