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NUMERICAL CALCULATION OF MAGNETIC FIELD INDUCTION IN THE WORK AREA LINEAR PERMANENT MAGNET MOTOR WITH A MASSIVE CONDUCTOR IN BY THE INTEGRAL EQUATIONS METHOD

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In work based on Maxwell's equations using the method of secondary sources of the mathematical model for the calculation of the magnetic field in the working area coaxial linear motor with permanent magnets considering losses due to eddy currents in massive elements of the engine has been developed.

Keywords: linear motors, permanent magnets, magnetic induction,

Using of permanent magnets as the source of the magnetic field in electromechanical devices are advantageous because the power consumption of the device is reduced. Design of new, energy-efficient and energy-saving devices requires a detailed study of the processes occurring in them, their mathematical description and computer modeling of the impact of technological and structural parameters of electrical devices are connected to electromagnetic and mechanical processes occurring in them. An important step in the simulation of non-stationary processes in electromechanical linear engines is the calculation of the magnetic field in the working area of the device.

In work [6] it is described calculation of transition processes in the magnitofugalnikh engines. Electromagnetic processes in linear engines which design has axial symmetry are considered. Calculation was carried out by method of the integrated equations taking into account influence of vortex currents, but without dynamics of the massive core movement.

In work [3] the engine design with windings of alternating current on the stator

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and windings of a direct current on an anchor is considered. The mathematical model of electromechanical process in the coaxial and linear engine on the basis of an integrated equations method, but without taking note of vortex currents on process is developed.

In work [2] the analysis methods of a dynamic condition electromagnetic vibrator with permanent magnets on the basis of the decision of Kirchhoff and Dalamber the equations system are developed. It is established analytical dependences of flux linkage and electromagnetic force on the current magnitude of a stator winding and shift of mobile part of the vibrator. Numerical modeling was carried out in the program COMSOL Multiphysics 3.5a complex where the magnetistatics problem was solved.

Research purpose. To develop an integrated equations method for calculation of an electromagnetic field taking into account existence of vortex currents in massive magnetic conductors of the coaxial and linear engine with permanent magnets.

Modeling demands the solution of three-dimensional regional tasks for Maxwell's equations in unlimited non-uniform area:

$$rot\vec{H} = \vec{\delta}\,,\ rot\vec{E} = -\frac{\partial \vec{B}}{\partial t},\ div\vec{B} = 0\,,\ div\vec{\delta} = 0\,,\ \vec{B} = \mu\vec{H}\,,\ \vec{\delta} = \gamma\left(\vec{E} + \left[\vec{V}, \vec{B}\right]\right), \quad (1)$$

At, $\vec{E}$ – the electric field, V/m; $\vec{H}$ – magnetic field A/m; $\vec{B}$ – magnetic induction, T; $\vec{\delta}$ – current density, A/m^3; γ – conductivity, Sm/m; μ – absolute permeability, H/m; $\vec{V}$ – speed of the anchor, m/s;

Formulation of the problem. Fig. 1 shows a simplified scheme of coaxial linear motor [1] [2]. It consists of coaxially arranged circular coils D_{w2}, and permanent magnets D_w toroidal steel bodies D_1, D_2, D_3 [1], [2] with the set specific conductivity γ_1, γ_2, γ_3. An engine anchor consists of a massive no ferromagnetic core conductor D_1, ferromagnetic conductive rings D_2 with absolute permeability μ_2 and oppositely magnetized along the axis of Oz permanent magnets D_w of magnetization $\vec{J}$ and springs with rigidity k. Stator is made of massive ferromagnetic conductor D_3 with absolute permeability μ_3, which we also accept independent of magnetic field intensity. Change range of magnetic field intensity in magnetic conductors lies within linear part of curve magnetization, allows to count separately a stator field and an anchor field. Value of induction of an electromagnetic field in any point is represented superposition of fields of an anchor and the stator. In grooves of the stator D_3 coils are placed D_{w2}, which are powered by a voltage source $u_2 = u_2(t)$ or current source with current $i_{w2} = i_{w2}(t)$.

