Mathematical model of electromagnetic processes in Tesla transformers

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In this article the electromagnetic processes occurring in Tesla transformer and developed mathematical model of these processes are reviewed. During the development of mathematical model the experimentally determined values of key parameters developed in the research of Tesla transformer model were used. Mathematical model usage allows to determine the reasonable values of basic parameters of Tesla transformer and to make a design of resonance single –wire electric power transmission system.

Tesla transformer, resonance, oscillation circuit, electric power transmission

One of the main tasks of further farm electrification development is electric power supply of stand-alone farm enterprises and small entities distant from the centralized transmission network. Taking into account the fact that electric power is transmitted by conduction current via wires the losses of electric power at active wire barrel are unavoidable. At the end of XIX century N. Tesla developed high-frequency transformer named after him, and on the basis of it he proposed the mode of electric power transmission by single wire and even without wires. But these inventions have not been principally used in the context of a range of reasons.

However, the analysis of scientific works in electric power branch revealed that N. Tesla mode of electric power transmission by single wire via high-frequency transformer is potentially productive especially regarding the aforesaid task [6]. Unfortunately, the analysis of previously drawn up scientific works indicated that physics of electromagnetic processes during electric power transmission by single wire via Tesla transformer has not been sufficiently researched, and principle of single wire electric power transmission requires further scientific justification.

Research objective is an analysis of electromagnetic processes at transmitting and receiving Tesla transformers which are the main functional units of resonance system for electric power transmission by single wire.

Materials and research methodology

In order to carry out calculated research Tesla transformer physical model developed by authors is considered, principle diagram of which is shown on fig.1 and methods of computer modeling with application of Matlab/Simulink software package are used.

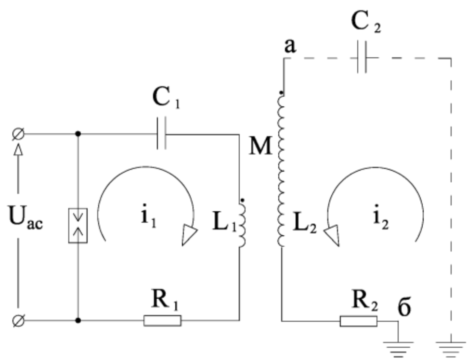

Fig.1 Principle diagram of Tesla transformer

Research results. As it can be seen from figure, Tesla transformer consists of two resonance circuits. High-voltage capacitor C_1 running low through discharge device to initial wiring with self-inductance L_1 as well as resistance R_1 are constituent elements of initial wiring. Secondary circuit is a disconnected secondary wiring with self-inductance L_2 and resistance R_2, the one end *a* of which is disconnected and another end *b* is earthed. This circuit is also featured by equivalent capacitance C_2 connecting the end of secondary circuit wiring with earth, refer to fig.1.

Analysis of electromagnetic processes at Tesla transformers shall be as follows. The first working phase of Tesla transformers is a charge of high-voltage capacitor C_1 via inductance L_1 of initial circuit coil and its self-equivalent resistance from alternating voltage source U_{ac} of current i_1. When the voltage is achieved its level at capacitor C_1 by spark gap sample the second working phase of Tesla transformer takes place, where the spark gap breaks down and capacitor C_1 runs low to initial wiring, and current i_1 is induced in the secondary wiring. The secondary circuit of Tesla transformer consists of series-connected inductance L_2 of secondary wiring with its self-equivalent resistance R_2 and capacitance C_2 between the end of secondary wiring and earthing and produces secondary resonance circuit. It is worthy of note that initial and secondary circuits of Tesla transformer are interconnected by mutual induction M and for this reason when the spark gap is closed the power is transmitted either from initial circuit to secondary one or vise versa. In continues until the spark gap is disconnected. Following it the charging process of capacitance C_2 and its discharge to the transformer repeat.

Power stored in capacitances C_1 and C_2 of two resonance circuits is determined correspondingly by the following relation:

$$W_1 = \frac{C_1 U_1^2}{2}, \quad W_2 = \frac{C_2 U_2^2}{2} \qquad (1)$$

On the basis of these power equations we obtain the following:

$$U_2 = \sqrt{\frac{C_1}{C_2} U_1}, \qquad (2)$$

When the mutual coupling coefficient value is minimum, resonance frequencies of initial and secondary circuits are the following:

$$f_{res} = \frac{1}{2\pi\sqrt{L_1 C_1}} = \frac{1}{2\pi\sqrt{L_2 C_2}} \qquad (3)$$

Analyzing specified relations we obtain required condition with regard to parameters dependency for initial and secondary circuits of Tesla transformer ensuring maximum power transmission from initial to secondary circuit:

$$L_1C_1 = L_2C_2 \tag{4}$$

According to the second Kirchoff law for initial and secondary circuits we obtain the following voltage balance equations:

$$R_1 i_1 + \frac{1}{C_1}\int i_1 dt + L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = 0 \tag{5}$$

$$R_2 i_2 + \frac{1}{C_2}\int i_2 dt + L_2\frac{di_2}{dt} + M\frac{di_1}{dt} = 0 \tag{6}$$

