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CONSTRUCTION OF A LINEAR SURFACE ACCORDING TO THE CALCULATED TRAJECTORY OF THE MOVEMENT OF A MATERIAL PARTICLE ALONG IT

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Abstract. *The construction of a linear surface, which ensures the movement of a particle along a slope line, is considered. A property of such lines is a constant angle between the tangent line drawn to the curve at any point and the horizontal plane (the angle of elevation of the curve), as well as the parallelism of the main normal of the curve to the horizontal plane.*

Currently, studies of the movement of agricultural materials on working surfaces have been carried out. They showed the possibility of solving the inverse problem - designing a surface that would ensure a given trajectory of the particle's movement.

The purpose of the study is to construct a linear surface along a given trajectory of movement of a material particle under the action of its own weight.

A system of equations is obtained that describes the movement of a material particle along a linear gravitational surface.

Differential equations are solved. Specific examples are given.

A linear surface, which, with a known coefficient of friction, would ensure the movement of a particle along a helical line given by the angle of elevation and a constant curvature, as well as a linear surface, which, with a known coefficient of friction, would ensure the accelerated movement of a particle along a surface with a constant angle was constructed.

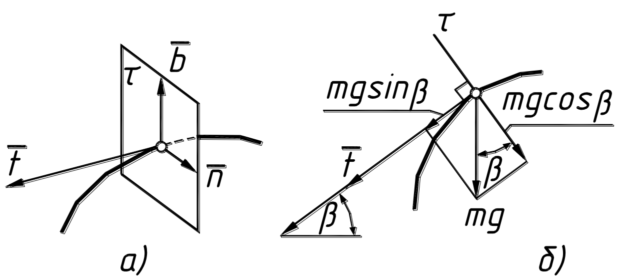
Key words: ***linear surface, trajectory of movement of a material particle, accompanying Frenet trihedron***

Introduction. The article considers the construction of a linear surface that ensures the movement of a particle along a slant line. A property of such lines is a constant angle β between the tangent line drawn to the curve at any point and the horizontal plane (the angle of rise of the curve), as well as the parallelism of the main normal of the curve to the horizontal plane.

Analysis of recent research and publications. Finding the trajectories of movement of material particles along gravitational surfaces is considered in works [2, 3]. Research concerns the movement of agricultural materials on working surfaces. These works also indicate the possibility of solving the inverse problem - the construction of such a surface that would ensure the given trajectory of the particle's movement.

The purpose of the study is to construct a linear surface along a given trajectory of movement of a material particle under the action of its own weight.

Research materials and methods. Let a material particle move along a spatial curved line. Let's attach to the particle the corresponding Frenet trihedron, whose ortho $\bar{t}$ is tangent to the curve, and orthos of the main normal $\bar{n}$ and binormal $\bar{b}$ are in the normal plane τ (Fig. 1, *a*). It is known from sketch geometry that it is possible to choose such a direction of projection in which the normal plane τ is projected into a straight line. As can be seen from Fig. 1b, which shows this case, the weight force *mg* (*m* – is the mass of the particle, kg; *g*=9,81 m/s^2), can be decomposed into two components: one component is directed along the orth $\bar{t}$ (it causes the movement of the particle), the other is in the normal plane τ.



a *b*

Fig. 1. The accompanying trihedron of the Frenet curve is the trajectory of the particle movement:

a – the normal plane τ is located arbitrarily in space; *b* – the normal plane τ is projected in a straight line

Now we will choose the direction of projection so that the ort $\bar{t}$ is projected to a point. This case is shown in Fig. 2. As it was said earlier, the angle β=*const*, so the direction of the main normal $\bar{n}$ at all points of the trajectory is parallel to the horizontal plane. Two forces act on a material particle in the normal plane: the component of the weight force $mg\ cos\beta$ and the centrifugal force mv^2k (*v* – is the velocity of the particle, *k*

– is the curvature of the trajectory). Let's start constructing the surface. Let's draw a plane μ, tangent to the future surface through the ort $\bar{t}$.