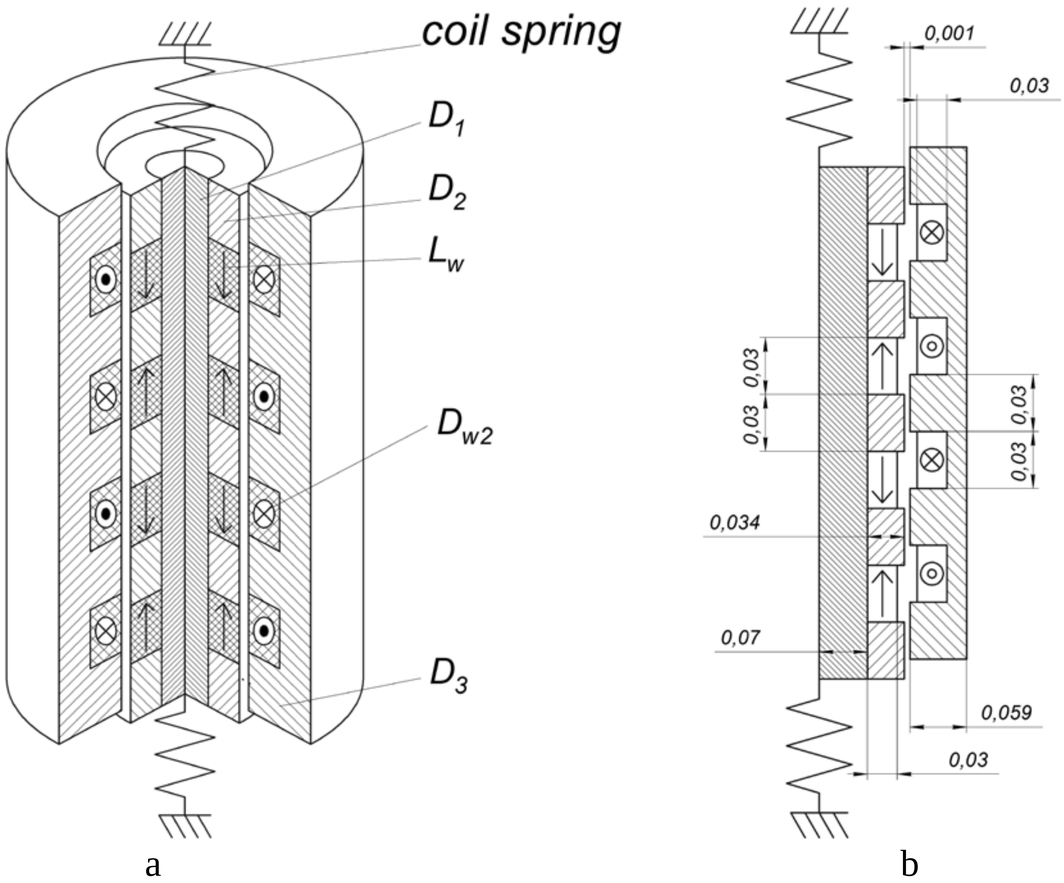

Fig. 1. A simplified scheme and cross section coaxial linear motor with permanent magnets

Considering geometrical features of a design of the coaxial and linear engine a task it is possible to consider in a wasp symmetric approach. The axis of system is combined with an axis of cylindrical system of coordinates (r, α, z).

The winding consists of identical coils which ends are connected that a field passer from them that magnetic fluxes from separate disk coils were added to each other in stator teeth N_{w_2}. Quantity rounds of each coil of the stator w_2

When a variable voltage $u_2(t)$ at a fixed stator winding pulsating magnetic fields of low frequency are made, which in the interaction with the field of permanent magnets, located on a movable anchor, and eddy currents in massive conductors leads to the emergence force, acting on the anchor and makes it to the oscillation with amplitude $\Delta z_{\max}$.

We consider known geometrical parameters of the device (Fig. 1. b), and electrical and magnetic properties of materials: γ_1, γ_2, γ_3 - conductivity material core, rings and stator; μ_1, μ_2, μ_3 – absolute magnetic permeability of core, ferromagnetic rings and stator; k – stiffness of coil spring; m – the mass of anchors.

The calculation of electromagnetic field is based on the solution of the boundary value problem for scalar and vector magnetic potentials, which is recorded on the basis of Maxwell's equations, which is formulated in the form:

$$\Delta\vec{A} = -\mu_0\vec{\delta}_w, \quad Q \in D_w, \quad D_w = D_{w2}; \tag{2}$$

$$\Delta\vec{A} = -\mu\vec{\delta}, \quad Q \in D,\ D = D_1 + D_2 + D_3; \tag{3}$$

$$\Delta\vec{A} = 0, \quad Q \in D^-; \tag{4}$$

$$\vec{A}^-(Q,t)\times\vec{n}_Q = \vec{A}^+(Q,t)\times\vec{n}_Q,\ Q \in \ell, \ell = \ell_1 \cup \ell_2; \tag{5}$$

