UDC 514.18

COMPUTER MODEL OF A PARTICLE ON A SLOPING ROUGH DISK

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Abstract. *The aim - to develop Maple-model of a particle on old rough drive in polar coordinates.*

Based on the proposed method of forming laws of motion of particles on rough surfaces in the interior of coordinates projected on orts accompanying trihedron was founded maple-model of a particle on the old disk. Due developed computer tools has become possible to perform online research trajectory-kinematic characteristics of a particle on the old disk. This enabled us to analyze the motion of particles in rough old drive in polar coordinates.

Keywords: ***material particle surface is rough, sloping disk***

Topicality. In scattering machines for fertilization by gravity particles coming from the hopper cars on inclined planes, then falls on the rotating disc, where the centrifugal force rises from it and provides free movement in the air before falling to the surface of the field [1]. Understanding the patterns of movement of particles in three dimensions allows purposefully to calculate structural and kinematic parameters of operating these machines.

Analysis of recent research and publications. Analytical output law of motion of particles on the rough surface is reduced to a compilation of differential equations of 2nd order. Sequence analytical output of differential equations and methods of solution is very time-consuming. [2]

Computer simulation of a particle on the surface allows to remove bulky analytic transformation and interactively to provide for the necessary computational experiments on the analysis of a particle at different baseline throwing it on any rough surface. But the development of computer models of a particle on the surface needs to address a number of theoretical and practical nature. First, is the development of general algorithm for automatic withdrawal of differential equations law of motion of particles on any surface that is randomly located in space; trajectory analysis, kinematic characteristics of a particle not only in time, but depending on the position of the particles on the surface and the direction of its movement on the surface; illustrate the results of research in the form of numerical data, graphics and motion simulations particles on the surface [3].

The aim - to developMaple-model of a particle on old rough drive in polar coordinates.

Materials and methods. A characteristic feature of the model of a particle on a plane is a parametric setting it in a rectangular coordinate system. But there are other systems parametric setting plane. Thus, the study of a particle on a plane that rotates around the axis perpendicular to it, using a polar coordinate system default flat compartment [1, 52]. Although the trajectory-kinematic characteristics of a particle will not depend on the choice of the coordinate plane setting, but the kind of analytical transformations formation law of motion of particles in a given inclined plane in polar coordinates will vary.

The results of research. Write parametric equations old flat disk as:

$$\boldsymbol{R}(u,v) = \boldsymbol{R}[v\,cos(u)\,cos(\xi), v\,sin(u), -v\,sin(u)\,sin(\xi)], \quad (1)$$

де $u \in [0; 2\pi]$, $v \in [0; v_n]$ - independent curvilinear coordinates flat disk;
ξ - horizontal rotation angle of a flat disk around the axisOy.

The first quadratic form$ds^2$of flat disk$\boldsymbol{R}(u,v)$ is:

$$ds^2 = v^2 du^2 + dv^2, \quad (2)$$

whereE, F, G - 1st coefficients quadratic forms under equal:

$$E = v^2, F = 0, G = 1. \quad (3)$$

Since the ratio *F = 0*, then the flat disc (1) is orthogonal to *u, v* - coordinate line. Substituting expressions *u = u (t), v = v (t)* desired trajectory of particles in the inner *u, v*- coordinates the equation (3) flat disk ***R*** *(u, v),* where we get the particle trajectory *r (t)* as:

$$\boldsymbol{r}(t) = \boldsymbol{r}\big[v(t)\,cos\big(u(t)\big)\,cos(\xi), v(t)\,sin\big(u(t)\big), -v(t)\,sin\big(u(t)\big)\,sin(\xi)\big]. \quad (4)$$

