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TERMS OF OCCURRENCE OF SOLUTIONS OF BOUNDARY PROBLEMS SLABOZBURENYH (CASE $k = -1$)

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The proposed scheme of coefficient of Existence conditions slabozburenyh boundary value problems for impulsive systems at fixed times.

Slabozburena linear inhomogeneous boundary value problems, homogeneous boundary value problem of impulsive, ortoproektor, row, pseudoinverse matrix, generalized Green's operator.

Problem. The relevance of this topic is due, above all, the importance of the practical application of the theory of boundary value problems in the theory of nonlinear oscillations, motion stability theory, control theory, a number of geophysical problems. On the other hand received a paper new results substantially complementary research on the theory of nonlinear oscillations for weakly perturbed boundary value problems.

Results. Consider slabozburenu linear inhomogeneous boundary value problem of impulsive type:

$$\begin{cases} \dot{z} = A(t)z + \varepsilon A_1(t)z + f(t), \quad t \neq \tau_i, \\ \Delta z|_{t=\tau_i} - S_i z = a_i + \varepsilon A_{1i} z(\tau_i - 0), \\ lz = \alpha + \varepsilon l_1 z. \end{cases} \quad . \qquad (1)$$

Suppose generating boundary problem that results from (1) in $\varepsilon = 0$:

$$\begin{cases} \dot{z} = A(t)z + f(t), \quad t \neq \tau_i, \\ \Delta z|_{t=\tau_i} - S_i z = a_i, \\ lz = \alpha. \end{cases} \quad , \qquad (2)$$

there are no solutions at random inhomogeneities $f(t) \in C([a,b]/\{\tau_i\}_I)$, $a_i \epsilon \mathbb{R}^n, \alpha \epsilon \mathbb{R}^m$. This means that there is a critical case $(rank\ Q = n_1 < n)$ and solvability criteria:

$$P_{Q_d^*} \left\{ \alpha - l \int_a^b K(\cdot,\tau) f(\tau) d\tau - l \sum_{i=1}^p \overline{K}(\cdot,\tau_i)\, a_i \right\} = 0, d = m - n_1,$$

boundary value problem (2) because of the arbitrariness of heterogeneous members is not satisfied.

The question is whether using linear perturbation make boundary problem (2) solvable and, if so, what should be perturbed terms $\varepsilon A_1(t)$,εA_{1i} and εl_1To the boundary problem (1) small solution for any inhomogeneities $f(t)$, a_i and α. To answer this question you can use $(d \times r)$-dimensional matrix:

$$B_0 = P_{Q_d^*}\left[l_1 X_r(\cdot) - l\int_a^b K(\cdot,\tau)A_1(\tau)X_r(\tau)d\tau - l\sum_{i=1}^{p}\overline{K}(\cdot,\tau_i+0)\times A_{1i}X_r(\tau_i-0)\right], \quad (3)$$

constructed coefficients on the output differential boundary value problem.

Application method Vishika-Lyusternika allows you to find effective coefficient of the boundary conditions of the problem (2) in a Laurent series in the small parameter pn-powers ε with a finite number of terms containing negative degree ε.

We prove a theorem that allows to solve the problem. $Q = lX(\cdot) - (m \times n)$-dimensional matrix; $P_{Q^*} - (m \times m)$-dimensional matrix (ortoproektor) that designs $\mathbb{R}^m$ to $N(Q^*)$, $P_{Q^*}: \mathbb{R}^m \to N(Q^*)$;$Q^+$ – only for the pseudo-Moore Penrose to Q $(n \times m) -$ dimensional matrix; $P_{B_0} - (r \times r) -$ dimensional matrix (ortoproektor) that designs $\mathbb{R}^r$ the zero-space $N(B_0)$ $(d \times r) -$ dimensional matrix $B_0, P_{B_0}: \mathbb{R}^r \to N(B_0)$; $P_{B_0^*} - (d \times d) -$ dimensional matrix (ortoproektor) that designs $\mathbb{R}^d$ the zero-space $N(B_0^*)$ $(r \times d) -$ dimensional matrix B_0^*=B_0^T, $P_{B_0^*}: \mathbb{R}^d \to N(B_0^*)$.