$$\mu_0^{-1}\vec{n}_Q \times rot\vec{A}^-(Q,t) = \mu^{-1}\vec{n}_Q \times rot\vec{A}^+(Q,t),\ Q \in \ell, \ell = \ell_1 \cup \ell_2; \tag{6}$$

$$\mu_0^{-1}\vec{n}_Q \times rot\vec{A}^-(Q,t) - \mu_0^{-1}\vec{n}_Q \times rot\vec{A}^+(Q,t) = \mu_0^{-1}\vec{n}_Q \times \vec{J},\ Q \in Lw; \tag{7}$$

$$\vec{\delta}_w(Q,0) = \vec{\delta}_w^0(Q),\ \vec{\delta}(Q,0) = \vec{\delta}^0(Q); \tag{8}$$

$$\vec{A}(\infty) = 0, \tag{9}$$

At, (4)-(6) equations for the vector potential, (7)-(9) boundary conditions, (10) initial conditions, and the value of the vector potential at infinity (11)

At, $\vec{\delta}_w$ – the current density in the stator winding; $\vec{\delta}$ – the density of eddy currents in massive conductors; j=1,2,3 – index of massive object $\vec{A}^+(Q)$, $\vec{A}^-(Q)$ – the limit values of the vector potential upon approaching to the point Q to border ℓ from inside and outside; $\vec{n}_Q$ – external to the border ℓ unit vector.

Solution to the problem leads to system of differential equations in partial derivatives recorded for an infinite area. It is expedient to pass the decision of such equations by method of the integrated equations as it allows to reduce area of search of the decision to the section of massive bodies and their borders.

Using the theory of secondary sources [4] [5] boundary problem (4)-(11) is reduced to a system of integral-differential equations relative to current density of the magnetization $\sigma(Q,t)$ on the border of cores, eddy current density $\delta(Q,t)$ in massive conductors[1], [4]:

$$\sigma(Q,t)+\frac{\chi}{\pi}\oint_{l}\sigma(M,t)P(Q,M)dl_M+\frac{\mu}{\mu_0}\frac{\chi}{\pi}\oint_{D}\delta(M,t)P(Q,M)ds_M=$$

$$=-\frac{\chi}{\pi}\oint_{l_w}J(M)P(Q,M)dl_M-\frac{\chi}{\pi}\oint_{D_{w2}}\delta_{w2}(M,t)P(Q,M)ds_M,\ Q\in l=l_1+l_2, \quad (10)$$

$$\frac{\partial}{\partial t}\oint_{l}\sigma(M,t)T(Q,M)dl_M+\frac{\delta(Q,t)}{\gamma\lambda}+\frac{\mu}{\mu_0}\frac{\partial}{\partial t}\oint_{D}\delta(M,t)T(Q,M)ds_M=$$

$$=-\frac{\partial}{\partial t}\oint_{D_{w2}}\delta_{w2}(M,t)T(Q,M)ds_M+\Phi(Q,t),\ \mathrm{Q}\in D=D_1+D_2+D_3, \quad (11)$$

$$\delta(Q,0)=\delta^{(0)}(Q),\ \delta_w(Q,0)=\delta_w^{(0)}(Q),\ \sigma(M,0)=\sigma^{(0)}(M), \quad (12)$$

where $\sigma(Q,t)$ – the instantaneous value of density currents simple layer magnetization in ferromagnetic bodies point Q boundary $l=l_1+l_2$, l_1 – boundary ferromagnetic massive anchor rings, l_2 – massive border ferromagnetic stator (fig. 1, a); $\delta(Q,t)$ – instantaneous eddy current density at the point of massive cross-section of conductors; $D=D_1+D_2+D_3$; $\sigma(M,t)$, $\delta(M,t)$ – the same point M ; γ – the electrical conductivity material core rings and the stator $\chi=(\mu^+-\mu^-)/(\mu^++\mu^-)$, μ^+, μ^- – absolute magnetic permeability of ferromagnetic materials anchor and stator rings when approaching a point respectively inside and outside;

$$P(Q,M)=\vec{e}_z\left[\vec{n}_Q\times\vec{b}(Q,M)\right]=n_z(Q)b_r(Q,M)-n_r(Q)b_z(Q,M),$$

$$b_r(Q,M)=\frac{z_Q-z_M}{r_Q\sqrt{(r_Q+r_M)^2+(z_Q-z_M)^2}}\left[-K(k)+\frac{r_Q^2+r_M^2+(z_Q-z_M)^2}{(r_M-r_Q)^2+(z_Q-z_M)^2}E(k)\right],$$

