

UDC 631.312

## MOVEMENT OF THE PARTICLE ON THE SURFACE OF THE CONVEYOR BELT, ARBITRARILY ORIENTED IN SPACE

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**Abstract.** *The movement of the material on the inclined belt of the conveyor takes place during transportation or its frictional cleaning. For an inclined moving plane (slide), the angle of its inclination to the horizontal plane is decisive.*

*The absolute motion of a particle is the sum of two motions - the portable belt and the relative particle along the belt, so it is affected by the angle between the vectors of the greatest inclination of the plane and the transfer velocity of the plane (tape).*

*The purpose of the study is to determine the motion of a material particle on the conveyor belt for the case when the angle between the vector of the line of greatest inclination of the conveyor plane and the direction of its transfer speed is arbitrary.*

*To do this, the conveyor belt element was depicted as a rectangle with an axis of symmetry drawn along the direction of translational movement. In the initial position, the plane was placed horizontally, so the angle of greatest inclination is absent. In the future, the plane was given an arbitrary location in space due to alternate rotation around the sides bounding its compartment or around the axes of symmetry of the compartment, which is equivalent.*

*The relative and absolute motions of the material particle along the moving web of the conveyor are considered for the case when the line of the greatest inclination of the web plane makes an arbitrary angle with the direction of the portable motion of the web. A system of differential equations of motion is compiled and solved. The obtained results are illustrated graphically.*

*It is established that the nature of the relative motion of a particle on an inclined plane moving rectilinearly and uniformly depends on the direction of the vector of the line of the greatest inclination and the value of the angle of inclination of this plane. If the angle of inclination is less than the angle of friction, then the lateral feed of the particle will eventually stop either on the curved section of the trajectory or on a straight line that is parallel to the line of greatest inclination. The stopping place of the particle depends on the value of the initial velocity. At an angle of inclination of the plane equal to the angle of friction, the particle during the movement along the curved section of the trajectory reduces its initial velocity by half and then moves in a straight line and evenly. If the angle of inclination of the plane is greater than the angle of friction, the particle in relative motion along the curvilinear section of the trajectory first reduces the velocity, and when*

*approaching a rectilinear section, its velocity increases and continues to increase on a rectilinear section of the trajectory.*

**Key words:** *material particle, conveyor, inclined plane, plane inclination angle, particle velocity*

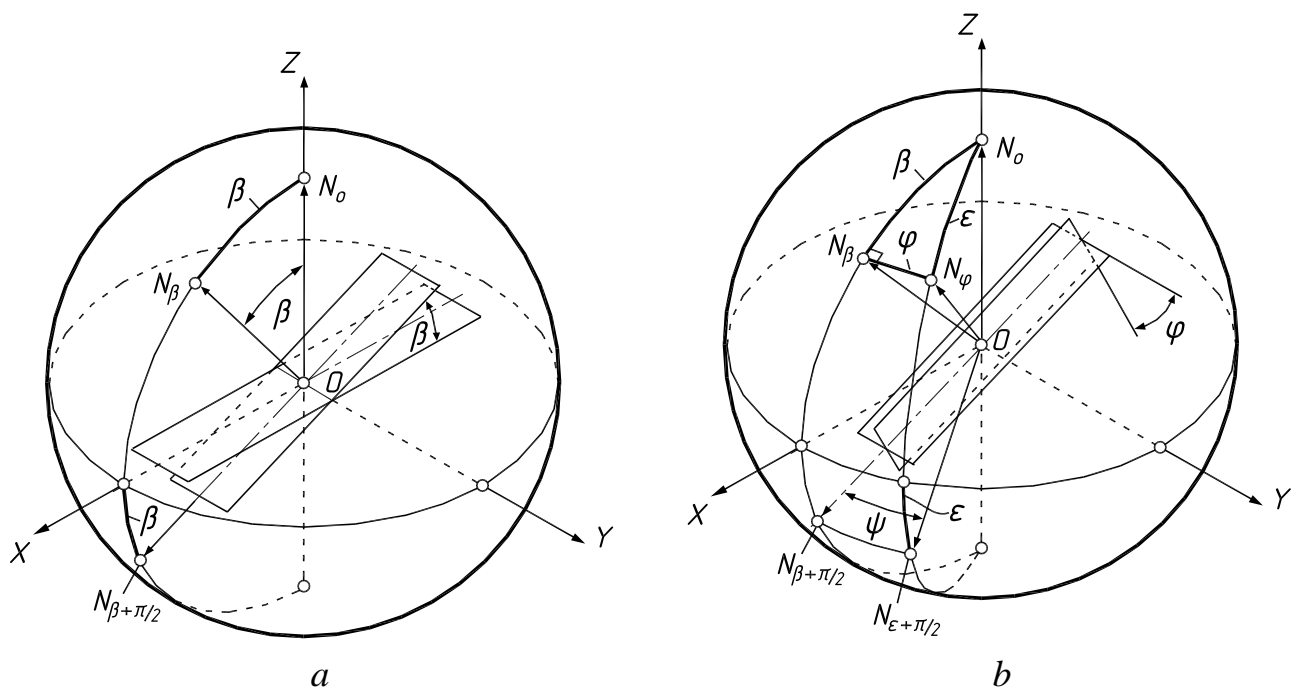
**Introduction.** The movement of the material on the inclined belt of the conveyor takes place during transportation or its frictional cleaning [1]. For an inclined moving plane (slide), the angle of its inclination to the horizontal plane is crucial. If it is less than the angle of friction of the particle in the plane, then the initial relative velocity of the particle decreases over time to zero and it then moves at an absolute speed equal to the transfer speed of the belt. If the angle of the tape is greater than the angle of friction, the absolute motion of the particle at each time is the sum of two movements - the portable motion of the belt and the relative motion of the particle on the belt.

