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ALGORITHM FOR CONSTRUCTING OF TOLERANCE FOR LINEAR SYSTEMS WITH VARIABLE STRUCTURE

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The results of numerical calculation of tolerance on the parameters for parametric linear systems with variable structure. Considered problem statement calculating tolerances covered algorithms practical stability.

Tolerances on parameters, stability, evaluation criteria, systems with variable structure.

When designing a structure-defined control systems often need to estimate the parameters of tolerance that would ensure the functioning of the real dynamic object [[redacted]–3]. In particular, as parameters, can be considered the initial conditions or other quantities characterizing the features of studied system. Thus, for systems with variable structure [[redacted]] as parameters to consider switching points − a moments of time, determining the change equation one another. Within this framework proposed construction areas tolerances for linear systems using algorithms make practical stability [[redacted]].

The aim of research — development of constructive algorithms regions of parameters tolerances for linear systems with variable structure methods practical stability.

Materials and methods research. In this paper apply mathematical methods of analysis of practical stability and sensitivity, based on the method of Lyapunov functions. In contrast to classical performances, stability studies carried out on a finite time interval, and dynamic constraints on variation in state parametric system specified.

Consider the linear system with variable structure

$$\frac{dx}{dt} = A^{(i)}(t)x + G^{(i)}(t)\alpha,\ \ t \in [t_{i-1}, t_i),\ \ i = 1,2,\ldots,N\ , \tag{1}$$

where $A^{(i)}(t)$, $G^{(i)}(t)$, $t \in [t_{i-1}, t_i)$, $i = 1,2,\ldots,N$ — matrix elements of integrable dimension $n \times n$ and $n \times m$ respectively, and the initial conditions are fixed: $x(t_0) = x_0$.

Let $\bar{\alpha}$, $\bar{x} = x(t, \bar{\alpha})$ − estimated value of the vector of parameters and trajectories of the system (1). Then for vectors spread $y = x - \bar{x}$, $\beta = \alpha - \bar{\alpha}$, we obtain the system

$$\frac{dy}{dt} = A^{(i)}(t)y + G^{(i)}(t)\beta,\ \ t \in [t_{i-1}, t_i),\ \ i = 1,2,\ldots,N\ , \tag{2}$$

but with zero initial conditions $y(t_0) = 0$.

The task of evaluating the tolerances on the parameters is formulated as follows: define a set of acceptable parameters $G(\beta)$ so that the trajectory of system (2) were the set of admissible states Φ_t, $t \in [t_0, T]$.

Definition. We say that the system (2) is $(G(\beta), \Phi_t, t_0, T)$ — evaluation parameters tolerances β if $y(t, \beta) \in \Phi_t$, , for any $\beta \in G(\beta)$.

The following specification testing problem set $G(\beta)$ is fully embedded in the formulation of problems of practical stability of parametric systems with variable structure [2]. Thus, for fixed initial conditions problem of estimating parameters tolerances is a problem $(G_0^x, G_0^\alpha, \Phi_t, t_0, T)$ — stability in the space of vectors of spread y, β, as set G_0^x consists of one zero point.

Specify a set $G(\beta)$ of structurally if an ellipsoid $G(\beta) = \{\beta : \beta^* G \beta \le c^2\}$, and dynamic restriction to linear

$$\Phi_t = \Gamma_t = \{y : |l_s^*(t)y| \le 1, s = 1,2,\ldots,N\},\ t \in [t_0, T], \tag{3}$$

quality tolerances will be determined (G, Γ_t, t_0, T) — assessment.