$$b_z(Q,M)=\frac{1}{r_Q\sqrt{(r_Q+r_M)^2+(z_Q-z_M)^2}}\left[K(k)+\frac{r_M^2-r_Q^2+(z_Q-z_M)^2}{(r_M-r_Q)^2+(z_Q-z_M)^2}E(k)\right],$$

$$T(Q,M)=\sqrt{r_M/r_Q}\,f(k),\ f(k)=\left(\frac{2}{k}-k\right)K(k)-\frac{2}{k}E(k),\ k^2=\frac{4r_Qr_M}{(r_Q+r_M)^2+(z_Q-z_M)^2},$$

r_Q, z_Q, r_M, z_M – coordinates Q and M respectively; $K(k)$, $E(k)$ – complete elliptic integrals of the first and second kind; $\Phi(Q,t)=\gamma\left[\vec{V}(Q,t)\times\vec{B}(Q,t)\right]\vec{e}_z$ – term, responsible for the impact velocity of anchor on the distribution of the density of eddy currents; $V(Q,t)$ – speed of the anchor relative to the stator; $\vec{B}(Q,t)$ – magnetic induction, which is caused by external currents, relative to the anchor.

$$\vec{B}(Q,t)=\frac{\mu_0}{2\pi}\int_{D_{w2}}\delta_{w2}(M,t)\vec{b}(Q,M)ds_M+\frac{\mu_0}{2\pi}\int_{D_2}\delta_2(M,t)\vec{b}(Q,M)ds_M+$$

$$+\frac{\mu_0}{2\pi}\int_{l_2}\sigma_2(M,t)\vec{b}(Q,M)dl_M\ ;$$

$\delta_w^{(0)}(Q)$, $\sigma^{(0)}(M)$, $\delta^{(0)}(Q)$ – Initial density of current in the stator winding, eddy current and magnetizing current. Initial values of density of currents in a winding of the stator, vortex currents and currents of magnetization which are defined with to switching mode.

Calculation field of permanent magnets, which are magnetized parallel to the axis Oz, of the magnets is reduced to calculating the field of simple layer of density currents $\sigma_J=\left[\vec{J}\times\vec{n}\right]\vec{e}_z$ on the boundary l_w of permanent magnets [6], where $\vec{n}$ – external normal to the boundary of the magnet that is taken into account in the derivation of equation (11).

To examine reliability of the received data performed comparison of the results of numerical solution of the system of integral-differential equations (12)-(13) using (16)-(17) with the distribution of the magnetic field in the working area of the linear motor obtained through the software system COMSOL Multiphysics 3.5a.

Research results.

1. For a single pole division geometric parameters of which are given in Fig. 1, b $\mu_1 = \mu_0$, $\mu_2 = \mu_3 = 100\mu_0$, $\gamma_1 = \gamma_2 = \gamma_3 = 7,6 \cdot 10^6$ Ohm^{-1}/m, without permanent magnet was conducted calculation of r-value of the magnetic field in the gap (Fig. 2) by changing the density of the current in the coil by law:

$$\delta(t) = \delta_{\max} \sin(\pi t)$$

The maximum peak value current density $\delta_{\max} = 3 \cdot 10^6$ A/m^2. The choice of a control point for comparison of results of calculations is caused by its proximity to an anchor tooth corner that assumes the greatest heterogeneity of a field

