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# SETTING CONDITIONS FOR OPTIMALITY CONTROLS TECHNICAL SYSTEMS IN A CLOSED AREA OF ADMISSIBLE VALUES

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*In the paper the generalized formulation of optimal control technical system. The problem is solved by the method of dynamic programming. As a result, the optimal control is obtained in the form of feedback. On the basis of the Bellman equation and Conditions maximum of Hamilton shows the relationship between dynamic programming method and the maximum principle. The conditions of optimum control in a closed area of allowable values.*

***Functional, control, optimization, maximum principle, dynamic programming, a closed area.***

**Problem.**A large number of technical systems working with regulators, configured using certain methods [1]. Important methods of setting regulators are methods of optimal control. They allow you to build process control (management) system so that certain indicators of functioning acquired minimum or maximum values. These figures are called criteria optimization and is usually presented as integrated or terminal functional.

One of the problems in the synthesis of optimal control is the consideration of restrictions on the control. Solution of the problem of optimal control as a function of the phase coordinate system and then use nonlinear element of "saturation" allows you to find quasi-optimal control [2]. Another way to consider restrictions on the control variation in the control weights standing with controls in the structure criterion [3]. The latter method requires considerable processing power of computers as every step necessary to perform cyclic control procedure of weighting coefficients of variation, followed by a test limit on the value of control. Thus, the first method of incorporation do not allow limitations found



optimal control, and the second - is associated with a significant number of calculations to be performed in real-time.

**Analysis of recent research.**To find the optimal control technology systems used a variety of methods. The use of a particular method is primarily due to the nature of the problem, which is solved. We give a brief analysis methods for optimal control.

The oldest method of optimal control is the calculus of variations [4], the origin and development of which is associated with the names of Euler, Lagrange, Poisson Veyershtrassa, Jacobi, Legendre and other scientists. Leonhard Euler derived the equations, which is a necessary condition for the integral functional. In a further development of the calculus of variations associated with the generalization of Euler, study necessary conditions for an extremum of the functional and installation of its type (minimum or maximum). The specificity of the calculus of variations is that the function that delivers extreme functional sought in the class of piecewise smooth functions. In addition, the area of controls is transparent control function can take any values. Modern drives technical systems can not create any control signals (effort, time), and efforts were mathematicians to overcome these limitations is to search optimal control in a closed area. As noted in the book YP Petrov [5] In 1913, the Russian mathematician NN Garnett in [6] performed a generalization of the fundamental theorem of variations. It is proved that if extreme functional in a closed area there is achieved in a class of piecewise smooth functions, it can be reached only on curves, composed of segments of extremals and pieces maximum permissible area.

In the 60's of last century begin to appear other methods of optimal control. In the Soviet school of mathematics LS Pontryagin has been hypothesized whereby optimal control delivers maximum of Hamilton (any variation problem can be represented in the form of Hamilton equations [7]). Subsequently, this hypothesis was proved for linear systems, it is called "maximum principle". The maximum principle possible to solve the problem that can not be solved by means of the calculus of variations, such as the problem of the maximum speed. It should be noted that the solution of this problem for a dynamic system of n-th order, which has only real roots of the characteristic equation has at most (n-1) switches, ie transitions from the upper boundary of the lower controls [8]. This theorem is called "Theorem of n-times" proved AA Feldbaum.

Difficulty using the maximum principle is that it is necessary to look for conjugate functions that are the expression of optimal control. Priori information for this very small: some tasks can be set only type of conjugate functions (polynomial, trigonometric, polynomial-trigonometric etc.). Thus, the maximum principle gives only "quality" information on optimal control. "Quantitative" information about the optimal management should seek using other methods, such as phase plane method [9].

A powerful method for solving optimal control not only technical, but also economic, social and other natural systems are dynamic programming [10]. The author of this method is an American mathematician R. Bellman, which established the principle of optimality. Based on this principle was found functional equation, which is a prerequisite for optimal control. Functional Bellman equation is represented as inhomogeneous differential equation in partial derivatives. For integral quadratic optimization criterion Solutions functional Bellman equation should look for in a quadratic form. For other criteria of interpretation Bellman equation presents certain difficulties associated with the lack of recommendations for presentation Bellman function. If the dynamical system described by a small number of phase coordinates, it is advisable to use a discrete form of dynamic programming method [10]. However, if the phase coordinates many, the use of discrete dynamic programming requires large amounts of computer memory. This problem R. Bellman called "curse of dimensionality".

The advantage of the method of dynamic programming is the possibility of finding the optimal control as feedback (calculus of variations and maximum principle possible to find only the best software control, that control a function of time). This is the advantage of dynamic programming has led to the widespread use of this method for the synthesis of optimal controls [5, 7].