In order to obtain voltage value for secondary circuit U_2 in capacitance C_2 it is necessary to solve the stated set of equations. In ideal case it is easy to reach when $R_{1=}$ $R_{2=}0$. On this assumption we get the following expression for calculation of voltage at the ends of transformer secondary winding:

$$U_2(\,) = \frac{2kU_1}{\sqrt{(-T)^2 + 4k^2T}}\sqrt{\frac{L_2}{L_1}}\sin\left(\frac{w_2+w_1}{2}t\right)\times\sin\left(\frac{w_2-w_1}{2}t\right), \tag{7}$$

From equation (6) for secondary circuit voltage there can be made a conclusion that secondary voltage with high oscillation frequency shall be subject to amplitude modulation of low oscillation frequency for initial circuit.

Development of Simulink simulation model for Tesla transformer experimental pattern

The following values of Tesla transformer parameters have been applied during calculation: R_1=15.4·10^{-3} *Om*, L_1=6.9·10^{-6} *henry*, C_1=5.26·10^{-9} *F*, R_2=18.5 *Om*, L_2=4.88·10^{-3} *H*, C_1=7.45·10^{-12} *F*, *k=0.308*.

Developed simulation model for analysis of Tesla transformer electromagnetic processes under Matlab/Simulink software environment shown in fig.2 includes R, L, C, M element units, measurement units for electrical values as well as virtual oscilloscopes for calculation results visualizing.

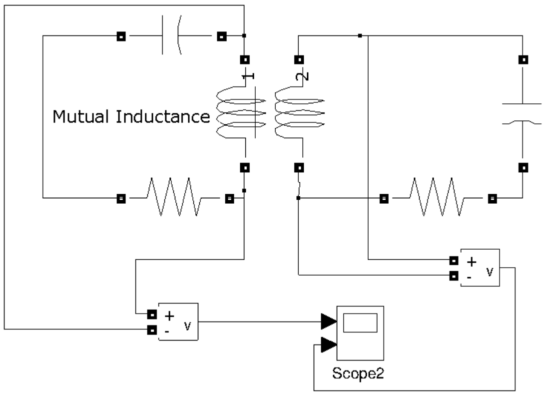

Fig.2 Tesla transformer simulation model developed under Simulink software environment

Outcomes of voltage calculation in initial and secondary circuits when capacitance L_1 is discharged are shown on fig.3. The voltage in initial wiring with inductance L_1 (top part of diagram) and voltage in secondary wiring with inductance L_2 (bottom part of diagram) are shown thereof. As it is evident from the aforesaid owing to dissipative processes in circuits the voltage in these circuits decreases within approximately 3 ms.

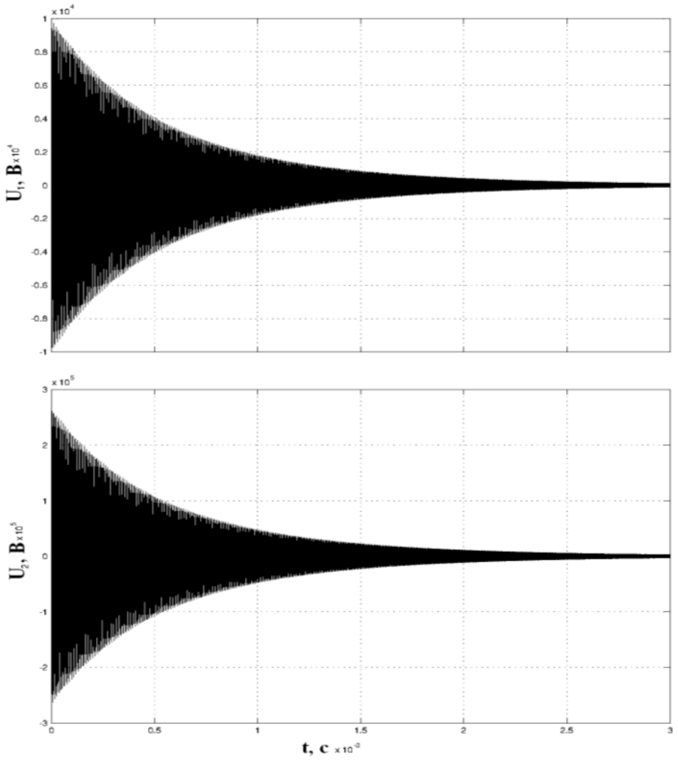

Fig.3 Voltage of initial and secondary wiring of Tesla transformer

In order to describe in detail the wave form of voltage the dependency diagrams for voltage in Tesla transformer windings within the period of $4 \cdot 10^{-5}$ sec. are shown in larger scale on Figure 4. It is clear that the voltage behavior in course of time coincides with the equation (6) and has two characteristic frequencies.

Analysis of influence caused by mutual-coupling coefficient *k* between initial and secondary circuits of Tesla transformer shown that it affects the resonance frequency of initial and secondary circuits as well as numerically determines power transmission speed from one circuit to another and vice versa.