**Analysis of recent research and publications.** The trajectory of the relative motion of a particle on an inclined plane depends on the vector of the line of its greatest inclination to the horizontal plane. The absolute motion of a particle is the sum of two motions - the portable motion of belt and the relative motion of particle on the belt, so it is affected by the angle between the vectors of the greatest inclination of the plane and the carrying speed of the plane (belt). In [1] we consider the case when these vectors coincide. In addition to this case, Vasylenko in [2] also investigated the motion of a particle on an inclined hill with a lateral motion of the belt, which corresponds to a right angle between these vectors.

**The purpose of the article.** Investigate the motion of a material particle on the conveyor belt for the case when the angle between the vector of the line of greatest inclination of the conveyor plane and the direction of its transfer velocity is arbitrary.

**Materials and methods.** The conveyor belt element is represented as a rectangle with an axis of symmetry drawn along the direction of translational movement. In the initial position, the plane is placed horizontally, so the angle of greatest inclination is absent. In the future, the plane will be given an arbitrary location in space by alternately rotating around the sides bounding its compartment or around the axes of symmetry of the compartment, which is equivalent. To find the angles that define the position of the plane

in space, it is convenient to use the formulas of spherical trigonometry. Suppose that in the initial position, when the rectangular compartment of the plane is horizontal, its point of intersection of the axes of symmetry is in the center of a sphere of unit radius (Fig. 1, a). The unit normal vector of the plane  $N_0$  coincides with the axis  $OZ$ . Rotate the compartment around the  $OY$  axis at an angle  $\beta$ . Then the normal vector of the plane will move from the position  $N_0$  to the position  $N_\beta$ , describing the arc of a large circle, which in the sphere of unit radius is numerically equal to the angle  $\beta$  ((Fig. 1, a). The line of the greatest inclination of the plane in the new position will coincide with the axis of symmetry of the compartment, and the angle of the greatest inclination will be  $\beta$  (in Fig. 1, and it is marked below the circle - the equator of the sphere).



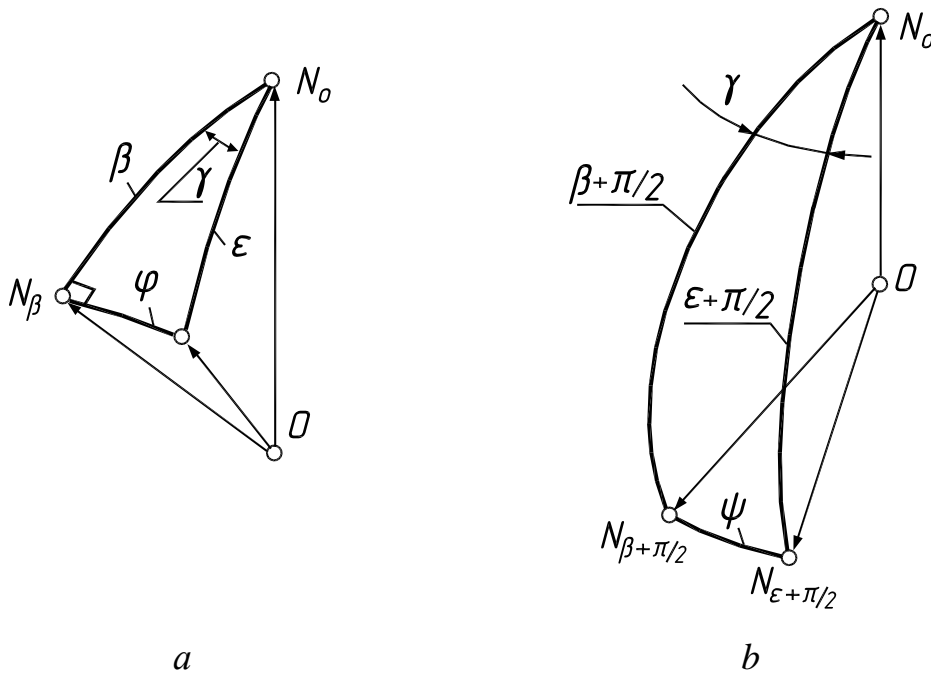
**Fig. 1. Placement and rotation of the rectangular compartment in the spherical coordinate system:**

*a* the first rotation of the compartment around the axis  $OY$  at an angle  $\beta$ ;

*b* the second rotation of the compartment around the longitudinal axis of symmetry at an angle  $\varphi$

The next rotation of the rectangular compartment is made around its longitudinal axis of symmetry at an angle  $\varphi$  (рис. 1,б). (Fig. 1, b). The normal vector of the plane will move from the position  $N_\beta$  to the position  $N_\varphi$ , while describing the corresponding arc on the

sphere equal to the angle  $\varphi$ . After this rotation, the deviation of the normal vector of the plane from the vertical position (OZ axis) will be the angle  $\varepsilon$ . Obviously, the same angle will be the angle of greatest inclination of the plane in the new position, ie after the second turn. In fig. 1, it is indicated below the equator in the corresponding vertical plane passing through the axis OZ. In this position of the plane, the angle of its greatest inclination will no longer coincide with the axis of symmetry, but will form a certain angle  $\psi$  (Fig. 1, b). Our task is to find two angles:  $\varepsilon$  and  $\psi$ . To do this, consider two spherical triangles:  $\Delta N_0 N_\beta N_\varphi$  і  $\Delta N_0 N_{\beta+\pi/2} N_{\varphi+\pi/2}$ . The first triangle is right-angled with a right angle at the vertex  $N_\beta$  (Fig. 2, a).