Drive from the equation ***R*** *(u, v)* and trajectory *r (t)* particles to define it:

vector tangent $\boldsymbol{\tau}(t)$ and trajectory $\boldsymbol{r}(t)$:

$$\boldsymbol{\tau}(t) = \frac{d}{dt}\boldsymbol{r}(t) = \boldsymbol{\tau}\begin{bmatrix} cos(\xi)\left(\frac{d}{dt}v(t)\,cos\big(u(t)\big) - v(t)\frac{d}{dt}u(t)\,sin\big(u(t)\big)\right), \\ \frac{d}{dt}v(t)\,sin\big(u(t)\big) + v(t)\frac{d}{dt}u(t)\,cos\big(u(t)\big), \\ sin(\xi)\left(v(t)\frac{d}{dt}u(t)\,sin\big(u(t)\big) - \frac{d}{dt}v(t)\,cos\big(u(t)\big)\right) \end{bmatrix} \quad (5)$$

speed $V(t)$ of particle:

$$V(t) = |\boldsymbol{\tau}(t)| = \sqrt{v(t)^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2}; \quad (6)$$

curvature *k (t)* trajectory *r (t)* particles:

$$k(t) = \frac{2\frac{d}{dt}u(t)\left(\frac{d}{dt}v(t)\right)^2 + v(t)\left(\frac{d}{dt}v(t)\frac{d^2}{dt^2}u(t) - \frac{d}{dt}u(t)\frac{d^2}{dt^2}v(t)\right) + v(t)^2\left(\frac{d}{dt}u(t)\right)^3}{\left(v(t)^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2\right)^{3/2}}; \quad (7)$$

normal vector $\boldsymbol{N}(t)$ to the surface of $\boldsymbol{R}(u,v)$ along the trajectory $\boldsymbol{r}(t)$:

$$\boldsymbol{N}(t) = \boldsymbol{N}[-v(t)\,sin(\xi), 0, -v(t)\,cos(\xi)]; \quad (8)$$

cosines of the angles ε and η between the respective vectors $\boldsymbol{n}$, $\boldsymbol{N}$ i $\boldsymbol{G}$:

$$C\varepsilon(t) = cos(\varepsilon) = cos(\widehat{\boldsymbol{n},\boldsymbol{N}}) = 0, \quad (9)$$

$$C\eta(t) = cos(\eta) = cos(\widehat{\boldsymbol{G},\boldsymbol{N}}) = cos(\xi). \quad (10)$$

Analytical expressions normal vector ***n*** *(t)* the trajectory of a particle and its acceleration ***w*** *(t)* is somewhat cumbersome, and therefore is not given.

Obtained above expressions can determine the centrifugal force $F_C(t)$ and normal reaction force F_N particle along its trajectory $\boldsymbol{r}(t)$:

$$F_C(t) = m\,V(t)^2\,k(t) = \frac{m\left(2\frac{d}{dt}u(t)\left(\frac{d}{dt}v(t)\right)^2 + v(t)\left(\frac{d}{dt}v(t)\frac{d^2}{dt^2}u(t) - \frac{d}{dt}u(t)\frac{d^2}{dt^2}v(t)\right) + v(t)^2\left(\frac{d}{dt}u(t)\right)^3\right)}{\left(v(t)^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2\right)^{\frac{3}{2}}}, \quad (11)$$

$$F_N(t) = mg\,cos(\varepsilon) + F_C\,cos(\eta) = mg\,cos(\xi). \quad (12)$$

To generate the law of motion of the particles is necessary to determine the time *t* orts $\boldsymbol{R}'_u \equiv \boldsymbol{u}$ i $\boldsymbol{R}'_v \equiv \boldsymbol{v}$local coordinate system$\boldsymbol{OuvN}$, which is adjacent to the *u*, *v*-coordinate lines of a flat disk$\boldsymbol{R}(u,v)$:

$$\boldsymbol{R}'_u = \frac{d}{dv}\boldsymbol{R}(u,v) = \boldsymbol{R}'_u[-v(t)sin(u(t))cos(\xi),\ v(t)\,cos(u(t)),\ v(t)sin(u(t))sin(\xi)] \quad (13)$$

$$\boldsymbol{R}'_v = \frac{d}{dv}\boldsymbol{R}(u,v) = \boldsymbol{R}'_v[cos(u(t))\,cos(\xi), sin(u(t)), -cos(u(t))\,sin(\xi)]; \quad (14)$$

value of acceleration$W(t)$:

$$W(t) = \sqrt{\left(\left(\frac{d^2}{dt^2}v(t)\right)^2 - v(t)\left(\frac{d}{dt}u(t)\right)^2\right)^2 + 4\frac{d}{dt}u(t)\frac{d}{dt}v(t)\left(v(t)\frac{d^2}{dt^2}u(t) + \frac{d}{dt}u(t)\frac{d}{dt}v(t)\right) + \left(v(t)\frac{d^2}{dt^2}u(t)\right)^2} \quad (15)$$

cosines of the angles between the vector ***w** (t)* and vectors$\boldsymbol{R}'_u$ i $\boldsymbol{R}'_v$:

$$Cwu(t) = cos(\widehat{\boldsymbol{w},\boldsymbol{R}'_u}) = \frac{2\frac{d}{dt}u(t)\frac{d}{dt}v(t) + v(t)\frac{d^2}{dt^2}u(t)}{W(t)}, \quad (16)$$

$$Cwv(t) = cos(\widehat{\boldsymbol{w},\boldsymbol{R}'_v}) = \frac{\frac{d^2}{dt^2}v(t) - v(t)\left(\frac{d}{dt}u(t)\right)^2}{W(t)}; \quad (17)$$

cosines of the angles between the vector **G**[0,0, -1] and vectors$\boldsymbol{R}'_u$ i $\boldsymbol{R}'_v$:

$$CGu(t) = cos\left(\widehat{\boldsymbol{G},\boldsymbol{R}'_u}\right) = -\,sin(u(t))\,sin(\xi), \quad (18)$$

$$CGv(t) = cos\left(\widehat{\boldsymbol{G},\boldsymbol{R}'_v}\right) = cos(u(t))\,sin(\xi); \quad (19)$$

cosines of the angles between the vector ***τ** (t)* and vectors$\boldsymbol{R}_u$ i $\boldsymbol{R}'_v$:

$$C\tau u(t) = cos(\widehat{\boldsymbol{\tau},\boldsymbol{R}'_u}) = \frac{v(t)\frac{d}{dt}u(t)}{\sqrt{v(t)^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2}}, \quad (20)$$

$$C\tau v(t) = cos(\widehat{\boldsymbol{\tau},\boldsymbol{R}'_v}) = \frac{\frac{d}{dt}v(t)}{\sqrt{v(t)^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2}}. \quad (21)$$

Then the law of motion of particles in the projection of ***u*** and ***v*** orty local coordinate system ***OuvN*** to rough flat disc ***R (u, v):***

$$\begin{cases} Ou := m\left(2\frac{d}{dt}u(t)\frac{d}{dt}v(t) + v(t)\frac{d^2}{dt^2}u(t)\right) = -mg\sin(u(t))\sin(\xi) - \frac{mf\ v(t)\frac{d}{dt}u(t)\cos(\xi)}{\sqrt{v(t)^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2}} \\ Ov := m\left(\frac{d^2}{dt^2}v(t) - v(t)\left(\frac{d}{dt}u(t)\right)^2\right) = mg\ \cos(u(t))\sin(\xi) - \frac{mf\ \frac{d}{dt}v(t)\cos(\xi)}{\sqrt{v(t)^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2}} \end{cases} \quad (22)$$

The initial terms of the solution of differential equations are:

$$Oi = \frac{d}{dt}u(0) = \frac{V_o \sin(\alpha_o)}{v_o}, u(t_0) = u_o, \frac{d}{dt}v(0) = V_o\cos(\alpha_o), v(t_0) = v_o, \quad (23)$$

whereα_o - the angle between the vector of initial velocity $V_o$and the axis$Oz \equiv \boldsymbol{v}$;

V_o - initial velocity$V(t_0)$particles at the beginning $t_0 = 0$its movement;

$u(t_o) = u_o, v(t_o) = v_o$- inside *u*, *v*-coordinates of the position of the particle.