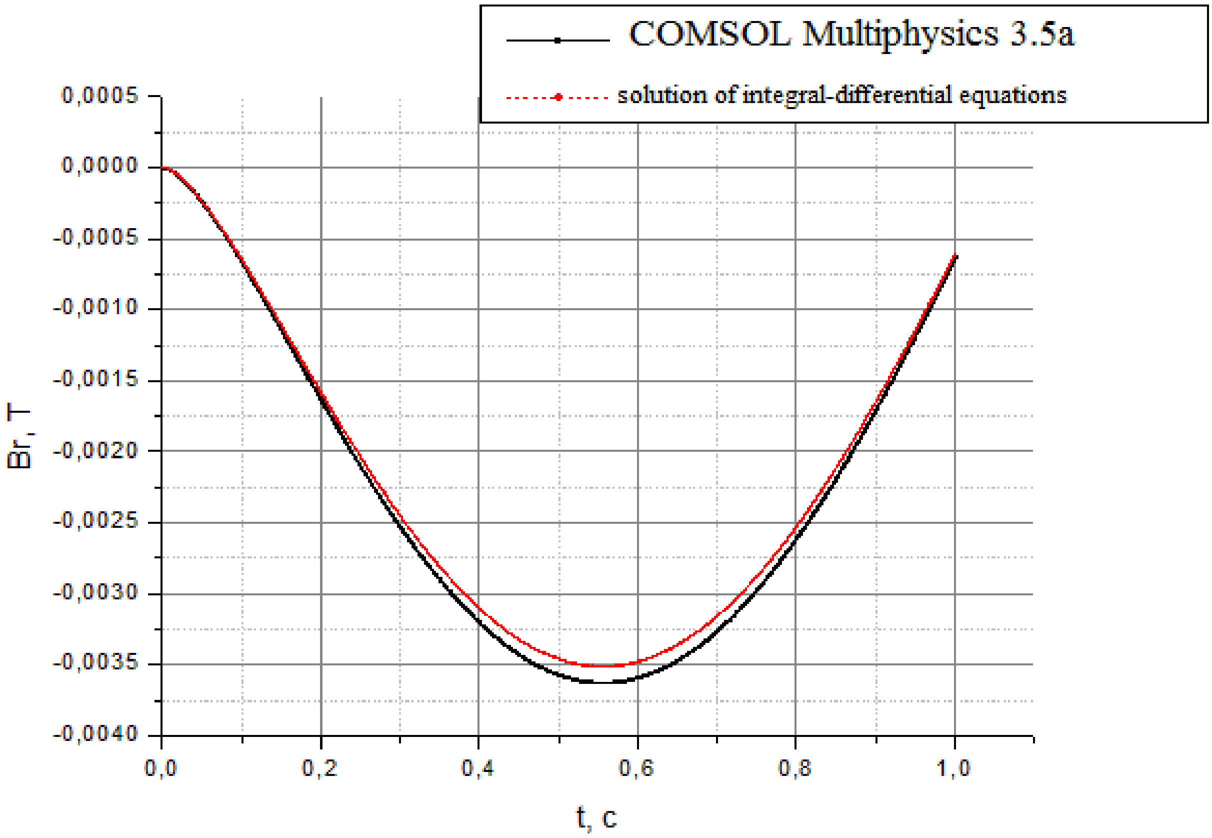

Fig. 2. The dependence of the r-value of the magnetic field on time at coordinates (0,1045;-0,014).

Mean square error is less than 3%.

2. Has the calculation of r-components of the magnetic field in the control point coordinates (0.1045, -0.014), conditioned only permanent magnet of magnetization $\vec{J}$=950 kA/m. When using the method of integral equations r-component of the magnetic field $B_r = -0.230$ T, and the calculation using program

complex COMSOL Multiphysics 3.5a $B_r = -0.242$ T, a difference of 5% is within acceptable error.

3. For numerical calculations simplification of further numerical experiment on influence of longitudinal regional effect on distribution of an electromagnetic field and loss on vortex currents was made here (Fig. 3) that will simplify the numerical solution of the system of integral-differential equations. For given geometrical parameters (Fig. 1. b) and $\mu_1 = \mu_0$, $\mu_2 = \mu_3 = 100\mu_0$, $\gamma_1 = \gamma_2 = \gamma_3 = 7{,}6 \cdot 10^6$ Ohm^{-1}/m, the maximum peak value current density $\delta_{max} = 3 \cdot 10^6$ A/m, with magnetization of permanent magnets $\vec{J}$=950 kA/m calculation of instantaneous values of the density of eddy currents in massive conductors depending on the number pairs of poles was provide. The study was conducted for the number of pole pairs $n = 2$, to $n = 20$.