**Fig. 2. Spherical triangles on a sphere of unit radius:**

*a* a right triangle formed by the normal of the flat compartment when it turns;

*b* an oblique triangle to determine the angle  $\psi$

Spherical triangles are characterized by the fact that not only the vertices but also the sides are measured by angles. The vertex specifies the dihedral angle between the planes passing through the radius vector of the sphere, and the side the central angle between the radius vectors. In a right triangle (Fig. 1, a), the angle at the vertex  $N_\beta$  is straight, and the

angles correspond to the angles of successive rotations of the planes  $\beta$  and  $\varphi$ . According to the known formulas of spherical trigonometry we find the angle  $\varepsilon$ :

$$\cos \varepsilon = \cos \beta \cos \varphi. \quad (1)$$

We will also find the dihedral angle  $\gamma$ , which we will need later:

$$\operatorname{tg} \varphi = \sin \beta \operatorname{tg} \gamma \quad \text{звідки} \quad \cos \gamma = \frac{\sin \beta \cos \varphi}{\sqrt{\sin^2 \varphi + \cos^2 \varphi \sin^2 \beta}}. \quad (2)$$

From the oblique triangle (Fig. 2, b) we find the angle  $\psi$  on the known sides  $\beta+90^\circ$  and  $\varepsilon+90^\circ$  and the dihedral angle between them::

$$\cos \psi = \cos(\beta + 90^\circ) \cos(\varepsilon + 90^\circ) + \sin(\beta + 90^\circ) \sin(\varepsilon + 90^\circ) \cos \gamma. \quad (3)$$

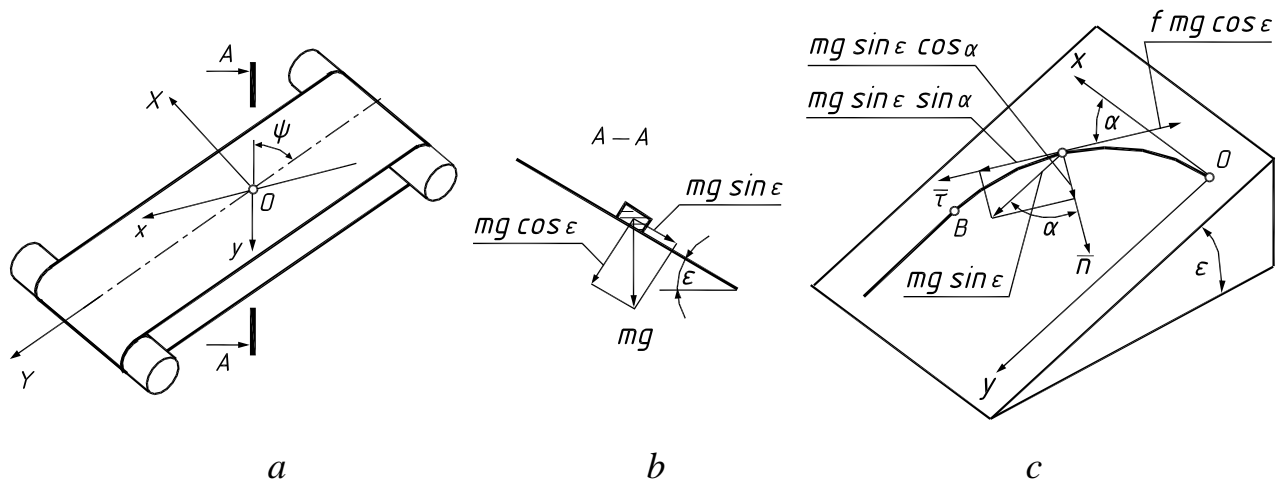
Substitute the expression  $\cos \gamma$  from (2) to (3) and after the transformations we obtain:

$$\cos \psi = \frac{\sin \beta}{\sqrt{\sin^2 \varphi + \cos^2 \varphi \sin^2 \beta}}. \quad (4)$$

Formulas (1) and (4) give the desired values of the angles by alternately rotating the rectangular compartment of the plane around its axes of symmetry at angles  $\beta$  and  $\varphi$ . or example, at angles of rotation  $\beta=5^\circ$ ,  $\varphi=10^\circ$  the angle of greatest inclination of the compartment plane to the horizon will be  $\varepsilon=11,2^\circ$ , and the line of greatest inclination in the plane of the compartment will be with its longitudinal axis of symmetry angle  $\psi=63,3^\circ$ .

**Results and discussion.** In fig. 3, and the rectangular compartment of the plane is shown in the form of a conveyor belt. At the point  $O$  of the particle hitting the conveyor, two coordinate systems are constructed in the plane of its belt: the  $OXY$  system is located so that its  $OY$  axis coincides with the direction of transfer of the web, and the  $Oxy$  system is oriented so that its  $Oy$  axis is directed along the line of greatest inclination. The angle of greatest inclination  $\varepsilon$  is shown in Fig. 3, b in the cross section of the belt by the vertical plane  $A-A$ , which passes through the vector of the greatest inclination of its plane. In the case when the second rotation of the plane at an angle  $\varphi$  will not be (Fig. 1, b), ie  $\varphi=0$ , then according to (4)  $\psi=0$  and both systems in Fig. 3, and will coincide. This means that the transfer motion will occur along the line of greatest inclination, which in this case will

coincide with the axis of symmetry  $OY$  of the conveyor and the value of the angle of greatest inclination according to (1) will be  $\varepsilon = \beta$ .