To generate lawprojected on orts ***T*** i ***P*** trihedron Darboux vector ***P OTPN*** define a tangent vector product$\boldsymbol{\tau}(t)$ та нормалі $\boldsymbol{N}(t)$:

$$\boldsymbol{P}(t) = \boldsymbol{\tau}(t) \times \boldsymbol{N}(t) =$$

$$\boldsymbol{P}\begin{bmatrix} v(t)\cos(\xi)\left(\frac{d}{dt}v(t)\sin(u(t)) + v(t)\frac{d}{dt}u(t)\cos(u(t))\right), \\ v(t)\cos(\xi)\left(\frac{d}{dt}v(t)\sin(u(t)) + v(t)\frac{d}{dt}u(t)\cos(u(t))\right), \\ v(t)\cos(\xi)\left(\frac{d}{dt}v(t)\sin(u(t)) + v(t)\frac{d}{dt}u(t)\cos(u(t))\right) \end{bmatrix}. \quad (24)$$

Cosine anglesψ, χ i φbetween vectors$\boldsymbol{P}, \boldsymbol{n}$ i $\boldsymbol{G}$is:

$$C\psi(t) = \cos(\widehat{\boldsymbol{P},\boldsymbol{G}}) = -\frac{\sin(\xi)\left(v(t)\frac{d}{dt}u(t)\sin(u(t)) - \frac{d}{dt}v(t)\cos(u(t))\right)}{\sqrt{v(t)^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2}}, \quad (25)$$

$$C\chi(t) = \cos(\widehat{\boldsymbol{P},\boldsymbol{n}}) = -1, \quad (25)$$

$$C\varphi(t) = \cos(\widehat{\boldsymbol{G},\boldsymbol{\tau}}) = -\frac{\sin(\xi)\left(v(t)\frac{d}{dt}u(t)\sin(u(t)) - \frac{d}{dt}v(t)\cos(u(t))\right)}{\sqrt{v(t)^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2}}. \quad (26)$$

Then the law of motion of particles in a disk inclined rough projections on orty ***T*** and ***P*** trihedron Darboux ***OTPN:***

$$\begin{cases} OT = m\left(v(t)^2 \frac{d}{dt}u(t)\frac{d^2}{dt^2}u(t) + \frac{d}{dt}v(t)\left(\frac{d^2}{dt^2}v(t) + v(t)\left(\frac{d}{dt}u(t)\right)^2\right)\right) = \\ mg\left(cos(u(t))\frac{d}{dt}v(t) - sin(\xi)\left(v(t)\,sin(u(t))\frac{d}{dt}u(t)\right) + mf\;cos(\xi)\sqrt{v(t)^2\left(\frac{d}{dt}u(t)\right)^2 + \left(\frac{d}{dt}v(t)\right)^2}\right) \\ OP = m\left(\frac{d}{dt}v(t)\left(2\frac{d}{dt}u(t)\frac{d}{dt}v(t) + v(t)\frac{d^2}{dt^2}u(t)\right) - v(t)\frac{d}{dt}u(t)\left(\frac{d^2}{dt^2}v(t) - v(t)\left(\frac{d}{dt}u(t)\right)^2\right)\right) = \\ mg\;sin(\xi)\left(\frac{d}{dt}v(t)\,sin(u(t)) + v(t)\frac{d}{dt}u(t)\,cos(u(t))\right) \end{cases} \quad (27)$$

To solve this system of differential equations can only be approximated.

Figure constructed trajectory ***r*** *(t)* particles on the rough drive for different values of the angle of throwing $\alpha_o = 0°, 30°, 60°, 90°$, if the initial velocity is $V_o = 4\ m/s$, the friction coefficient $f = 0.3$, starting position $u_o = \pi$, $v_o = 2$ and $\xi = 30°$ angle of inclination of the disk to plane OxyNone of the particles on the disk stops because $f = 0.3 < \tan(\xi = 30°)$. Rectilinear trajectory of particles thrown at an angle $\alpha_o = 0°$ (up from the center of the disc), will go through it - the point *O*. If the initial position of the particle to the horizontal drive generators- $u_o = \pi$, $v_o = 2$, then the trajectory of the initial conditions the particles will already be on the other side of the disc diameter (Figure, b). The trajectories of particles thrown in one direction $\alpha_o = 120°$, but with different initial velocity $V_o = 2, 4, 6, 8\ m/s$, shown in Figure c.