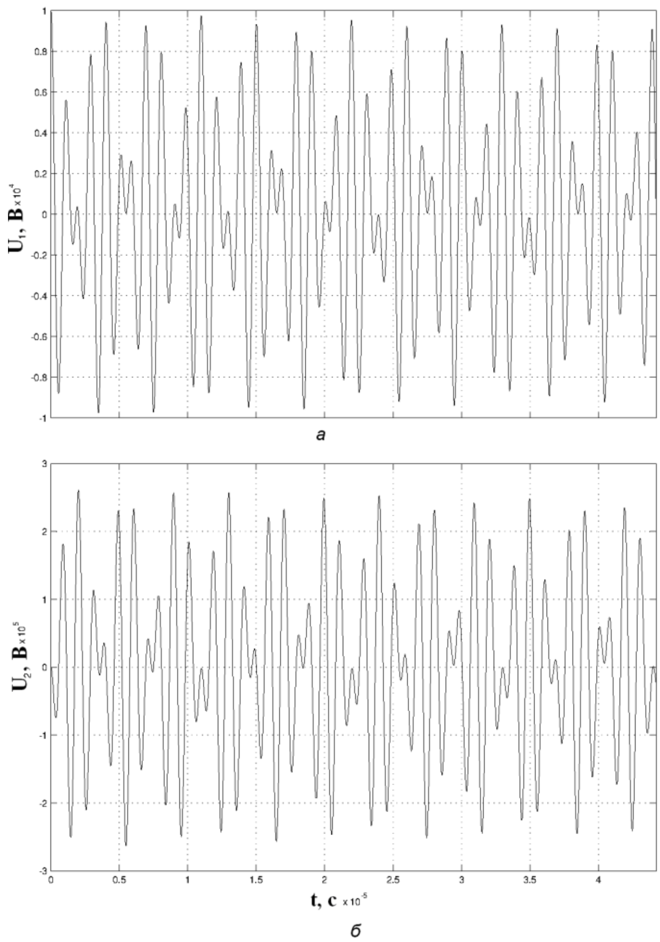

Fig.4. Wave form of voltage in initial and secondary circuits of Tesla transformer

The lower value of mutual-coupling coefficient *k* is, the longer period of time the power transmission from initial circuit to secondary one takes and vice versa – increase of *k* value results in extension of the time period between voltage and current nodes. In order to protect the arrester from overheating and further functional loss when developing exploratory prototypes mutual-coupling coefficient shall be taken up less than 0.1. Two resonance circuits will always have some difference in frequency in working mode with mutual induction since high frequencies cause intercircuit capacitance. Dependency of influence caused by mutual induction and mutual-coupling coefficient *k* on resonance frequencies of initial and secondary circuits should be described by the following relations:

$$f_1 = \frac{f_r}{\sqrt{1+k}},\ f_2 = \frac{f_r}{\sqrt{1-k}},\ \text{where} \quad (7)$$

f_1, f_2 – working frequency of initial and secondary circuit correspondingly considering the influence of mutual induction,

f_r – resonance frequency of the circuits without consideration of mutual induction.

Graphical analytic curve of impedance depending on frequency at different values of mutual-coupling coefficient *k* are shown on Figure 5.

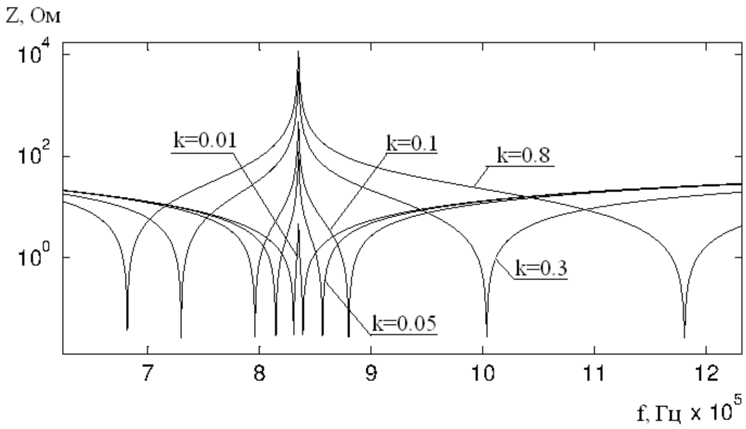

Fig. 5. Frequency response of Tesla transformer at different values of mutual-coupling coefficient *k*

Mathematical model allows to determine the most reasonable parameter values of Tesla transformer at which the maximum value of the voltage could be achieved in its the secondary winding.

Conclusions

The developed mathematical model of electromagnetic process occurred in Tesla transformer allows to determine the most reasonable parameter values of Tesla transformer using the criteria of peak voltage for open circuit in its the secondary winding. The developed model could be used in the design and improvement of single-wire system for electric power transmission using the resonance method. Upon that the experimentally determined parameters of created physical model of Tesla transformer are used in mathematical model whereby the minimal electric power consumption shall be ensured in winding of transmitting and receiving Tesla transformers.

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