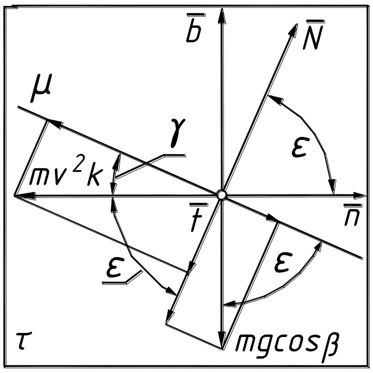


Fig. 2. Decomposition of the acting forces in the normal plane of the accompanying trihedron of the trajectory

In Fig. 2, it is projected in a straight line. Since the constructed surface is linear, the rectilinear surface will be the result of the intersection of the normal plane τ and the tangent plane μ. In order for the particle to move exactly along the given line, it is necessary that at each point of the trajectory the projections of the force of gravity and the centrifugal force on the tangent plane μ are balanced. Based on Fig. 2, we can write:

$$mv^2k\sin\varepsilon = mg\cos\beta\cos\varepsilon, \quad (1)$$

where ε - is the angle between the main normal $\bar{n}$ of the trajectory and the normal $\bar{N}$ to the surface. We project onto the orth $\bar{t}$ the forces that give the particle acceleration $\frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds}$ (t –is time, s – is the length of the arc of the trajectory):

$$mv\frac{dv}{ds} = mg\sin\beta - fR, \quad (2)$$

where f – coefficient of friction; R – is the pressure exerted by the particle on the surface. After projecting the centrifugal force and the weight force on the normal to the surface, we obtain the pressure force R (see Fig. 2):

$$R = mg\cos\beta\sin\varepsilon + mv^2k\cos\varepsilon. \quad (3)$$

By substituting (3) into (2) and reducing the resulting equation, as well as equation (1) by the mass *m*, we obtain a system of equations that describes the movement of a material particle along a linear gravitational surface:

$$\begin{cases} v\dfrac{dv}{ds} = g\sin\beta - f(g\cos\beta\sin\varepsilon + v^2 k\cos\varepsilon); \\ v^2 k\sin\varepsilon = g\cos\beta\cos\varepsilon. \end{cases} \quad (4)$$

System (4) includes four unknown quantities: dependencies $v=v(s)$; $\varepsilon=\varepsilon(s)$; $k=k(s)$ and the angle β (the coefficient f is assumed to be known). Therefore, some values must be set, and the rest must be found from system (4). If we project the surface according to a predetermined trajectory of the movement of the particle, this means that the trajectory itself must be set by the angle β and its curvature $k=k(s)$, and the dependences $v=v(s)$ and $\varepsilon=\varepsilon(s)$ can be found from the solution system connection (4).

Research results and their discussion. Let's consider examples.

Example 1. Construct a linear surface that, with a known coefficient of friction f would ensure the movement of a particle along a helical line given by the elevation angle β and a constant curvature *(k=const)*.

From the second equation of system (4), we write:

$$v^2 = \frac{g}{k}\cos\beta\,\mathrm{ctg}\,\varepsilon. \quad (5)$$

We differentiate equation (5) with respect to the parameter *s* and write it in the form:

$$v\frac{dv}{ds} = -\frac{g\cos\beta}{2k\sin^2\varepsilon}\frac{d\varepsilon}{ds}. \quad (6)$$

Substituting (5) and (6) into the first equation of system (4), after simplifications we obtain the differential equation:

$$-\frac{\cos\beta}{2k\sin^2\varepsilon}\frac{d\varepsilon}{ds} = \sin\beta - f\frac{\cos\beta}{\sin\varepsilon}. \quad (7)$$

The differential equation (7) must be integrated by numerical methods, but under the condition that the surface is perfectly smooth *(f=0)*, the variables are separated and after integration we obtain the dependence $\varepsilon=\varepsilon(s)$ in the final form:

$$\mathrm{ctg}\,\varepsilon = 2\mathrm{tg}\beta\cdot ks \text{ або } \varepsilon = \mathrm{arcctg}(2\mathrm{tg}\beta\cdot ks). \quad (8)$$

