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CONSTRUCTION minimal surfaces USING ISOTROPIC curve, WHAT IS WITH SURFACE Catenoid

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Abstract.*In the article the design of minimal surfaces using isotropic curve that lies on the surface Catenoid. Used Catenoid analytical condition referring to the isometric coordinates.*

Keywords: isotropic curve, minimum surface line element surface Catenoid

Formulation of the problem. Design and analytical description of minimal surfaces is an important issue geometric modeling derived from their use in the design of surface engineering forms and architectural designs. Minimal surfaces leads to a geometric problem: find surface that passes through a closed curve (contour) and has the smallest area [8]. In particular, the minimum surface and the surface of constant average curvature used in the design of transitional membranes to connect two cylindrical pipes of various diameters with a minimum size of surface area [6] and in many other forms of technical design problems.

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Analysis of recent research. The theory of minimal surfaces has many ways of analytical description. Several scientists studied the minimal surface facilities and tensor calculus of variations [1, 6]. The main problem of analytical description of minimal surfaces is the difficulty associated with finding their parametric equations. In particular, the mathematical description of a minimal surface used point frame of reference of the surface contour in the shape of a diamond or parallelogram [2]. In [5] researched design minimal surface passing through the predetermined curve. But the analytical description of the surface close to the minimum, was represented by elliptic functions.

Construction of continuous frame minimal surfaces associated with finding analytical description of isotropic curves zero length. [8] In the thesis [4] found ways of constructing isotropic spatial curves Weierstrass and Schwarz formulas [8], and based on the equations of isotropic curves constructed minimal and they are connected to the surface. In the thesis [3] rohlyanuto construction of minimal surfaces using Bezier curves isotropic and isotropic used straight sides defining characteristic polygons. So, to solve the problem of designing a continuous frame of minimal surfaces is an important way to expand education isotropic curves.

The purpose of research. Find analytical description of isotropic curves that lie on the surface Catenoid using analytical condition that the coordinate lines Catenoid form isometric system. Based on these curves build isotropic minimal surface and are attached to them.

Results. Two families of coordinate lines surface, referred to the isometric system, are displayed as infinitely small squares [3]. In [7] an algorithm for finding parametric equations meridian surface of revolution, in which the surface will be referred to the isometric coordinates.

Consider Catenoid - surface rotation axis catenary Oz Who asked parametric equations:

$$\begin{aligned} X(t;v) &= \mathrm{ch}(t)\cdot\cos v; \\ Y(t;v) &= \mathrm{ch}(t)\cdot\sin v; \\ Z(t;v) &= t, \end{aligned} \tag{1}$$

where: $t \in R,\ v \in [0;\ 2\pi)$.

Find the coefficients of the first quadratic form Catenoid the formulas [9]

$$\begin{aligned} E &= \left(\frac{\partial X}{\partial t}\right)^2 + \left(\frac{\partial Y}{\partial t}\right)^2 + \left(\frac{\partial Z}{\partial t}\right)^2; \\ F &= \frac{\partial X}{\partial t}\frac{\partial X}{\partial v} + \frac{\partial Y}{\partial t}\frac{\partial Y}{\partial v} + \frac{\partial Z}{\partial t}\frac{\partial Z}{\partial v}; \\ G &= \left(\frac{\partial X}{\partial v}\right)^2 + \left(\frac{\partial X}{\partial v}\right)^2 + \left(\frac{\partial X}{\partial v}\right)^2, \end{aligned} \tag{2}$$

Differentiating expression (1), after the change, according to (2), we get: $E = G = \mathrm{ch}^2(t);\ F = 0$. Then the line element Catenoid assigned to isothermal system is:

$$ds = \mathrm{ch}^2(t)\cdot(dv^2 + dt^2). \tag{3}$$

Expanding on the factors, expression (3) we get:

$$ds = \mathrm{ch}^2(t)\cdot(dv - i\cdot dt)(dv + i\cdot dt),$$

where: i - Imaginary unit.

Equating to zero the right side of the last equality, we get after integration:

$$v = i\cdot t + C \quad , \tag{4}$$

or

$$v = -i\cdot t + C, \tag{5}$$

where: C - Arbitrary constants of integration.

The expressions on the right side of equations (4) and (5) called Darboux coordinates (Darboux) [8].