**Fig. 3. Graphic illustrations to the location of the plane in space and the movement of particles along it:**

- a* the conveyor belt with two coordinate systems on it;
- b* decomposition of the force of gravity of the particle in the vertical plane;
- c* the scheme of action of forces on a particle at lateral giving of material on an inclined plane

Getting on the conveyor belt with a certain initial velocity, the particle will make a complex movement: relative (sliding on the belt) and portable (rectilinear motion of the belt itself). It is possible to stop the relative motion of the particle (in the case when the angle of greatest inclination of the plane is less than the angle of friction); then the absolute motion will be equal to the portable motion of the canvas. With the described complex motion of the particle, the transfer motion is rectilinear, so Coriolis acceleration does not occur, which allows us to consider the dynamics of the particle in each motion separately. If the conveyor is an integral part of the unit, then when it rotates there is a Coriolis acceleration, which must be taken into account, as is done, for example, in [3]

Therefore, the trajectory of relative motion can be considered on the example of the motion of a particle in a stationary plane. A fragment of such a plane is shown in Fig. 3, c. If a particle hits a plane with initial velocity  $v_0$  in the direction of the axis  $Ox$ , it will move along a curvilinear trajectory to a certain point  $B$ , and then its motion will be rectilinear

and parallel to the axis  $Oy$  (the line of greatest inclination of the plane). If the angle  $\varepsilon$  is less than the angle of friction, the particle will stop after some time.

Find the trajectory of the particle with the described entry into the plane, which is also called the lateral feed of the material [2]. We will take into account only the force of friction; air resistance is ignored. The system of differential equations of particle motion is considered in the projections on the ords  $\bar{\tau}$  and  $\bar{n}$  the accompanying trihedron of the trajectory. Its position relative to the  $Oxy$  system will be determined by the angle  $\alpha$ , formed by the orth  $\bar{\tau}$  (tangent to the trajectory) with the  $Ox$  axis and the length  $s$  of the trajectory. The system of differential equations will be written [2]:

$$m \frac{dv}{dt} = F_{\tau}; \quad mkv^2 F_n, \quad (5)$$

where  $m$  – particle mass;

$v$  – particle velocity;

$k$  – the curvature of the trajectory is a quantity inverse to the radius of curvature:

$k=1/\rho$ .

The right-hand side of equations (5) indicates the forces applied to the particle. Such forces are the friction force  $fmg\cos\varepsilon$  ( $f$  – coefficient of friction,  $g=9,81 \text{ m/s}^2$ ), directed along the tangent to the trajectory to the side opposite to the movement and the driving force  $mgsin\varepsilon$ , directed along the line of greatest inclination. It must be decomposed into triangular orthograms through the angle  $\alpha$  (Fig. 3, c).

We pass in the expression of acceleration from the time variable  $t$  to the arc

coordinate  $s$ :  $\frac{dv}{dt} = \frac{dv}{ds} \frac{ds}{dt} = v \frac{dv}{ds}$ , since  $\frac{ds}{dt} = v$ . Let's make another substitution of the

variable, moving from the arc coordinate  $s$  to the angle  $\alpha$ :  $\frac{dv}{ds} = \frac{dv}{d\alpha} \frac{d\alpha}{ds} = k \frac{dv}{d\alpha}$ , since

$\frac{d\alpha}{ds} = k$  is by definition. So,  $\frac{dv}{dt} = vk \frac{dv}{d\alpha}$ . With all this in mind, we write system (5) in

expanded form:

$$\begin{aligned} mkv \frac{dv}{d\alpha} &= mg \sin \varepsilon \sin \alpha - fmg \cos \varepsilon; \\ mkv^2 &= mg \sin \varepsilon \cos \alpha. \end{aligned} \quad (6)$$

Equation of system (6) is reduced by the mass  $m$ . From the second equation we find:

$$k = \frac{g \sin \varepsilon \cos \alpha}{v^2}. \quad (7)$$

Substitution (7) in the first equation (6) gives a differential equation in which changes can be divided:

$$\frac{dv}{v} = \operatorname{tg} \alpha d\alpha - f \operatorname{ctg} \varepsilon \frac{d\alpha}{\cos \alpha}. \quad (8)$$

After integration (8) we obtain:

$$v = \frac{c}{\cos \alpha} \left[ \frac{1 + \cos \alpha - \sin \alpha}{1 + \cos \alpha + \sin \alpha} \right]^{f \operatorname{ctg} \varepsilon}, \quad (9)$$

