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INVESTIGATION BY POTENTIAL PLANE DEFORMATIONS IN PROBLEMS OF LINEAR VISCOELASTICITY

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This article considers to research potential methods by plane strain problems in linear viscoelasticity. *For a problems about plane deformations of viscoelastic material of the Abelian type the system of second type boundary-time integral equations is got. The algorithm of decision of this system is offered.*

Potential method successfully solved many applied problems of mathematical physics. This paper develops a potential method for solving boundary value problem of linear viscoelasticity.

The purpose of research - to provide boundary integral equations for the initial boundary value problem of viscoelastic deformation of the flat area of the moving boundary.

Materials and research methods. Theoretical provisions of mathematical physics and potential theory methods have been used. Viscoelastic materials with rheological Abel's kernels offered by Yu. Rabotnov are considering.

Let the solid medium consisting of a homogeneous, isotropic, viscoelastic ageless with continuous memory fills cylindrical body cross section $D(0)$. It is believed that the material is ageless (its properties do not change over time), and memory - fading. Gave only small deformations such an environment, exploring the problem of plane deformation.

Random geometric point M of the plane viscoelastic region $D(\tau)$ in time moment $\tau > 0$ coincides with the radius vector $\vec{y}(\tau) = \vec{e}^k y_k(\tau)$. The total pressure of time period $0 < \tau < t$ for tension in the material particle caused deformities in time τ and the subsequent relaxation of tensions, pressure tensor is determined by:

$$\Pi(\vec{y}(t),t)=\int_0^t\{E[kh(t-\tau)-\frac{2}{3}\mu q(t-\tau)]div\ \vec{v}(\vec{y}(\tau),\tau)+ \\ +2\mu q(t-\tau)Def\ \vec{v}(\vec{y}(\tau),\tau)\}d\tau, \quad (1)$$

where μ - instantaneous elastic shear modulus; $\lambda = k-\frac{2}{3}\mu$ - instantly became Lame-elastic; k - instantaneous bulk modulus of deformation; E - unit tensor; $Def\ \vec{v}(\vec{y}(\tau),\tau)=\frac{1}{2}\left(\nabla\vec{v}+(\nabla\vec{v})^T\right)$ - tensor of deformation speed; $\vec{v}\equiv\vec{v}(\vec{y}(\tau),\tau)$ - particle velocity vector.

Functions $q(t)$ and $h(t)$ characterize the rheological properties of the material are called nuclei shear and bulk relaxation respectively; defined in $t\geq 0$. We assume $h(t)=0$, as most viscoelastic relaxation bulk material is not significant. We admit that viscoelastic material behavior is consistent with the rheological model by Yu. Rabotnov: $q(t)=\dfrac{ct^{\alpha-1}}{\Gamma(\alpha)}$, (2)

where $c>0,\ \alpha\in(0,\ 1)$ - the parameters of the material; $\Gamma(\alpha)$ - gamma function.

Results. Speed and displacement of particles occurring in the material body for a period of relaxation and derivatives of the coordinates of displacement to the second order inclusive feel small quantities and the initial time $t=0$ the body free from stress. So can take approximately

$$\vec{v}(M,\tau)\approx\vec{v}(\vec{y},\tau)\approx\frac{\partial u(\vec{y},\tau)}{\partial\tau},\quad \overset{\tau}{\nabla}\approx\nabla,$$

where indicated: $\vec{y}\equiv\vec{y}(t),\ \vec{u}(\vec{y},\tau)=\vec{e}^k u_k$ - displacement vector, $(y_1,y_2;t)$ - main independent variables (Euler's variables).