One method of finding the optimal control is the method of moments, the development of which is associated with the name of academician NN Krasovskii [12]. In computing terms, this method is more complicated than other methods, but it allows you to find the extremes "non-standard" functional, such as the rules of vector-control functions [12].

All methods above to establish the necessary conditions for optimality dynamic process control systems. Research needed for achieving a functional extremum held VF Krotov [13]. He proved a theorem that establishes three conditions for optimal control. The first of the conditions coincides with R. Bellman functional equation.

In solving problems of optimal control quite often have different complications. They may be related to the computational complexity problem or fundamental difficulty solving it. For some optimal control problems the exact solution is unimportant closer to the exact solution (exact or rough) gives good results in driving, while clarifying the optimal control law does not lead to a significant improvement of the system (a significant increase or decrease functional). These and other reasons have prompted researchers to develop approximate methods of optimal control. Review and analysis of these methods can be found in [14, 15], so we will not dwell on them in detail.

Note that for optimal control of lifting machinery researchers used different methods: calculus of variations [16, 17], the principle of maximum LS Pontryagin [9, 16, 18], dynamic programming [2], the method of moments [19] and approximate methods [20].

**The purpose of research.**The aim of this work is to establish conditions for optimality controls systems taking into account technical constraints on the value of control.

To achieve this goal it is necessary to solve the following problem:

1. Run a generalized statement of the problem of optimal control of dynamic system;
2. To solve the problem of optimal control using dynamic programming method, ie to find control in the form of feedback;
3. Write a necessary condition of optimality according to the principle of maximum and compare it with the functional Bellman equation, ie to establish the relationship between these methods;
4. Based on the analysis to give practical recommendations for optimal control as feedback while respecting the constraints management, ie closed area controls.

**Results.**Problem optimal control should include the following elements:

1) Mathematical model of the motion of a dynamical system. It is usually represented as a system of ordinary differential equations and partial differential equations. At least one equation must be heterogeneous, that means being able to physically control the movement of the system;

2) initial and final conditions of movement. These conditions set the initial state of the system and end - desirable;

3) optimization criterion establishes additional requirements for the implementation of motion, such as lower power consumption, duration of motion, improve system reliability and so on;

4) restrictions on the controls that are usually presented in the form of inequality;

5) constraints on the phase coordinates, their highest time derivative, integral constraints, which include phase coordinates and a variety of linear and nonlinear combinations of these restrictions. Mathematically constraints can be represented as equalities or inequalities.

Later will result mathematical formulation of the problem of optimal control, which includes all the elements given above except the last. Note that the last restrictions in some cases, be taken into account at the stage of optimal control with digital speed control systems (microcontroller board computers, etc.).

The equations of motion of the system we present in normal form:

$$\dot{x}_i = \sum_{j=1}^{n} a_{ij} x_j + k_j u, \quad i = 1,\dots,n, \tag{1}$$

where xi - i-th phase coordinate system (dot over a symbol means differentiation with respect to time); aij - coefficients; ki - some factors that show the impact of the change management phase coordinates xi; n - order system (total order differential equations describing the dynamics of the system).

Boundary conditions of the system are:

$$\begin{cases} x_i(t_0) = x_{i0}; \\ x_i(T) = x_{iT}, \end{cases} \tag{2}$$

where t0 and T - start and end time of the system, respectively.

The criterion for optimization of the system is a functional integral:

$$J = \int_{t_0}^{T} \left( \sum_{i=1}^{n} \delta_i x_i^2 + \left(1 - \sum_{i=1}^{n} \delta_i \right) u^2 \right) dt \to \min, \tag{3}$$

where δi - and a bearing weight, which shows the importance of a component in the integrand criterion (3), and should always be performed such inequality:

$$\sum_{i=1}^{n} \delta_i \le 1. \tag{3}$$

The restrictions imposed on the management function written in the following form:

$$u_{\min} \le u \le u_{\max}, \tag{4}$$

where umin and umax - respectively the lower and upper limits of acceptable control area.

*Solving optimal control problem by dynamic programming.*To find the optimal control use dynamic programming method. To do this, write the main functional Bellman equation [10]:

$$\min \left[ \sum_{i=1}^{n} \delta_i x_i^2 + \left(1 - \sum_{i=1}^{n} \delta_i \right) u^2 + \sum_{i=1}^{n} \frac{\partial S}{\partial x_i} \left( \sum_{j=1}^{n} a_{ij} x_j + k_j u \right) \right] = 0, \tag{5}$$

where S - Bellman function (minimum criterion (3)), ie min J = S.

