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PROBLEMS THE GUARANTEED SENSITIVITY FOR SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS.

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The algorithms optimization of dynamic systems using the function of sensitivity. The problem of guaranteed sensitivity covered practical stability criteria. Done field evaluation parameters for each value of which does not violate the restrictions on the function of sensitivity.

Keywords: guaranteed sensitivity, sensitivity functions, parameters, resistance.

When solving a number of applied problems associated with the design, management and optimization of real systems [1,3,5], often need to analyze the sensitivity of the system relative to various kinds of perturbations parameters. To ensure the normal operation of the system, in particular, impose limits on sensitivity function and evaluate the corresponding area of initial conditions [2]. It is essential that the requirements for sensitivity performed not only on settlement modes, and in view of the spread of values of real parameters. This approach applies to a class of problems the guaranteed sensitivity of the proposed algorithms for solving practical stability [2,3] for equations sensitivity.

The aim of research ─ development of design methods for solving problems on the basis of the guaranteed sensitivity algorithms practical stability of parametric systems.

Materials and methods research. The paper used mathematical methods of the theory of stability, sensitivity and optimization.

Consider first the linear system of differential equations:

$$\frac{dx}{dt} = A(t)x + G(t)\alpha, \ t \in [t_0, T], \tag{1}$$

with initial conditions

$$x(t_0) = x_0 = x_0(\alpha),\ t_0 = t_0(\alpha). \tag{2}$$

Here α − m - dimensional vector of parameters selected from some set G_α; $x = x(t,\alpha)$ − vector of phase coordinates dimension n; $A(t)$, $G(t)$ − matrix of dimension $n \times n$ and $n \times m$ respectively.

The equation for the sensitivity $[2,5]$ of the system (1) are as follows:

$$\frac{du^{(i)}(t,\alpha)}{dt} = A(t)u^{(i)}(t,\alpha) + g^{(i)}(t),\ t \in [t_0, T], \tag{3}$$

where $u^{(i)}(t,\alpha) = \dfrac{\partial x(t,\alpha)}{\partial \alpha_i}$ − the vector dimension n sensitivity functions; $g^{(i)}(t)$ − n-dimensional vector corresponding to i- column matrix $G(t)$ ($i = 1,2,...,m$).

Consider the problem of minimizing functional on the final state of the system (1) for vector parameters α:

$$J\left(\alpha^{(0)}\right) = \min_{\alpha \in G_\alpha} \Phi(x(T,\alpha)) \tag{4}$$

if restrictions on the function of sensitivity $[1-3]$:

$$\Phi_t = \Gamma_t = \left\{u(t,\alpha):\ \left|\sum_{i=1}^{m} l_s^{(i)^*}(t) u^{(i)}(t,\alpha)\right| \le 1, s = 1,2,...,N\right\},\ t \in [t_0, T]; \tag{5}$$

$$\Phi_t = \Psi_t = \{u(t,\alpha):\ \psi(u(t,\alpha),t) = \psi\left(u^{(1)}(t,\alpha), u^{(2)}(t,\alpha),...,u^{(m)}(t,\alpha)\right) \le 1\},\ t \in [t_0, T]. \tag{6}$$

Here $l_s^{(i)}(t)$, $i = 1,2,...,m$, $s = 1,2,...,N$ − known continuous vector function dimension n; $u^*(t,\alpha) = \left(u^{(1)^*}(t,\alpha),...,u^{(m)^*}(t,\alpha)\right)$ − the vector dimension $n \cdot m$; $\psi(u(t,\alpha),t)$ − scalar function continuous in their arguments with partial on the elements of the vector $u(t,\alpha)$; set Ψ_t − convex, closed and contains an internal zero point to any $t \in [t_0, T]$.

To solve the problem (4) applies gradient descent scheme $[1]$. In order to account restrictions (5), (6) is introduced for consideration of initial conditions set for sensitivity functions $G_0 = \left\{u(t_0):\ \sum_{i=1}^{m} u^{(i)^*}(t_0) B_i u^{(i)}(t_0) \le c^2\right\}$ and algorithms apply practical stability $[3]$ in space of these functions.

In contrast to the problems of limited sensitivity, there limits on sensitivity functions will be performed for any α of a plurality

$$G_1^{\alpha} = \left\{ \alpha : \sum_{i=1}^{m} \left(\frac{dx_0(\alpha)}{d\alpha_i} - \left(\tilde{A}x_0(\alpha) + \tilde{G}\alpha\right) \frac{dt_0(\alpha)}{d\alpha_i} \right)^{*} B_i \left(\frac{dx_0(\alpha)}{d\alpha_i} - \left(\tilde{A}x_0(\alpha) + \tilde{G}\alpha\right) \frac{dt_0(\alpha)}{d\alpha_i} \right) \le c^2 \right\}.$$

From these positions can be considered nonlinear system linearization and hold it in the vicinity of any settlement movement.

Conclusions

Based on practical criteria for sustainability of parametric algorithms solving problems guaranteed sensitivity. If the dynamic limits on sensitivity functions performed numerical estimation initial field conditions implicitly defines the set of allowable spread parameters of the original system.

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