By substituting (8) into (5), we obtain the dependence of the change in the velocity of particle movement *v=v(s)*:

$$v^2 = 2g\sin\beta\cdot s = 2gH, \tag{9}$$

where $H = \sin\beta\cdot s$ - the height to which the particle descends during movement. The obtained result corresponds to Galileo's theorem, according to which the speed of a particle when moving along a perfectly smooth surface does not depend on the shape of the trajectory, but on the height *H*. From dependence (8), it can be concluded that at the initial moment of motion *(s=0)* the angle $\varepsilon = 90^o$, i.e the rectilinear surface is parallel to the horizontal plane. Next, the angle ε decreases, approaching zero, which means that the surface is gradually approaching the surface of the binormals of the helical line.

For a real surface $f \neq 0$. In this case, as shown in work [3], with the help of numerical integration, the particle is first accelerated, and then its motion stabilizes and the trajectory is a line close to a helical one. It is obvious that the increase in speed occurs up to a certain limit, until the component of the force of gravity is balanced by the force of friction. In the future, the speed will be constant and the left side of the first equation of system (4) will be equal to zero. Taking into account the mentioned solution of system (4) gives the result:

$$\sin\varepsilon = f\mathrm{ctg}\beta; \qquad v^2 = \frac{g\sin\beta}{fk}\sqrt{1 - f^2\mathrm{ctg}^2\beta}. \tag{10}$$

The next stage is the design of the surface. The parametric equations of the slope line given by the elevation angle β and the curvature *k=k(s)* have the form [1]:

$$x = \cos\beta\int\cos\left(\frac{1}{\cos\beta}\int kds\right)ds; \quad y = \cos\beta\int\sin\left(\frac{1}{\cos\beta}\int kds\right)ds; \quad z = s\sin\beta. \tag{11}$$

Since *k=const*, the integration of equations (11) leads to the parametric equations of the helical line:

$$x = \frac{\cos^2\beta}{k}\sin\frac{ks}{\cos\beta}; \qquad y = -\frac{\cos^2\beta}{k}\cos\frac{ks}{\cos\beta}; \quad z = s\sin\beta. \tag{12}$$

The helical line (12) is located on a cylinder of radius $r = \dfrac{\cos^2\beta}{k}$ and has a pitch $h = \dfrac{\pi}{k}\sin 2\beta.$ To construct a helical surface, it is necessary to draw a straight line at an angle

$\gamma = 90^0 - \arcsin(f\text{ctg}\beta)$ (Fig. 2) to the main normal, which is parallel to the horizontal plane, in the normal plane of each point of the helical line. Omitting the derivation, we present the parametric equations of the described surface:

$$\begin{aligned} X &= \frac{\cos^2\beta}{k}\sin\frac{ks}{\cos\beta} - u\left(f\text{ctg}\beta\sin\frac{ks}{\cos\beta} + \sqrt{1-f^2\text{ctg}^2\beta}\sin\beta\cos\frac{ks}{\cos\beta}\right); \\ Y &= \frac{\cos^2\beta}{k}\cos\frac{ks}{\cos\beta} - u\left(f\text{ctg}\beta\cos\frac{ks}{\cos\beta} - \sqrt{1-f^2\text{ctg}^2\beta}\sin\beta\sin\frac{ks}{\cos\beta}\right); \\ Z &= -\left(s\sin\beta + u\sqrt{1-f^2\text{ctg}^2\beta}\cos\beta\right), \end{aligned} \qquad (13)$$

where *s*, *u* – are variable parameters of the surface, respectively, the length of the arc of the trajectory and the rectilinear generator.

In Fig. 3, *a* helical linear surface is constructed according to equations (13). The trajectory of the particle is shown by a double line. Initial data: $k=0,1$; $\beta=20^0$; $f=0,3$. Calculated values according to formulas (10): $\varepsilon=55,5^0$; $v=7,96$ *m/s*. The trajectory of the movement is a spiral line on a cylinder with a radius $r=8,8$ *m* and a pitch $h=20,2$ *m*. The constructed surface differs from the surface considered in the work [3, p.327] in that the rectilinear generators are tangential to the axis of the spiral line while in the specified work they cross the axis.