While we assume that the restrictions on control (4) does not exist. Then for the solution of the equation (5) prodyferentsiyuyemo his left side for u and compare the resulting zero:

$$2u \left(1 - \sum_{i=1}^{n} \delta_i \right) + \sum_{i=1}^{n} \frac{\partial S}{\partial x_i} k_j = 0. \tag{6}$$

From equation (6) we obtain the optimal control function for the open field controls, ie without restrictions (4):

$$\underset{u_{opt}\in(-\infty;\infty)}{u_{opt}} = \frac{-1}{2\left(1-\sum_{i=1}^{n}\delta_i\right)}\sum_{i=1}^{n}\frac{\partial S}{\partial x_i}k_j. \tag{7}$$

The second partial left side of equation (5) for the control u is as follows:

$$2\left(1-\sum_{i=1}^{n}\delta_i\right)\geq 0. \tag{8}$$

Inequality (8) means that except in the case $\sum_{i=1}^{n}\delta_i = 1$ control function (7) gives the left side of equation (5) minimum.

Substituting the obtained expression (7) in equation (5), we have:

$$\sum_{i=1}^{n}\delta_i x_i^2 + \frac{\left(\sum_{i=1}^{n}\frac{\partial S}{\partial x_i}k_j\right)^2}{4\left(1-\sum_{i=1}^{n}\delta_i\right)} + \sum_{i=1}^{n}\frac{\partial S}{\partial x_i}\left(\sum_{j=1}^{n}a_{ij}x_j - \frac{k_j}{2}\frac{\sum_{i=1}^{n}\frac{\partial S}{\partial x_i}k_j}{\left(1-\sum_{i=1}^{n}\delta_i\right)}\right) = 0. \tag{9}$$

Since criterion (3) - quadratic-linear, it is necessary to seek function S as a quadratic form:

$$S = S(x_i) = \sum_{i,\ j=1}^{n}A_{ij}x_i x_j, \tag{10}$$

where Aij - certain factors that need to be found. Find the partial derivatives of (10) for the phase coordinates:

$$\frac{\partial S}{\partial x_i} = 2A_{ii}x_i + \sum_{i=2}^{n}A_{ij}x_i. \tag{11}$$

Substituting the expressions in equation (7), and we write:

$$\underset{u_{opt}\in(-\infty;\infty)}{u_{opt}} = \frac{-1}{2\left(1-\sum_{i=1}^{n}\delta_i\right)}\sum_{i=1}^{n}\left(2A_{ii}x_i + \sum_{i=2}^{n}A_{ij}x_i\right)k_j. \tag{12}$$

Once the coefficients Aij are found optimal control function is defined.

*Hamiltonian and communication maximum principle of dynamic programming.* We use the maximum principle approach for the analysis of optimal control problem (1) - (4). To do this, we write the Hamiltonian function:

$$H = \psi_0\left(\sum_{i=1}^{n}\delta_i x_i^2 + \left(1-\sum_{i=1}^{n}\delta_i\right)u^2\right) + \psi_i\left(\sum_{j=1}^{n}a_{ij}x_j + k_j u\right), \tag{13}$$

where ψ0 and ψi - conjugate variables. According to the maximum principle, the optimal control function must deliver maximum Hamilton.

Apart from the constraints (4), then it is necessary to find the partial derivative of the Hamiltonian for the control u and got equated to zero causing obtain:

$$\frac{\partial H}{\partial u} = 2\psi_0\left(1-\sum_{i=1}^{n}\delta_i\right)u + \sum_{j=1}^{n}\psi_i k_j = 0. \tag{14}$$

From equation (14) we get:

$$\underset{u_{opt}\in(-\infty;\,\infty)}{u_{opt}} = \frac{-1}{2\psi_0\left(1-\sum_{i=1}^{n}\delta_i\right)}\sum_{j=1}^{n}\psi_i k_j. \tag{15}$$

The second partial derivative of the Hamiltonian (13) for control and equal to:

$$\frac{\partial^2 H}{\partial u^2} = 2\psi_0\left(1-\sum_{i=1}^{n}\delta_i\right). \tag{16}$$

If $\sum_{i=1}^{n}\delta_i < 1$ and ψ0 <0 (usually choose ψ0 = -1), the function of optimal control delivers maximum Hamiltonian. For closed area controls are:

$$\underset{u_{opt}\in[u_{\min};\,u_{\max}]}{u_{opt}} = \begin{cases} u_{\min}, \ npu \ \underset{u_{opt}\in(-\infty;\,\infty)}{u_{opt}} < u_{\min}; \\ \underset{u_{opt}\in(-\infty;\,\infty)}{u_{opt}}, \ npu \ u_{\min} \le \underset{u_{opt}\in(-\infty;\,\infty)}{u_{opt}} \le u_{\max}; \\ u_{\max}, \ npu \ \underset{u_{opt}\in(-\infty;\,\infty)}{u_{opt}} > u_{\max}. \end{cases} \tag{17}$$