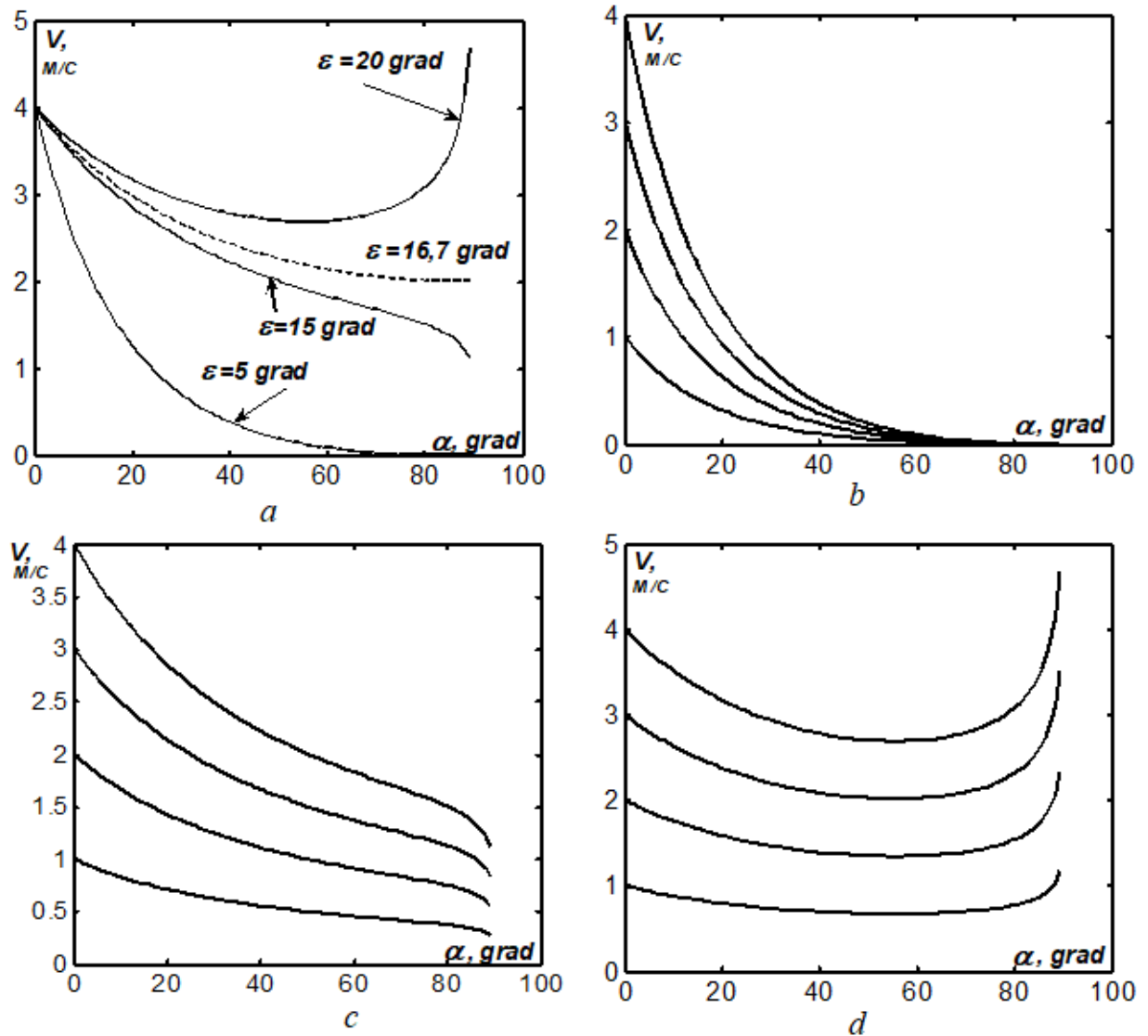
where  $c$  – is the integration constant. Based on the condition that for  $\alpha = \alpha_0$   $v = v_0$  we find the expression for the constant  $c$ :

$$c = v_0 \cos \alpha_0 \left[ \frac{1 + \cos \alpha_0 - \sin \alpha_0}{1 + \cos \alpha_0 + \sin \alpha_0} \right]^{-f \operatorname{ctg} \varepsilon}. \quad (10)$$

In particular, if  $v = v_0$  at  $\alpha = 0$ , the constant  $c = v_0$ . Substituting (9) into (7), we find the expression of curvature:

$$k = \frac{g \sin \varepsilon}{c^2} \cos^3 \alpha \left[ \frac{1 + \cos \alpha - \sin \alpha}{1 + \cos \alpha + \sin \alpha} \right]^{-2 f \operatorname{ctg} \varepsilon}. \quad (11)$$





**Fig. 4. Graph of the change in the velocity of the particle when it hits the initial velocity  $v_0$  on an inclined plane with a lateral feed of the material ( $f=0,3$ ):**

*a*  $v_0=4$  m/s at different angles  $\epsilon$  of the plane;

*b*  $\epsilon=5^\circ$  at different initial velocities;

*c*  $\epsilon=15^\circ$  at different initial velocities;

*d*  $\epsilon=20^\circ$  at different initial velocities

In fig. 4, and plots of velocity change by formula (9) at  $f=0,3$  and different values of the angle  $\epsilon$  are constructed. The initial velocity  $v_0=4$  m/c at  $\alpha_0=0$ , ie its direction at the initial moment is perpendicular to the line of greatest inclination of the plane. The angle  $\alpha$  varied from  $0^\circ$  to  $90^\circ$ . The graph shows that at  $\epsilon=5^\circ$  the particle stops at  $\alpha \approx 80^\circ$ , ie it does not go on a straight path. At  $\epsilon=15^\circ$  the particle enters a rectilinear trajectory and stops

later, because the angle  $\varepsilon$  is less than the angle of friction  $\arctg(0,3)=16,7^{\circ}$ . At  $\varepsilon=20^{\circ}$  the particle moving along the curve (corresponding to a change in the angle  $\alpha$ ) first slows down its motion, and then accelerates, moving to a straight path. The dashed line indicates the graph of the change in particle velocity for the case when the angle  $\varepsilon$  is equal to the angle of friction. It is known that in a rectilinear trajectory in this case the velocity of the particle is constant and equal to the initial value of  $v_0$ . In our case (for a curvilinear trajectory) the speed decreases and when entering a rectilinear section of the trajectory will continue to be constant. It is not possible to find its value at  $\alpha=90^{\circ}$  by formula (9), since we have an uncertainty of type  $0/0$ . We apply the limit to expression (9) for  $\alpha \rightarrow 90^{\circ}$  (the exponent  $\text{fctg } \varepsilon=1$ , since the angle  $\varepsilon$  is equal to the friction angles):

$$\lim_{\alpha \rightarrow 90^{\circ}} \frac{v_0}{\cos \alpha} \left[ \frac{1 + \cos \alpha - \sin \alpha}{1 + \cos \alpha + \sin \alpha} \right] = \frac{v_0}{2}. \quad (12)$$

Thus, when the material is applied to a plane set at an angle of friction to the horizon with an initial velocity  $v_0$  perpendicular to the line of greatest inclination, the particle during movement along a curvilinear trajectory will reduce the initial velocity by half and then move rectilinearly along the line of greatest inclination with velocity  $v_0/2$ .