Criterion 1. For $\langle G,\Gamma_t,t_0,T\rangle$ — evaluation parameters β of the system (2) is necessary and sufficient that performed inequality

$$c^2 \le \min_{i=1,2,...,N}\ \min_{t\in[t_{i-1},t_i]}\ \min_{s=1,2,...,N} \frac{1}{l_s^*(t)R(t)B^{-1}R^*(t)l_s(t)},$$

$$t\in[t_{i-1},t_i],\ i=1,2,\ldots,N,\ s=1,2,\ldots,N. \qquad (4)$$

Here

$$R(t)=X^{(i)}(t,t_{i-1})X^{(i-1)}(t_{i-1},t_{i-2})\ldots X^{(2)}(t_2,t_1)\tilde G^{(1)}(t_1)+X^{(i)}(t,t_{i-1})X^{(i-1)}(t_{i-1},t_{i-2})\ldots$$

$$X^{(3)}(t_3,t_2)\tilde G^{(2)}(t_2)+\ldots+X^{(i)}(t,t_{i-1})\tilde G^{(i-1)}(t_{i-1})+\tilde G^{(i)}(t),\quad \tilde G^{(i)}(t)=\int_{t_{i-1}}^{t}X^{(i)}(t,\tau)G^{(i)}(\tau)d\tau;\qquad t\in[t_{i-1},t_i],$$

$i=1,2,\ldots,N$. The fundamental matrix of solutions $X^{(i)}(t,t_{i-1})$ of the homogeneous system (1), normalized by moment t_{i-1}, is defined by the following matrix equation:

$$\frac{dX^{(i)}(t,t_{i-1})}{dt}=A^{(i)}(t)X^{(i)}(t,t_{i-1}),\ X(t_{i-1},t_{i-1})=E,\ t\in[t_{i-1},t_i]. \qquad (5)$$

Should be emphasized that relations (3) $l_s(t)$, $s=1,2,\ldots,N$ — given vector dimension n with piecewise continuous elements.

For the case of nonlinear constraints on vector spread y

$$\Phi_t=\Psi_t=\{y:\ \psi(y,t)\le 1\},\ t\in[t_0,T], \qquad . \qquad (6)$$

where $\psi(x,t)$ — continuously-differentiable along x, t scalar functions, quality tolerances will be determined $\langle G,\Psi_t,t_0,T\rangle$ — assessment.

Criterion 2. For $\langle G,\Psi_t,t_0,T\rangle$ — evaluation parameters β of the system (2) is necessary and sufficient that performed inequality

$$c^2\le \min_{i=1,2,...,N}\ \min_{t\in[t_{i-1},t_i]}\ \min_{y\in\Psi'_t}\frac{[g^*(y,t)y]^2}{g^*(y,t)R(t)B^{-1}R^*(t)g(y,t)},$$

$$g^*(y,t)y>0,\ y\in\Psi'_t,\ t\in[t_{i-1},t_i],\ i=1,2,\ldots,N, \qquad (7)$$

where $g^*(y,t)=grad_y^*\psi(y,t)$, $y\in\Psi'_t$, Ψ'_t − boundary of the closed convex set Ψ_t, $t\in[t_0,T]$.

In addition, these criteria with respect to the variable t differentiability condition for a function $\psi(x,t)$ can, in general, not be carried out at switching points t_i, $i = 1,2,\dots,N$.

For linear relation α type initial conditions: $x_0(\alpha) = X_0\alpha$, where X_0 − known matrix dimension $n \times m$, tolerance evaluation criteria (4), (7) take the form respectively

$$c^2 \le \min_{i=1,2,\dots,N}\ \min_{t\in[t_{i-1},t_i)}\ \min_{s=1,2,\dots,N} \frac{1}{l_s^*\tilde{R}(t)B^{-1}\tilde{R}^*(t)l_s},$$

$$t \in [t_{i-1},t_i),\ i = 1,2,\dots,N,\ s = 1,2,\dots,N;$$

$$c^2 \le \min_{i=1,2,\dots,N}\ \min_{t\in[t_{i-1},t_i)}\ \min_{\bar{y}\in\Psi_t'} \frac{\left[g^*(y,t)\bar{y}\right]^2}{g^*(y,t)\tilde{R}(t)B^{-1}\tilde{R}^*(t)g(y,t)},$$

$$g^*(y,t)\bar{y} > 0,\ \bar{y}\in\Psi_t',\ t\in[t_{i-1},t_i),\ i = 1,2,\dots,N;$$

with $\tilde{R}(t) = X^{(i)}(t,t_{i-1})X^{(i-1)}(t_{i-1},t_{i-2})\dots X^{(1)}(t_1,t_0)X_0 + R(t)$.