For further research is necessary to communicate the maximum principle of dynamic programming. Transform the functional Bellman equation (5) - submit it in the following way:

$$\max\left[-\left(\sum_{i=1}^{n}\delta_i x_i^2 + \left(1-\sum_{i=1}^{n}\delta_i\right)u^2\right) + \left(\sum_{i=1}^{n}\left(-\frac{\partial S}{\partial x_i}\right)\left(\sum_{j=1}^{n}a_{ij}x_j + k_j u\right)\right)\right] = 0, \tag{18}$$

Comparing the expressions (13) and (18) we conclude that:

$$\begin{cases} \psi_0 = -1; \\ \psi_i = -\frac{\partial S}{\partial x_i}. \end{cases} \tag{19}$$

Thus, the conjugate functions used in the maximum principle is partial Bellman functions for phase coordinates in the method of dynamic programming. Based on the results (19) in the expression of optimal control (17). It is necessary to take into account the expressions (11) for partial Bellman functions for phase coordinates. As a result of substitutions we obtain:

$$\underset{u_{opt}\in[u_{\min};\,u_{\max}]}{u_{opt}} = \begin{cases} u_{\min}, \ npu \ \dfrac{-1}{2\left(1-\sum\limits_{i=1}^{n}\delta_i\right)}\sum\limits_{i=1}^{n}\left(2A_{ii}x_i+\sum\limits_{i=2}^{n}A_{ij}x_i\right)k_j < u_{\min}; \\ \dfrac{-1}{2\left(1-\sum\limits_{i=1}^{n}\delta_i\right)}\sum\limits_{i=1}^{n}\left(2A_{ii}x_i+\sum\limits_{i=2}^{n}A_{ij}x_i\right)k_j, \\ npu \ \ u_{\min} \le \dfrac{-1}{2\left(1-\sum\limits_{i=1}^{n}\delta_i\right)}\sum\limits_{i=1}^{n}\left(2A_{ii}x_i+\sum\limits_{i=2}^{n}A_{ij}x_i\right)k_j \le u_{\max}; \\ u_{\max}, \ npu \ \dfrac{-1}{2\left(1-\sum\limits_{i=1}^{n}\delta_i\right)}\sum\limits_{i=1}^{n}\left(2A_{ii}x_i+\sum\limits_{i=2}^{n}A_{ij}x_i\right)k_j > u_{\max}. \end{cases} \tag{20}$$

As is evident from the reduced expression (20) optimal control is a complex function:

$$\underset{u_{opt}\in[u_{\min};\,u_{\max}]}{u_{opt}} = f(u_{\min}, u_{\max}, \ \delta_i, \ A_{ii}, \ A_{ij}, \ k_j, \ x_i). \tag{21}$$

However, analysis of the function (21) for each case makes it possible to simplify it.

**Conclusion.**Thus, in this paper it is proved that the optimal control restrictions in the form of feedback through the use of non-linear element of "saturation" does not lead to loss of optimality process. This control will still delivers maximum Hamiltonian and therefore optimal. This greatly simplifies the method of calculating the optimal control. It comes down to perform these operations: 1) optimal control problem statement; 2) record the necessary conditions of optimality - functional Bellman equation; 3) solution of the Bellman equation and the establishment of optimal control expression in the form of feedback; 4) "cut-off" pieces of optimal control functions that go beyond the permissible area controls by using nonlinear element of "saturation".

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*In Article obobschennoy Flag raising the problem of optimal control Tehnicheskoe system. Postavlennaya problem with razvyazana pomoshchju Dynamic programming method. As a result of Local Government will provide a optymalnoe obratnoy in video communications. Based on analysis Bellman equation and the maximum terms Hamilton function shows the Communications Between Dynamic programming method and the maximum principle. Established in terms of optimum control region zakrытoy permissible values.*

***Funktsyonal, control, optimization, the maximum principle, dynamycheskoe programming, zamknutaya region.***

*The generalized statement of the problem of optimal control of a technical system has been carried out in the article. The statemented problem has been solved by dynamic programming. This gave the optimal control in the form of feedback. Based on the Bellman equation and the condition of the Hamiltonian maximum analysis shows the relationship between the methods of dynamic programming and the maximum principle. The optimal control condition is set for in a closed feasible region.*

***Functional, control, optimization, maximum principle, dynamic programming, closed area.***

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## ANALYSIS cutting EQUIPMENT PLANTS FOR BEZPIDPIRNOHO mowing

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*The analysis of structures and working process of cutting machines for cutting plants bezpidpirnoho - belt-segment, segment-chain, rope-segment, and tape rotation, in which found that the best conditions of agricultural production meets the rotary cutting device, but none of them does not meet the requirements for cutting and chopping crop residues and grounded fundamentally two types - new bunk cutting machines with parallel and consistent work knives.*

***Cutting machine belt segment, segment chain, cable segment, belt, rotary, two-level unit.***