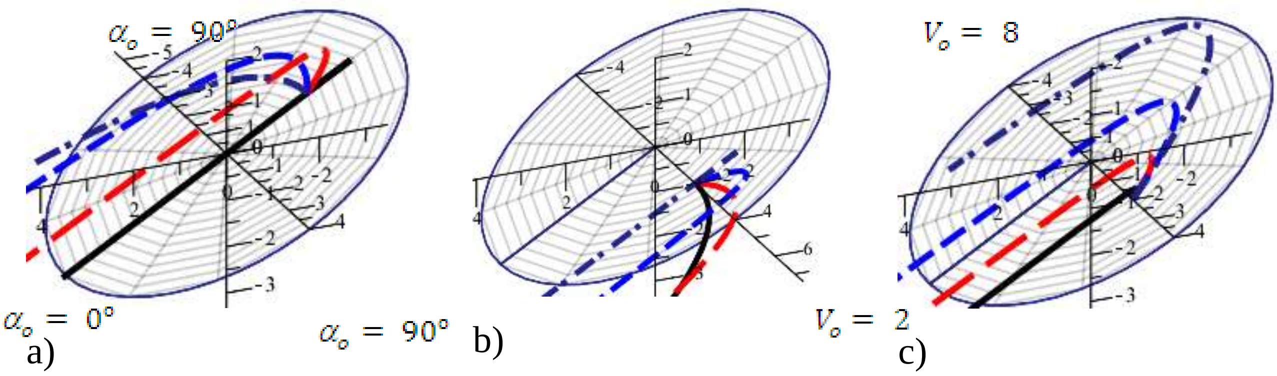

Figure. Trajectory $r(t)$ particles on the rough drive according to:

a) throwing angle α_o and the initial position $u_o = \pi$; b) throwing angle α_o iand the initial position $u_o = 1.5\pi$; c) the initial velocity V_o.

Conclusions

For the same initial conditions throwing particles, particle trajectories built on rough disk is congruent to the trajectories of particles on rough inclined plane. At the same time, the coordinate position trajectory lines of flat sections vary considerably.

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КОМП'ЮТЕРНА МОДЕЛЬ РУХУ ЧАСТИНКИ ПО ПОХИЛОМУ ШОРСТКОМУ ДИСКУ

А. В. Несвідомін

Анотація. *Мета дослідження - розробка Maple-моделі руху частинки по похилому шорсткому диску в полярній системі координат.*

На основі запропонованого способу формування законів руху частинок по шорстких поверхнях у внутрішніх їх координатах у проекціях на орти супроводжуючих тригранників була запропонована maple-модель руху *частинки по шорсткому диску. Завдяки розробленому комп'ютерному моделюванню стало можливим в інтерактивному режимі виконати дослідження траєкторно-кінематичних характеристик руху частинки по шорсткому диску. Це дало можливість аналізувати рух частинок по похилому шорсткому диску в полярних координатах.*

Ключові слова: ***матеріальна частинка, шорстка поверхня, похилий диск***

КОМПЬЮТЕРНАЯ МОДЕЛЬ ДВИЖЕНИЯ ЧАСТИЦЫ ПО НАКЛОННОМУ ШЕРОХОВАТОЙ ДИСКЕ

А. В. Несвидомин

Аннотация. *Цель исследования - разработка Maple-модели движения частицы по наклонному шероховатому диску в полярной системе координат.*

На основе предложенного способа формирования законов движения частиц по шероховатых поверхностях во внутренних их координатах в проекциях на орты сопровождающих трёхгранников была предложена maple-модель движения частицы по шероховатому диску. Благодаря разработанному компьютерному моделированию стало возможным в интерактивном режиме выполнить исследования траекторно-кинематических характеристик движения частицы по шероховатому диску. Это дало возможность анализировать движение частиц по наклонному шероховатому диску в полярных координатах.

Ключевые слова: ***материальная частица, шероховатая поверхность, наклонный диск***