Linear element (3) Catenoid determines the length of any curve that lies on the surface. So when substituted expressions (4) or (5) in a parametric equation Catenoid (1) obtain a parametric equation two families imaginary isotropic curves zero length. In particular, the substitution of (4) in equation (1) for each value C obtain a parametric equation isotropic imaginary curve that lies on the surface Catenoid:

$$x(t) = \mathrm{ch}(t) \cdot \cos(i \cdot t + C);$$
$$y(t) = \mathrm{ch}(t) \cdot \sin(i \cdot t + C); \qquad (6)$$
$$z(t) = t.$$

With a view to finding equations minimum and entailing minimal surface for complex variable (6) Type replacement [8] $t = u + i \cdot v$. Then we get the parametric equations of minimal surface [8] $X(u,v), Y(u,v), Z(u,v)$:

$$X(u,v) = \mathrm{Re}\{x(u + i \cdot v)\};\ Y(u,v) = \mathrm{Re}\{y(u + i \cdot v)\};\ Z(u,v) = \mathrm{Re}\{z(u + i \cdot v)\}; \quad (7)$$

and the associated surface $X^*(u,v), Y^*(u,v), Z^*(u,v)$:

$$X^*(u,v) = \mathrm{Im}\{x(u + i \cdot v)\};\ Y^*(u,v) = \mathrm{Im}\{y(u + i \cdot v)\};\ Z^*(u,v) = \mathrm{Im}\{z(u + i \cdot v)\}. \quad (8)$$

Separating real and imaginary parts for each function (6), have minimal surface equation:

$$X(u,v) = \cos(C - v) \cdot \cos(v) \cdot \mathrm{ch}^2(u) + \sin(C - v) \cdot \sin(v \cos\beta) \cdot \mathrm{sh}^2(u);$$
$$Y(u,v) = \sin(C - v) \cdot \cos(v) \cdot \mathrm{ch}^2(u) - \cos(C - v) \cdot \sin(v) \cdot \mathrm{sh}^2(u); \qquad (9)$$
$$Z(u,v) = u;$$

and the associated surface:

$$X^*(u,v) = -\frac{1}{2}\sin(C - 2v) \cdot \mathrm{sh}(2u);$$
$$Y^*(u,v) = \frac{1}{2}\cos(C - 2v) \cdot \mathrm{sh}(2u); \qquad (10)$$
$$Z^*(u,v) = v.$$

where: C - Arbitrary constants of integration.

Fig. 1 shows a minimal surface compartment, built by equations (9) at $C = \pi;\ u \in [-1;...1];\ v \in [-1;...1]$.

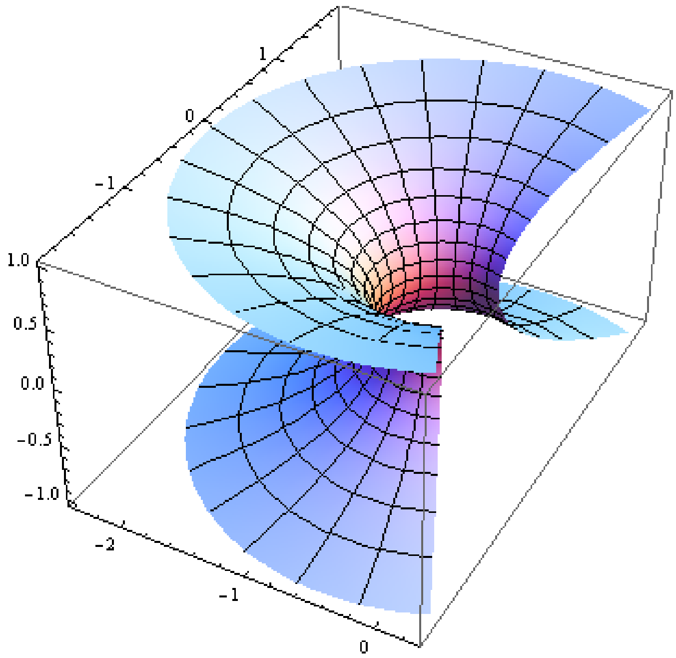

Fig. 1. Compartment minimum surface, based on equations (9).

Affiliate minimal surface is formed by equations (10) at $C = \pi;\ u \in [-1;...1];\ v \in [-1;...1]$ Has the shape of a screw conoid and built in Fig. 2 and Fig. 3, respectively.