In fig. 4, b and 4, c shows graphs of changes in velocity at  $\varepsilon=5^{\circ}$  and  $\varepsilon=15^{\circ}$  respectively. In the first case, the speed decreases to zero when the angle  $\alpha$  reaches a value of  $70^{\circ} - 80^{\circ}$ , and in the second does not reach zero. Therefore, at the inclination of the plane  $\varepsilon=5^{\circ}$  the particle will stop on a curvilinear trajectory, and at  $\varepsilon=15^{\circ}$  the particle will pass the curvilinear section and stop at a straight line trajectory. At an angle  $\varepsilon$ , equal to the angle of friction, the motion of the particle is determined. At an angle  $\varepsilon$  greater than the angle of friction, the particle will not stop. When moving along a curved section of the trajectory, its speed first decreases, and then increases (Fig. 4, d) and will increase further when moving in a straight line.

Find the parametric equations of the trajectory of relative motion. To do this, use the

known dependencies  $\frac{dx}{ds} = \cos \alpha$  and  $\frac{dy}{ds} = \sin \alpha$  and move on to a new variable:

$\frac{dx}{ds} = \frac{dx}{d\alpha} \frac{d\alpha}{ds} = k \frac{dx}{d\alpha}$ . For the same reasons  $\frac{dy}{ds} = k \frac{dy}{d\alpha}$ . Thus, you can write:

$$\frac{dx}{d\alpha} = \frac{\cos \alpha}{k}; \quad \frac{dy}{d\alpha} = \frac{\sin \alpha}{k} \quad (13)$$

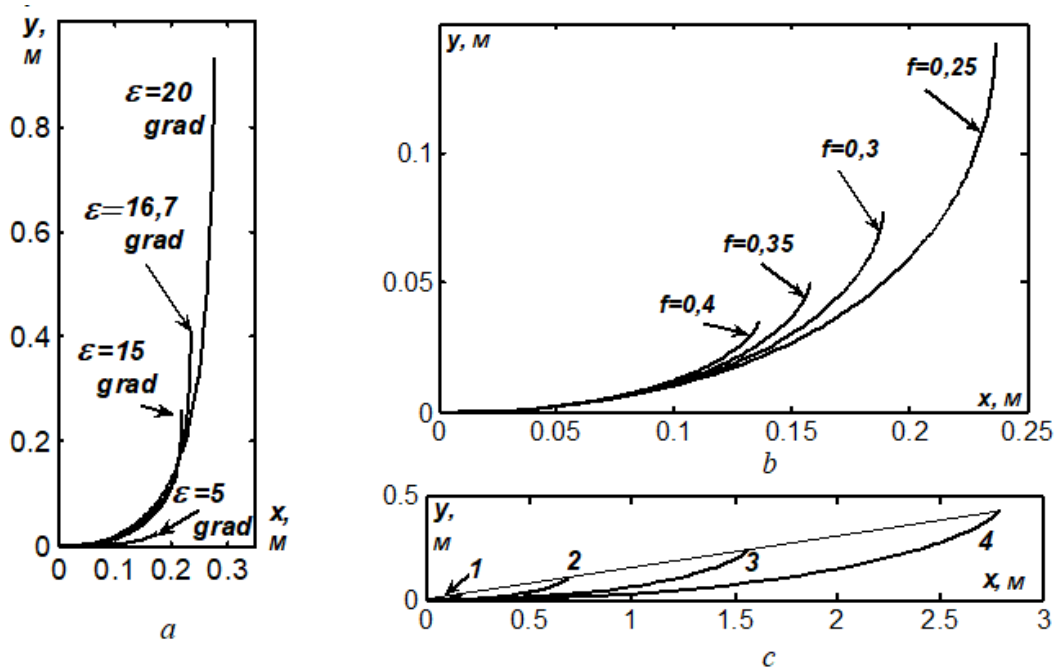
Substitute the expression of curvature (11) into (13) and integrate. After trigonometric transformations we obtain:

$$x = \frac{c^2 \left[ \frac{1 + \cos \alpha - \sin \alpha}{1 + \cos \alpha + \sin \alpha} \right]^{2 \operatorname{ctg} \varepsilon} (2f \cos \varepsilon + \sin \varepsilon \sin \alpha)}{g \cos \alpha (\sin^2 \varepsilon - 4f^2 \cos^2 \varepsilon)} + x_0;$$

$$y = \frac{c^2 \left[ \frac{1 + \cos \alpha - \sin \alpha}{1 + \cos \alpha + \sin \alpha} \right]^{2 \operatorname{ctg} \varepsilon} [2f \cos \varepsilon \sin \alpha + (1 + \sin^2 \alpha) \sin \varepsilon]}{4g \cos^2 \alpha (\sin^2 \varepsilon - f^2 \cos^2 \varepsilon)} + y_0, \quad (14)$$

where  $x_0, y_0$  - constant integrations. Their values are found provided that when  $\alpha = \alpha_0$   $x=0$  and  $y=0$ .

According to equations (14) in Fig. Figure 5 shows the trajectories of the particle under different initial conditions when changing  $\alpha = 0^\circ \dots 90^\circ$  and  $c = v_0$  at  $\alpha_0 = 0$ .



**Fig. 5. The trajectory of the particle on an inclined plane:**

- a* the trajectory of the particle at  $f=0.3$ ;  $v_0=1$  m/s and different angles  $\varepsilon$  of the plane;
- b* the trajectory of the particle at  $\varepsilon=10^\circ$ ;  $v_0=1$  m/s and different coefficients of friction;
- c* the trajectory of the particle at  $\varepsilon=5^\circ$ ;  $f=0.3$  and different initial speeds  $v_0$  (the value of the initial speed is indicated by a number)

In fig. 5, and constructed trajectories at different angles of inclination of the plane and equal other conditions. The trajectories differ mainly in the magnitude of the distance traveled. At  $\varepsilon=5^0$  the particle stops before reaching the angle  $\alpha=90^0$ .