If the motion of the object described by the system (1) with $t \ne t_i$, and at switching points t_i, $i = 1,2,\dots,N$, subject to the condition

$$x(t_i+0) = C^{(i)}x(t_i-0),\ i = 1,2,\dots,N_1-1, \tag{8}$$

depending on the type of constraints Φ_t, $t\in[t_0,T]$, we have the following estimates:

$$c^2 \le \min_{i=1,2,\dots,N_1}\ \min_{t\in[t_{i-1},t_i)}\ \min_{s=1,2,\dots,N} \frac{1}{l_s^*\bar{R}(t)B^{-1}\bar{R}^*(t)l_s},$$

$$t \in [t_{i-1},t_i),\ i = 1,2,\dots,N_1,\ s = 1,2,\dots,N; \tag{9}$$

$$c^2 \le \min_{i=1,2,\dots,N_1}\ \min_{t\in[t_{i-1},t_i)}\ \min_{\bar{y}\in\Psi_t'} \frac{\left[g^*(y,t)\bar{y}\right]^2}{g^*(y,t)\bar{R}(t)B^{-1}\bar{R}^*(t)g(y,t)},$$

$$g^*(y,t)\bar{y} > 0,\ \bar{y}\in\Psi_t',\ t\in[t_{i-1},t_i),\ i = 1,2,\dots,N_1. \tag{10}$$

Here

$$\bar{R}(t) = X^{(i)}(t,t_{i-1}+0)C^{(i-1)}X^{(i-1)}(t_{i-1}-0,t_{i-2}+0)C^{(i-2)}X^{(i-2)}(t_{i-2}-0,t_{i-3}+0)\dots C^{(1)}X^{(1)}(t_1-0,t_0)X_0 +$$
$$+X^{(i)}(t,t_{i-1}+0)C^{(i-1)}X^{(i-1)}(t_{i-1}-0,t_{i-2}+0)C^{(i-2)}X^{(i-2)}(t_{i-2}-0,t_{i-3}+0)\dots X^{(2)}(t_2-0,t_1+0)C^{(1)}$$
$$\tilde{G}^{(1)}(t_1-0) + X^{(i)}(t,t_{i-1}+0)C^{(i-1)}X^{(i-1)}(t_{i-1}-0,t_{i-2}+0)C^{(i-2)}$$
$$X^{(i-2)}(t_{i-2}-0,t_{i-3}+0)\dots X^{(3)}(t_3-0,t_2+0)C^{(2)}\tilde{G}^{(2)}(t_2-0) + X^{(i)}(t,t_{i-1}+0)C^{(i-1)}X^{(i-1)}(t_{i-1}-0,t_{i-2}+0)\dots$$
$$X^{(4)}(t_4-0,t_3+0)C^{(3)}\tilde{G}^{(3)}(t_3-0)+\dots+X^{(i)}(t,t_{i-1}+0)C^{(i-1)}$$
$$\tilde{G}^{(i-1)}(t_{i-1}-0) + \tilde{G}^{(i)}(t),\ t\in[t_{i-1},t_i),\ i = 1,2,\dots,N_1.$$

In particular, if the initial conditions of the system with discontinuous coordinates are fixed, you must first term in the expression $\bar{R}(\ )$ equated to zero and use the estimates (9) and (10).

Based on these studies can be conducted evaluation of tolerance in the case of non-linearity of both the system and the corresponding initial conditions.

Studies. For parametric linear systems with variable structure proved evaluation criteria of tolerance on the parameters in the given structure. Considered problem statement calculating allowances for fixed and linear initial conditions for specific types of dynamic constraints.

Conclusions

Based on the practical stability criteria for parametric systems with variable structure algorithms evaluation of tolerance of parameters associated with an increase in the efficiency of dynamic real objects.

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Приведены результаты численного расчета областей допусков на параметры для линейных параметрических систем с переменной структурой. Рассмотрены постановки задач расчета допусков, которые включаются в алгоритмы практической устойчивости.

Допуски на параметры, устойчивость, критерии оценки, системы с переменной структурой.

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