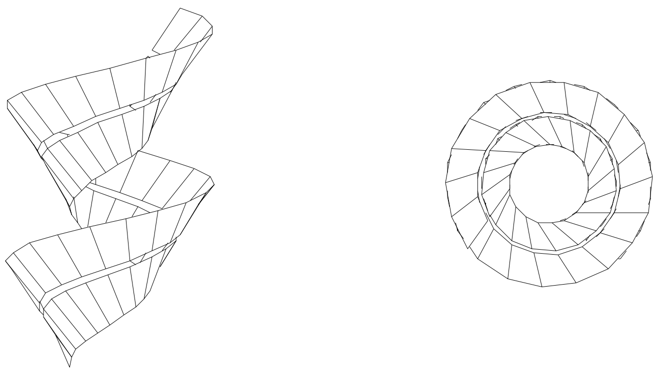


Fig. 3. A helical linear surface that ensures a constant speed of particle movement along it (on the left – frontal projection, on the right – horizontal)

Example 2. Construct a linear surface that, with a known coefficient of friction *f* would ensure the accelerated movement of a particle along the surface with a constant angle ε (ε = *const*).

The solution process will be the same as in the previous example. Differentiation of expression (5) taking into account that $\varepsilon = const$, $k = k(s)$ gives:

$$v\frac{dv}{ds} = -\frac{g\cos\beta\cos\varepsilon}{2k^2\sin\varepsilon}\cdot\frac{dk}{ds}. \tag{14}$$

After substituting (5) and (14) into the first equation of system (4), we obtain a differential equation that can be integrated:

$$\frac{dk}{k^2} = -\frac{2}{\cos\varepsilon}(\text{tg}\beta\sin\varepsilon - f)ds \quad \text{where} \quad k = \frac{\cos\varepsilon}{2(\text{tg}\beta\sin\varepsilon - f)s}. \tag{15}$$

Substitution of the expression of the curvature k from (15) into (5) gives the dependence of the change in the velocity of the particle:

$$v^2 = \frac{2g}{\sin\varepsilon}(\sin\beta\sin\varepsilon - f\cos\beta)s. \tag{16}$$

At $f=0$ expression (16) turns into expression (9). System (4) is solved because, given constants β, ε and f the curve of the trajectory in (15) and the speed of the particle movement in (16) were found. We look for the shape of the trajectory itself by integrating equations (11) while substituting the curvature expression from (15) into them:

$$\begin{aligned} x &= \frac{a\cos\beta}{1+a^2}\left[\frac{s}{a}\cos(a\ln s) + s\sin(a\ln s)\right]; \\ y &= \frac{a\cos\beta}{1+a^2}\left[\frac{s}{a}\sin(a\ln s) - s\cos(a\ln s)\right]; \\ z &= s\sin\beta, \quad \text{де} \quad a = \frac{\cos\varepsilon}{2(\sin\beta\sin\varepsilon - f\cos\beta)}. \end{aligned} \tag{17}$$

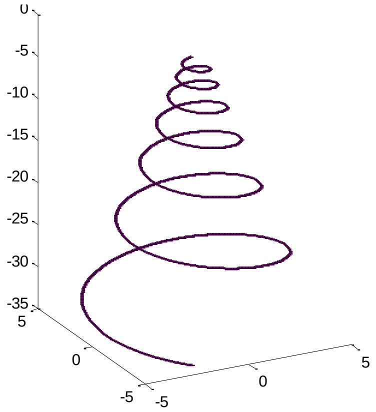


Fig. 4. A conical slope line, which is the trajectory of the movement of a material particle and at the same time the guide curve of a linear surface with a constant angle ε of the slope of the generating

In Fig. 4, a spatial curve is constructed according to equations (17), which is the calculated trajectory of the movement of a material particle and, at the same time, a guide curve for the constructed surface. The design of the surface itself is carried out in the same way as in the previous example. Initial data: $\varepsilon = 60^0$; $\beta = 20^0$; $f = 0,3$. Calculated values according to formulas (15), (16): $k = \frac{16,4}{s}$; $v = 0,563\sqrt{s}$.