Theorem. Let the boundary value problem (1) satisfies the above mentioned conditions so that there is a critical case $(rank\ Q = n_1 < n)$ and generating boundary value problem (2) with random inhomogeneities $f(t) \in C([a,b]/\{\tau_i\}_I)$, $a_i \epsilon \mathbb{R}^n, \alpha \epsilon \mathbb{R}^m$ has no solutions. Then, if the following conditions are satisfied:

$$P_{B_0} = 0, \quad P_{B_0^*} P_{Q_d^*} = 0, \quad (4)$$

then the boundary value problem (1) exists for arbitrary $f(t) \in C([a,b]/\{\tau_i\}_I)$, $a_i \epsilon \mathbb{R}^n, \alpha \epsilon \mathbb{R}^m$ unique solution, presented in the form of convergent in $\varepsilon \epsilon (0, \varepsilon_*]$ series:

$$z(t,\varepsilon) = \sum_{i=k}^{+\infty} \varepsilon^i z_i(t), \quad k = -1. \quad (5)$$

Proof. Substituting the series (5) the boundary value problem (1) and equate the coefficients of the same powers of ε. Consequently, the ε^{-1} to $z_{-1}(t)$ obtain homogeneous boundary value problem

$$\begin{cases} \dot{z}_{-1} = A(t)z_{-1}, \quad t \neq \tau_i, \\ \Delta z_{-1}|_{t=\tau_i} = S_i z_{-1}(\tau_i - 0), \\ l z_{-1} = 0. \end{cases} \tag{6}$$

Under the assumptions of the theorem, homogeneous boundary value problem (6) is r –parametric family $r = n - n_1$ solutions $z_{-1}(t, c_{-1}) = X_r(t)c_{-1}$Where $r -$ dimensional column vector $c_{-1} \epsilon \mathbb{R}^r$ constants will be determined in the next step of the solvability conditions for the problem $z_0(t)$.

At ε^0 come to $z_0(t)$ to the boundary value problem

$$\begin{cases} \dot{z}_0 = A(t)z_0 + A_1(t)z_{-1} + f(t), \quad t \neq \tau_i, \\ \Delta z|_{t=\tau_i} = S_i z_0(\tau_i - 0) + A_{1i} z_{-1}(\tau_i - 0) + a_i, \\ l z_0 = l_1 z_{-1} + \alpha. \end{cases} \tag{7}$$

The criterion of solvability of problem (7) has the form:

$$P_{Q_d^*}\left\{\alpha + l_1 X_r(\cdot)c_{-1} - l\int_a^b K(\cdot, r)(f(t) + A_1(\tau)X_r(\tau)c_{-1})d\tau - l\sum_{i=1}^p \overline{K}(\cdot, \tau_i)(a_i + A_{1i}X_r(\tau_i - 0)c_{-1})\right\} = 0.$$

Hence we obtain the algebraic relatively $c_{-1} \epsilon \mathbb{R}^r$ system:

$$B_0 c_{-1} = -P_{Q_d^*}\left\{\alpha - l\int_a^b K(\cdot, \tau)f(\tau)d\tau - l\sum_{i=1}^p \overline{K}(\cdot, \tau_i)\, a_i\right\}, \tag{8}$$

where

$$B_0 = P_{Q_d^*}\left\{l_1 X_r(\cdot) - l\int_a^b K(\cdot, \tau)A_1(\tau)X_r(\tau)d\tau - l\sum_{i=1}^p \overline{K}(\cdot, \tau_i)\, A_{1i}X_r(\tau_i - 0)\right\} \tag{9}$$