In fig. 5, b trajectories are constructed at different values of the coefficient of friction  $f$  and equal to other conditions. The trajectories differ both in the magnitude of the distance traveled and in the trail. Particles with a coefficient of friction  $f=0,4$  and  $f=0,35$  stop before reaching the angle  $\alpha=90^0$ .

In fig. 5, in constructed trajectories at different values of the initial velocity and equal other conditions. All particles will stop before reaching the angle  $\alpha=90^0$ , and their stopping points are located on a line passing through the origin. This means that no matter what initial velocity  $v_0$  we give particles with the same coefficient of friction in the direction of the axis  $Ox$ , none will not cross this line. Studies have shown that with increasing the angle of inclination  $\varepsilon$  the picture does not change significantly, just increases the angle of inclination of the line to the axis  $Ox$ .

The resulting trajectory (14) will be the trajectory of the relative motion of the particle with respect to the  $Oxy$  system. The absolute trajectory will be the sum of two movements: the relative particle in the plane of the canvas (sliding) and the portable conveyor belt. The angle  $\psi$  between the fixed  $OXY$  and the moving  $Oxy$  coordinate systems must be taken into account. Let's project these displacements on the axis of the fixed  $OXY$  system:

$$\begin{aligned}x_{a\delta c} &= x \cos \psi - y \sin \psi; \\y_{a\delta c} &= x \sin \psi + y \cos \psi \pm v_e t,\end{aligned}\tag{15}$$

where  $\pm v_e t$  – portable movement of the conveyor belt in the direction of the  $OY$  axis (with the sign "+") or in the opposite direction (with the sign "-"). The velocity of the belt  $v_e$  is known, so we need to know the time  $t$  during which the particle is in relative motion, because during the same time it is in translational motion.

The velocity  $v = \frac{ds}{dt}$  according to (9) is known in the expression. From another expression  $k = \frac{d\alpha}{ds}$  we find:  $ds = \frac{d\alpha}{k}$ . As a result of substitution of the last expression in the first we find:

$$dt = \frac{d\alpha}{k \cdot v} \quad (16)$$

Substitute in (16) the expression for the velocity  $v$  from (9) and the curvature  $k$  from (11) and after integration we obtain:

$$t = \frac{c}{g \sin \varepsilon} \int \frac{1}{\cos^2 \alpha} \left( \frac{1 + \cos \alpha - \sin \alpha}{1 + \cos \alpha + \sin \alpha} \right)^{f \operatorname{ctg} \varepsilon} d\alpha =$$

$$\frac{c(f \cos \varepsilon + \sin \varepsilon \sin \alpha)}{g \cos \alpha (\sin^2 \varepsilon - f^2 \cos^2 \varepsilon)} \left( \frac{1 + \cos \alpha - \sin \alpha}{1 + \cos \alpha + \sin \alpha} \right)^{f \operatorname{ctg} \varepsilon} + t_0, \quad (17)$$

where the integration constant  $t_0$  is determined from the condition that  $t=0$  at  $\alpha = \alpha_0$ :

$$t_0 = - \frac{c(f \cos \varepsilon + \sin \varepsilon \sin \alpha_0)}{g \cos \alpha_0 (\sin^2 \varepsilon - f^2 \cos^2 \varepsilon)} \left( \frac{1 + \cos \alpha_0 - \sin \alpha_0}{1 + \cos \alpha_0 + \sin \alpha_0} \right)^{f \operatorname{ctg} \varepsilon}. \quad (18)$$

Thus, all the formulas for finding the relative and absolute trajectories of the particle on the conveyor belt and other kinematic parameters are obtained.

**Conclusions and prospects for further research.** The nature of the relative motion of a particle on an inclined plane moving rectilinearly and uniformly depends on the direction of the vector of the line of greatest inclination and the magnitude of the angle of inclination of this plane. If the angle of inclination is less than the angle of friction, then the lateral feed of the particle will eventually stop either on a curved section of the trajectory or on a straight line that is parallel to the line of greatest inclination. The stopping place of the particle depends on the value of the initial velocity. At an angle of inclination of the plane equal to the angle of friction, the particle during the movement along the curved section of the trajectory reduces its initial velocity by half and then moves in a straight line and evenly. If the angle of inclination of the plane is greater than the angle of friction, the particle in relative motion along the curved section of the

trajectory first decreases the velocity, and when approaching a rectilinear section, its velocity increases and continues to increase on the rectilinear section of the trajectory.

### List or references

1. Босой Е. С., Верняев О. В., Смирнов И. И., Султан-Шах Е. Г. Теория, конструкция и расчет сельскохозяйственных машин: под. ред. Е.С. Босого. М.: Машиностроение, 1977. 568 с.
2. Василенко П. М. Теория движения частицы по шероховатым поверхностям сельскохозяйственных машин. К.: УАСХН, 1960. 283 с.
3. Налобіна О. О. Про рух вороху по стрічці транспортера льонокомбайна під час його повороту. Механізація сільськогосподарського виробництва: Зб. наукових праць Національного аграрного університету. 1999. Т. V. С. 304 – 308.

### References

1. Bosoy, E. S., Vernyayev, O. V., Smirnov, I. I., Sultan-Shakh, E. G. (1977). Teoriya, konstruktsiya i raschet sel'skokhozyaystvennykh mashin [Theory, design and calculation of agricultural machines]. Moscow: Mashinostroyeniye. 568.
2. Vasilenko, P. M. (1960). Teoriya dvizheniya chastitsy po sherokhovatym poverkhnostyam sel'skokhozyaystvennykh mashin [The theory of particle motion on rough surfaces of agricultural machinery]. Kyiv: UASKHN, 283.
3. Nalobina, O. O. (1999). Pro rukh vorokhu po strichtsi transportera lonokombaina pid chas yoho povorotu [About the movement of the heap on the conveyor belt of the flax harvester during its rotation]. Mekhanizatsiia silskohospodarskoho vyrobnytstva: Zb. naukovykh prats Natsionalnoho ahrarnoho universytetu, 5, 304 – 308.

