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DETERMINATION OF PARTICLE MOTION PARAMETERS OF BIOMASS During rotation digesters

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The mathematical model to determine the parameters of particle motion in a rotating biomass digesters, which allows you to set rational values of angular velocity digesters and its design parameters.

Biomass digesters, biogas, mixing, motion of a particle.

Formulation of the problem. The operation of biogas plants showed that promote contact with anaerobic bacteria biomass substrate is provided by the mixing of the substrate, but with intensive mixing must be avoided, as this can lead to poor anaerobic digestion at the expense of symbiosis atsetohennyh and methanogenic bacteria. In practice, the compromise achieved by slow rotation agitators or work within a short time. [1] At the same time, operating experience biogas reactors showed that almost impossible to remove bundles of biomass in a reactor in mineral and organic sediment floating biomass, indicating weaknesses in the operation of the mixing biomass [2, 3].

Improving the efficiency of biogas plants is one of the main directions of improving the process of biogas production, and therefore justify the methods of determining the parameters of particle motion in a rotating biomass digesters that will setrational values of angular velocity digesters and its design parameters needs further improvement.

Analysis of recent research. As a result of research we have patented a number of technical solutions that largely eliminate the separation of biomass by providing biomass mixing layers using embedded rotating biogas reactors. Defined as the level of immersion in the rotating liquid methane tanks and rate of filling, depending on its geometric parameters and density of the liquid, which is immersed rotating digesters in securing its location in a suspended state. [4]

It was also established that the power it takes to overcome the moment of resistance in the bearings depends on the level of organic biomass into digesters, its weight and performance of biogas and biomass [5, 6]. Power consumed by mixing biomass depends on characteristics of biomass (density, dry matter content, particle size of dry matter) and structural-kinematic characteristics digesters (angular velocity, the inner radius, length, geometric dimensions and placement of blades, mixers and walls inside the digesters) [7, 8].

However, to prevent the separation of biomass and ensuring its stirring by raising the mineral component of biomass that accumulates in the bottom of the methane tanks and dive organic component of biomass that accumulates at the top of digesters, you must know the parameters and the trajectory of mineral and organic particles of biomass by stirring blades and in volume rotary digesters.

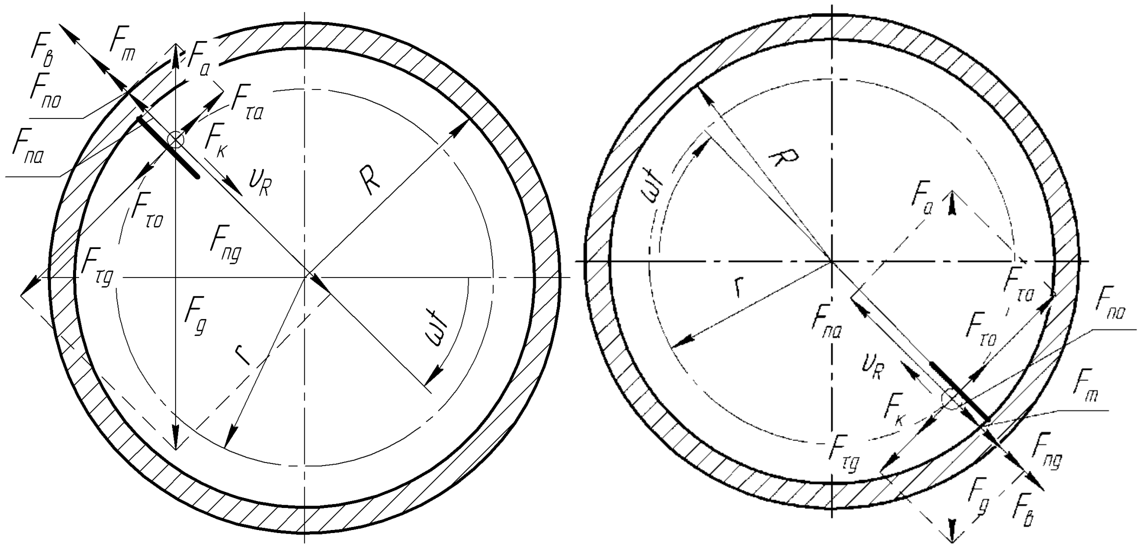
The purpose of research. Theoretically, set the trajectory and mineral and organic particles by stirring blades biomass and methane tanks in volume during its rotation.

Results. Diagram of the forces on the mineral (s) and organic (b) of the substrate particles that interact with mixing blades rotating digesters shown in Fig. 1: Fg - gravity, H; Fa - Archimedes force, N; FT - friction, N; Fk - Coriolis force of inertia, N; Fv - centrifugal force of inertia, N; Fno - the resistance of the substrate, which prevents movement of particles in the radial direction, N; Fто - the force that presses the blade to the piece due to the resistance of the substrate, N; r - the radius of the current position of the substrate particles, m; υR - relative velocity organic particles when driving on the shoulder blade, m / s; R - inner radius digesters, m; ωt - angle digesters, tips.