Quasi-static equations of motion $div\Pi(\vec{y},t)+\rho(\vec{y},t)\vec{m}(\vec{y},t)=\vec{0}$ such viscoelastic medium takes the following form:

$$\mu\Delta\vec{u}(\vec{y},t)+(\lambda+\mu)graddiv\vec{u}(\vec{y},t)-\mu\int_0^t q(t-\tau)[\Delta\vec{u}(\vec{y},\tau)+ \\ +\frac{1}{3}graddiv\vec{u}(\vec{y},\tau)]d\tau+\vec{f}(\vec{y},t;\vec{u})=\vec{0}, \quad (3)$$

here $\vec{f}(\vec{y},t;\vec{u}) = \rho_0 \vec{m}(\vec{y},t)[1 - div\vec{u}(\vec{y},t)]$, (4)

$\vec{m}(\vec{y},t)$ - intensity mass forces; $\rho_0 = \rho(\vec{y},0)$ - density of the material; Δ - Laplace's operator.

Let the contour points $L(t)$ to give vector of stresses $\vec{p}_n(\vec{x},t)$:

$$\vec{n} \cdot \Pi(\vec{x},t) = \vec{p}_n(\vec{x},t), \quad \vec{x} \in L(t), \quad t \geq 0, \tag{5}$$

where $\vec{n}$ - normal to the contour $L(t)$ of his point $\vec{x}$. With increasing time $t > 0$ the border $L(t)$ of field $D(t)$ moves. The law of contour change will describe with vector equation

$$\vec{x} = \vec{x}(l,t), \tag{6}$$

where l - the contour length of the $L(t)$ initial time point. The length of the arc $s(l,t)$ of the curve $L(t)$ calculated by the formula $s(l,t) = \int_0^l \left| \frac{\partial \vec{x}(l,t)}{\partial l} \right| dl$.

The solution of the problem (3), (5) sought as potential viscoelastic

$$\vec{u}(\vec{y},t) = \vec{u}[\vec{f}] + \sum_{k=1}^{2} \vec{e}^k \int_0^t d\tau \int_{L(\tau)} \vec{v}(l,\tau) \cdot \vec{v}^{(k)}(\vec{y} - \vec{x}; t - \tau) dl, \tag{7}$$

where $\vec{u}[\vec{f}] = \sum_{k=1}^{2} \vec{e}^k \int_0^t d\tau \iint_{D(\tau)} \vec{v}^{(k)}(\vec{y} - \vec{x}; t - \tau) \cdot \vec{f}(\vec{x},\tau) ds$, (8)

$\vec{v}(l,t)$ - arbitrary continuous vector function, $\vec{v}^{(k)}(\vec{y} - \vec{x}; t - \tau)$ - fundamental solution of equation (3).

As a result of the substitution of expression (7) in the boundary condition (5) the system of integral equations has got for the unknown component vector potential density $\vec{v}(l,t)$:

$$\pi v_1(l_0,t) + \int_{L(t)} \sum_{i=1}^{2} v_i(l,t) K_{1i}(l,l_0;t) \left| \frac{\partial \vec{x}(l,t)}{\partial l} \right| dl +$$

$$+ \int_0^t \tilde{k}(t-\tau) d\tau \int_{L(\tau)} \sum_{i=1}^{2} v_i(l,\tau) k_{1i}(l,l_0;t,\tau) \left| \frac{\partial \vec{x}(l,\tau)}{\partial l} \right| dl = \psi_1(l_0,t); \tag{9}$$

$$\pi \nu_2(l_0,t) + \int\limits_{L(t)} \sum_{i=1}^{2} \nu_i(l,t) K_{2I}(l,l_0;t) \left| \frac{\partial \vec{x}(l,t)}{\partial l} \right| dl +$$

$$+\int_0^t \tilde{k}(t-\tau)d\tau \int\limits_{L(\tau)} \sum_{i=1}^{2} \nu_i(l,\tau) k_{2i}(l,l_0;t,\tau) \left| \frac{\partial \vec{x}(l,\tau)}{\partial l} \right| dl = \psi_2(l_0,t).$$

Kernels expression $K_{ij}(l,l_0;t)$, $k_{ij}(l,l_0;t,\tau)$, function $\tilde{k}(t)$ and right parts $\psi_1(l_0,t)$, $\psi_2(l_0,t)$ of the system of equations founded.

For the numerical solution of the system of second type boundary-time integral equations (9) the method of "time steps" applies.