## РУХ ЧАСТИНКИ ПО ПОВЕРХНІ СТРІЧКИ ТРАНСПОРТЕРА, ДОВІЛЬНО ОРІЄНТОВАНОЇ У ПРОСТОРІ

**С. Ф. Пилипака, А. В. Несвідомін**

**Анотація.** *Рух матеріалу по похилій стрічці транспортера має місце при транспортуванні або його фрикційному очищенні. Для похилої рухомої площини (гірки) визначальне значення має кут її нахилу до горизонтальної площини.*

*Абсолютний рух частинки є сумою двох рухів – переносного стрічки і відносного частинки по стрічці, отже на нього впливає кут між векторами найбільшого нахилу площини і переносної швидкості самої площини (стрічки).*

*Мета дослідження - визначити рух матеріальної частинки по стрічці транспортера для випадку, коли кут між вектором лінії найбільшого нахилу площини транспортера і напрямом його переносної швидкості є довільним.*

*Для цього елемент стрічки транспортера був зображений у вигляді прямокутника із віссю симетрії, проведеною вздовж напрямку поступального переміщення. У початковому положенні площина була розміщена горизонтально, отже кут найбільшого нахилу відсутній. Надалі площині надавалося довільне розташування у просторі за рахунок почергового повороту навколо сторін, що обмежують її відсік або ж навколо осей симетрії відсіку, що рівнозначно.*

*Розглянуто відносний та абсолютний рухи матеріальної частинки по рухомому полотну транспортера для випадку, коли лінія найбільшого нахилу площини полотна складає довільний кут із напрямом переносного руху полотна. Складено і розв'язано систему диференціальних рівнянь руху. Отримані результати проілюстровано графічно.*

*Встановлено, що характер відносного руху частинки по похилій площині, що рухається прямолінійно і рівномірно, залежить від напрямку вектора лінії найбільшого нахилу і величини кута нахилу цієї площини. Якщо кут нахилу менший кута тертя, то при боковій подачі частинка з часом зупиниться або на криволінійній ділянці траєкторії або на прямолінійній, яка паралельна лінії найбільшого нахилу. Місце зупинки частинки залежить від величини початкової швидкості. При кутові нахилу площини, рівному кутові тертя, частинка під час руху по криволінійній ділянці траєкторії зменшує свою початкову швидкість вдвічі і далі рухається прямолінійно і рівномірно. Якщо кут нахилу площини більший кута тертя, то частинка у відносному русі по криволінійній ділянці траєкторії спочатку зменшує швидкість, а при наближенні до прямолінійної ділянки її швидкість зростає і продовжує зростати на прямолінійній ділянці траєкторії.*

**Ключові слова:** *матеріальна частинка, транспортер, похила площина, кут нахилу площини, швидкість частинки*

## **ДВИЖЕНИЕ ЧАСТИЦЫ ПО ПОВЕРХНОСТИ ЛЕНТЫ ТРАНСПОРТЕРА, ПРОИЗВОЛЬНО ОРИЕНТИРОВАННОЙ В ПРОСТРАНСТВЕ**

**С. Ф. Пилипака, А. В. Несвидомин**

**Аннотация.** *Движение материала по наклонной ленте транспортера имеет место при транспортировке или его фрикционной очистке. Для наклонной движущейся плоскости (горки) определяющее значение имеет угол ее наклона к горизонтальной плоскости.*

*Абсолютное движение частицы является суммой двух движений – переносной ленты и относительной частицы по ленте, так что на него влияет угол между векторами наибольшего наклона плоскости и переносной скорости самой плоскости (ленты).*

*Цель исследования – определить движение материальной частицы по ленте транспортера для случая, когда угол между вектором линии наибольшего наклона плоскости транспортера и направлением его переносной скорости является произвольным.*

*Для этого элемент ленты транспортера был изображен в виде прямоугольника с осью симметрии, проведенной по направлению поступательного перемещения. В исходном положении плоскость была расположена горизонтально, так что угол наибольшего наклона отсутствует. В дальнейшем плоскости предоставлялось произвольное расположение в пространстве за счет поочередного поворота вокруг сторон, ограничивающих ее отсек или вокруг осей симметрии отсека, что равнозначно.*

*Рассмотрены относительное и абсолютное движения материальной частицы по подвижному полотну транспортера для случая, когда линия наибольшего наклона плоскости полотна составляет произвольный угол с направлением переносного движения полотна. Составлена и решена система дифференциальных уравнений движения. Полученные результаты проиллюстрированы графически.*

*Установлено, что характер относительного движения частицы по наклонной плоскости, движущейся прямолинейно и равномерно, зависит от направления вектора наибольшего линии наклона и величины угла наклона этой плоскости. Если угол наклона меньше угла трения, то при боковой подаче частица со временем остановится либо на криволинейном участке траектории, либо на прямолинейной, параллельной линии наибольшего наклона. Место остановки частицы зависит от величины начальной скорости. При угле наклона плоскости, равном углу трения, частица во время движения по криволинейному участку траектории уменьшает свою начальную скорость вдвое и далее движется прямолинейно и равномерно. Если угол наклона плоскости больше угла трения, то частица в относительном движении по криволинейному участку траектории сначала сбавляет скорость, а при приближении к прямолинейному участку ее скорость возрастает и продолжает расти на прямолинейном участке траектории.*

**Ключевые слова:** *материальная частица, транспортер, наклонная плоскость, угол наклона плоскости, скорость частицы*