The coefficients of the first quadratic form of minimal surfaces (9) and the associated surface (10), found the formulas (2), equal to:

$$E = G = ch^2(2u);\ F = 0. \qquad (11)$$

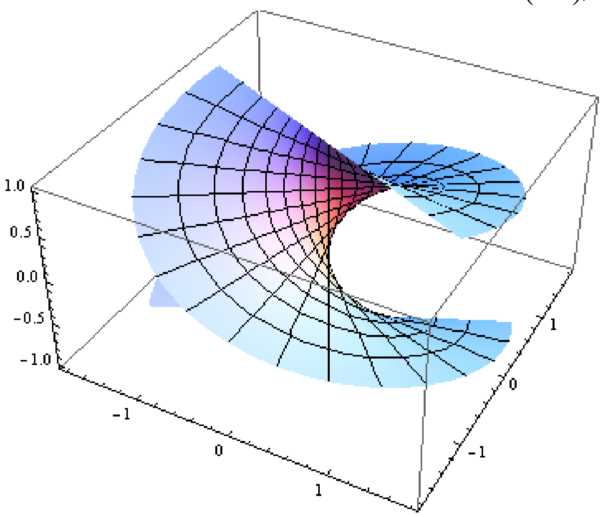

Fig. 2. Axonometry attached minimal surface, based on equations (10).

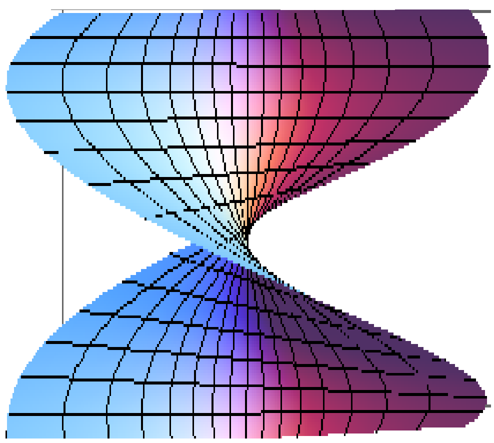

Fig. 3. Front projection attached minimal surface.

The coefficients of the second quadratic form of minimal surfaces (9) and the associated surface (10), found by the known formulas of differential geometry [9], the average curvature transform expression

$H = \dfrac{E \cdot N - 2 \cdot F \cdot M + G \cdot L}{2(E \cdot G - F^2)}$ For each of the specified surfaces to scratch. When substitution of (5) in equation (1) for each value C obtain another parametric equation isotropic imaginary curve that lies on Catenoid:

$$x(t) = \text{ch}(t) \cdot \cos(-i \cdot t + C);$$
$$y(t) = \text{ch}(t) \cdot \sin(-i \cdot t + C); \quad (12)$$
$$z(t) = t.$$

Using for complex variable (12) replacement $t = u + i \cdot v$, You can find the formulas (7) and (8) parametric equations of minimal surface and attached to it. Parametric equations of these minimal surfaces of different surfaces defined by equations (9) and (10) only arbitrary constant C Have the same first and second coefficients of quadratic forms, that allow simultaneous transfer of one another in a coordinate system.

Expression (3) can be decomposed into factors as:

$$ds = \text{ch}^2(t) \cdot (dt - i \cdot dv)(dt + i \cdot dv) \quad . \quad (13)$$

Equating to zero right-hand side of (13), after integration we get:

$$t = i \cdot v + C \text{ or } t = -i \cdot v + C . \quad (14)$$

Substituting (14) in a parametric equation Catenoid (1), we obtain the equation two families imaginary isotropic curves zero length. For each C found by isotropic curves can build minimal surfaces and are attached to them. Then, found parametric equations and minimal surfaces attached, compared to expressions (9) and (10) respectively, variable t and v "Swap". But have formed the surface level of the coefficients of the first and second fundamental form with minimal surfaces (9) and (10), ie parallel transfer tolerate each other.

Conclusion. On the surface Catenoid can construct two families of isotropic lines and each line put in line with minimum surface and attached to it. The method of analytical description isotropic imaginary curve that lies on the surface Catenoid, can manage the process of formation of minimal surfaces at separating real and imaginary parts of the function of complex variable.

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Abstract.*In the work dannoy osuschestvleno konstruyrovanye mynymalnыh with surfaces Using yzotropnoy curve,kotoraja nahodytsya on the surface katenoyda.Yspolzovano analytycheskoe terms and conditions of,something koordynatnыe LINES katenoyda obrazuyut yzometrycheskuyu system.*

Keywords: yzotropnaya opens up, mynymalnaya surface, linear element surface, katenoyd

Annotation.*In paper the design of minimal surfaces using isotropic curve that lies on the surface of the catenoid. It uses analytical condition that the coordinate lines catenoid form isometric system.*

Key words: ***isotropic curve, The minimum surface, line element surface, catenoid***

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IMPLEMENTATION OF TRACKING POTENTIAL HAZARDS OF THE AGRICULTURAL ENTERPRISE BASED ON A RISK-BASED APPROACH

Voinalovych AV, OA Gnatyuk, Ph.D.

Abstract. *The structure tracking potential hazards in the agricultural sector and apply methods of evaluating industrial risks. The algorithm definition production risks and limits their performance*