To solve the latter at random $f(t) \in C([a, b]/\{\tau_i\}_I)$, $a_i \epsilon \mathbb{R}^n, \alpha \epsilon \mathbb{R}^m$ necessary and sufficient to fulfill the condition $P_{B_0^*} P_{Q_d^*} = 0$. If you require additional $rank\, B_0 = r \Leftrightarrow P_{B_0} = 0$, The system (8) uniquely cheeky relatively $c_{-1} \epsilon \mathbb{R}^r$:

$$c_{-1} = -B_0^+ P_{Q_d^*}\{\alpha - l\int_a^b K(\cdot, \tau)f(\tau)d\tau - l\sum_{i=1}^p \overline{K}(\cdot, \tau_i)\, a_i\}.$$

Boundary value problem (7) in (4) has r –parametric family $(r = n - n_1)$ solutions:

$$z_0(t, c_0) = X_r(t)c_0 + \overline{z_0}(t),$$

where $c_0 - r -$ dimensional vector of constants to be clearly defined in the next step of the solvability conditions for a boundary value problem $z_1(t)$;$\bar{z_0}(t)$ – particular solution of problem (7):

$$\bar{z_0}(t) = \left(G\begin{bmatrix} A_1(\tau)z_{-1}(\tau, c_{-1}) + f(\tau) \\ A_{1i}z_{-1}(\tau_i - 0) + a_i \end{bmatrix}\right)(t) + X(t)Q^+\{\alpha + l_1 z_{-1}(\cdot, c_{-1})\},$$

where $\left(G\begin{bmatrix} * \\ * \end{bmatrix}\right)(t)$ - Generalized Green's operator boundary value problem (7), which is:

$$\left(G\begin{bmatrix} A_1(\tau)z_{-1}(\tau, c_{-1}) + f(\tau) \\ A_{1i}z_{-1}(\tau_i - 0) + a_i \end{bmatrix}\right)(t) \stackrel{\text{def}}{=}$$

$$\stackrel{\text{def}}{=}$$

$$(\Big[\int_a^b K(t,\tau) * d\tau - X(t) \times Q^+ l \int_a^b K(\cdot,\tau) * d\tau,\ \textstyle\sum_{i=1}^p \overline{K}(\cdot,\tau_i) *$$

$$-X(t)Q^+ l \textstyle\sum_{i=1}^p \overline{K}(\cdot,\tau_i) * \Big]$$

$$\begin{bmatrix} A_1(\tau)z_{-1}(\tau, c_{-1}) + f(\tau) \\ A_{1i}z_{-1}(\tau_i - 0) + a_i \end{bmatrix}(t).$$

Continuing this process, by mathematical induction we prove that if (4) factors $z_i(t)$ series (5) are uniquely determined from the corresponding boundary value problems. The convergence of the series (5) also brought the traditional ways mazhoruvannya.

Conclusion. On the basis of the type Vishika-Lyusternika coefficient obtained Existence conditions of the boundary value problem (1) for arbitrary $f(t) \in C([a,b]/\{\tau_i\}_I)$, $a_i \epsilon \mathbb{R}^n, \alpha \epsilon \mathbb{R}^m$. For these conditions is constructed $(d \times r)$ – dimensional matrix (3).

References

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Proposals schema definitions coefficient uslovyy occurrence of solutions slabovozmuschennыh kraevыh problems for linear systems with ympulsnым Impact in fyksyrovannыe momentы time.

Slabovozbuzhdennaya lyneynaya neodnorodnaya kraevaya task odnorodnaya kraevaya problem with ympulsnым action, row, pseudoinverse matrices obobschennыy Green operator.

Proposed scheme for determining the coefficient conditions of decision of weakly perturbedlinear inhomogeneous boundary value problem for systems with impulse action in fixed times.

Poorlyraised linear non-uniform regional problem, homogeneous regional problem with pulse action, number, pseudo-returnable matrix, Green's generalised operator.