Conclusions and perspectives. A system of equations is obtained that describes the movement of a material particle along a linear gravitational surface. This made it possible to construct a linear surface which, with a known coefficient of friction, would ensure the movement of a particle along a helical line given by the angle of elevation and a constant curvature, as well as a linear surface which, with a known coefficient of friction, would ensure the accelerated movement of a particle along a surface with a constant angle.

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КОНСТРУЮВАННЯ ЛІНІЙЧАТОЇ ПОВЕРХНІ ЗА РОЗРАХУНКОВОЮ ТРАЄКТОРІЄЮ РУХУ МАТЕРІАЛЬНОЇ ЧАСТИНКИ ПО НІЙ

С. Ф. Пилипака, А. В. Несвідомін

Анотація. *Розглянуто конструювання лінійчатої поверхні, яка забезпечує рух частинки по лінії укосу. Властивістю таких ліній є сталий кут між дотичною прямою, проведеною до кривої в будь-якій точці, і горизонтальною площиною (кут підйому кривої), а також паралельність головної нормалі кривої до горизонтальної площини.*

Нині проведені дослідження руху сільськогосподарських матеріалів по робочих поверхнях. Вони показали можливість розв'язання оберненої задачі – конструювання такої поверхні, яка забезпечила б задану траєкторію руху частинки.

Мета дослідження - конструювання лінійчатої поверхні за заданою траєкторією руху матеріальної частинки під дією сили власної ваги.

Одержана система рівнянь, яка описує рух матеріальної частинки по лінійчатій гравітаційній поверхні.

Розв'язано диференціальні рівняння. Наведено конкретні приклади.

Побудована лінійчата поверхня, яка при відомому коефіцієнті тертя забезпечила б рух частики по гвинтовій лінії, заданій кутом підйому і сталою кривиною, а також лінійчата поверхня, яка при відомому коефіцієнті тертя забезпечила б прискорений рух частинки по поверхні із постійним кутом.

Ключові слова: ***лінійчата поверхня, траєкторія руху матеріальної частинки, супровідний тригранник Френе***

КОНСТРУИРОВАНИЕ ЛИНЕЙЧАТОЙ ПОВЕРХНОСТИ ПО РАСЧЕТНОЙ ТРАЕКТОРИИ ДВИЖЕНИЯ МАТЕРИАЛЬНОЙ ЧАСТИЦЫ ПО НЕЙ

С. Ф. Пилипака, А. В. Несвидомин

Аннотация. *Рассмотрено конструирование линейчатой поверхности, обеспечивающей движение частицы по линии откоса. Свойством таких линий является постоянный угол между касательной, проводимой к кривой в любой точке, и горизонтальной плоскостью (угол подъема кривой), а также параллельность главной нормали кривой к горизонтальной плоскости.*

В настоящее время проведены исследования движения сельскохозяйственных материалов по рабочим поверхностям. Они показали возможность решения обратной задачи – конструирование такой поверхности, которая бы обеспечила заданную траекторию движения частицы.

Цель исследования – конструирование линейчатой поверхности по заданной траектории движения материальной частицы под действием силы собственного веса.

Получена система уравнений, описывающая движение материальной частицы по линейчатой гравитационной поверхности.

Решены дифференциальные уравнения. Приведены конкретные примеры.

Построена линейчатая поверхность, которая при известном коэффициенте трения обеспечила бы движение частицы по винтовой линии, заданной углом

подъема и постоянной кривизной, а также линейчатая поверхность, которая при известном коэффициенте трения обеспечила бы ускоренное движение частицы по поверхности с постоянным углом.

Ключевые слова: ***линейчатая поверхность, траектория движения материальной частицы, сопроводительный трехгранник Френе***