The control point is in the section of the conductor of a ferromagnetic ring D_2

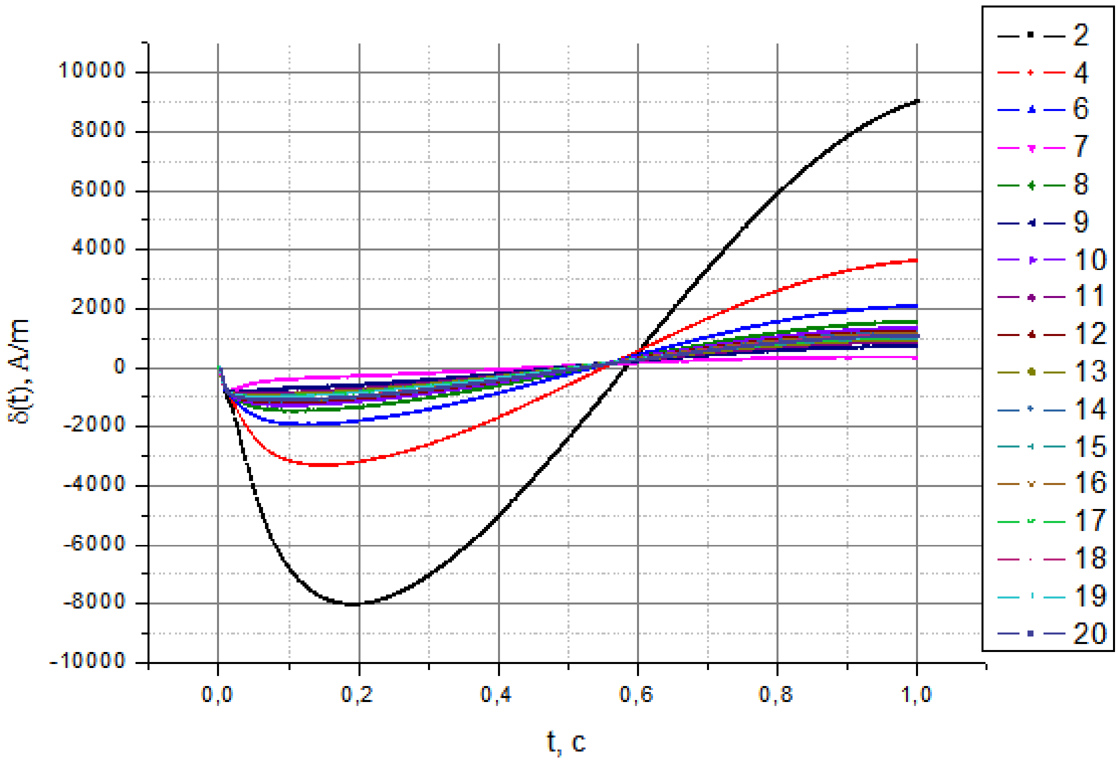

Fig. 3. The dependence of the instantaneous value of the density of eddy currents at coordinates (0.0276, 0.054) on the number of pole pairs

Established only in $n > 11$ the difference between values of density of vortex currents, at increase in number of couples of poles, doesn't exceed 5%

Conclusions

The method of numerical calculation of an electromagnetic field in the coaxial and linear engine with permanent magnets by method of the integrated equations is developed, it allows to consider influence of vortex currents in the massive carrying-out device elements.

Influence of longitudinal regional effect in the coaxial and linear engine needs to be considered, if number of couples of poles in it less than 11.

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ЧИСЕЛЬНИЙ РОЗРАХУНОК ІНДУКЦІЇ МАГНІТНОГО ПОЛЯ В РОБОЧІЙ ЗОНІ ЛІНІЙНОГО ДВИГУНА З ПОСТІЙНИМИ МАГНІТАМИ ЗА НАЯВНОСТІ МАСИВНИХ ПРОВІДНИКІВ МЕТОДОМ ІНТЕГРАЛЬНИХ РІВНЯНЬ

А.В. Жильцов, Д.С. Сорокін

В роботі на основі системи рівнянь Максвела з використанням методу вторинних джерел розроблено математичну модель для розрахунку індукції магнітного поля в робочій області коаксіально-лінійного двигуна з постійними магнітами з урахуванням втрат на вихрові струми в масивних елементах двигуна.

Ключові слова: індукція магнітного поля, лінійний двигун, постійні магніти

ЧИСЛЕННЫЙ РАСЧЕТ ИНДУКЦИИ МАГНИТНОГО ПОЛЯ В РАБОЧЕЙ ЗОНЕ ЛИНЕЙНОГО ДВИГАТЕЛЯ С ПОСТОЯННЫМИ МАГНИТАМИ ПРИ НАЛИЧИИ МАССИВНЫХ ПРОВОДНИКОВ МЕТОДОМ ИНТЕГРАЛЬНЫХ УРАВНЕНИЙ

А.В. Жильцов, Д.С. Сорокин

В работе на основе системы уравнений Максвелла с использованием метода вторичных источников разработана математическая модель для расчета индукции магнитного поля в рабочей области коаксиально-линейного двигателя с постоянными магнитами с учетом потерь на вихревые токи в массивных элементах двигателя.

Ключевые слова: индукция магнитного поля, линейный двигатель, постоянные магниты