To determine the relative speed of installation and the trajectory of the mineral particles of biomass based on Fig. 1 (a) is made differential equation of motion mineral substrate particles in the form of a point on a rotating pan digesters. Subject to existing mineral particle forces, equation of motion for pan will look like:

$$m\frac{d\upsilon_R}{dt} = F_в + F_m + F_{na} + F_{no} - F_{ng}, \quad (1)$$

where: Fv - chastyky centrifugal force of inertia, N; FT - friction particles, N; Fna - part Archimedes force acting on the particle vzovzh blade, N; Fno - the resistance of the substrate, which prevents movement of particles in the radial direction, N; Fng - component particles gravity acting along the scoops, N; υR - relative velocity while moving in a pan, m / s.



a) b)

Fig. 1. Scheme of forces in the mineral (s) and organic (b) of the substrate particles that interact with mixing blades:

Friction is determined by the forces of prytyskuyut to the blades:

$$F_m = f\left(F_{\tau o} + F_{\tau g} - F_{\tau a} - F_{\kappa}\right), \qquad (2)$$

Where: Fтo - the force that presses the blade to the piece due to the resistance of the substrate, N; Fтg - component of gravity particles that acts perpendicular to the blade, N; Fтa - Archimedes force component for a particle that acts perpendicular to the blade, N; Fk - Coriolis force of inertia chastyky, N; f - friction coefficient of the substrate material on the blade digesters, ratio. units.

Taking into account that:

$$F_{\text{в}} = m_{\text{ч}} r\omega^2;\ F_g = m_{\text{ч}} g; F_a = \rho_c g V_{\text{ч}};\ F_{\kappa} = 2m_{\text{ч}}\omega\frac{dr}{dt};$$
$$F_{no} = m_{\text{ч}} k_1\frac{dr}{dt};\ F_{\tau o} = m_{\text{ч}} k_1 r\omega, \qquad (3)$$

Where: mch - particle mass, kg; ω - angular velocity digesters, s-1; r - the radius of the current position of the substrate particles, m; *g* - Acceleration of gravity, m / s2; Vch - particles volume, m3; k1 - coefficient of proportionality in laminar flow around the particles substrate, p-1; k1 = 18η / (ρsdE2); k2 - ratio of the density of the particles and the substrate, k2 = ρs / ρch; ρs - density of the substrate, kg / m3; ρch - density particles, kg / m3; η - dynamic viscosity substrate, Pa; dE - equivalent to the diameter of the city.

Subject (3) and (2) the equations of motion of mineral particles on the surface of the substrate rotating blades digesters can be written as follows:

$$\frac{d^2r}{dt^2}+\left(2f\omega-k_1\right)\frac{dr}{dt}-\left(\omega^2+fk_1\omega\right)r= \\ =g\left[f\left(1-k_2\right)\cos\left(\omega t\right)+\left(k_2-1\right)\sin\left(\omega t\right)\right]. \tag{4}$$

This equation is linear second-order equations with constant coefficients and right side in a trigonometric polynomial [9]. According to him, homogeneous differential equation will look like:

$$\frac{d^2r}{dt}+\left(2f\omega-k_1\right)\frac{dr}{dt}-\left(\omega^2+fk_1\omega\right)r=0. \tag{5}$$

Characteristic equation of the homogeneous differential equation will look like:

$$\lambda^2+\left(2f\omega-k_1\right)\lambda+\left(-\omega^2-fk_1\omega\right)=0. \tag{6}$$

The roots of the characteristic equation will look like:

$$\lambda_1=\frac{k_1}{2}-f\omega-\sqrt{\omega^2\left(f^2+1\right)+\frac{k_1^2}{4}};\ \lambda_2=\frac{k_1}{2}-f\omega+\sqrt{\omega^2\left(f^2+1\right)+\frac{k_1^2}{4}}. \tag{7}$$

General solution of differential equations will have the form:

$$r=C_1\exp\left(\lambda_1 t\right)+C_2\exp\left(\lambda_2 t\right)+r_{ЧР}, \tag{8}$$

where: C1, C2 - constants of differential equations.

A partial solution of the inhomogeneous differential equation is written as trigonometric polynomial:

$$r_{ЧР}=M\cos\left(\omega t\right)+N\sin\left(\omega t\right), \tag{9}$$

the first derivative of which is as follows:

$$r_{ЧР}^{/}=-M\omega\sin\left(\omega t\right)+N\omega\cos\left(\omega t\right), \tag{10}$$

and the second derivative respectively as follows:

$$r_{ЧР}^{//}=-M\omega^2\cos\left(\omega t\right)-N\omega^2\sin\left(\omega t\right). \tag{11}$$

Equation (4) present in the form:

$$\left[-M\omega^2\cos\left(\omega t\right)-N\omega^2\sin\left(\omega t\right)\right]+\left(2f\omega-k_1\right)\times \\ \times\left[-M\omega\sin\left(\omega t\right)+N\omega\cos\left(\omega t\right)\right]-\left(\omega^2+fk_1\omega\right)\times \\ \times\left[M\cos\left(\omega t\right)+N\sin\left(\omega t\right)\right]=gf\left(1-k_2\right)\cos\left(\omega t\right)+g\left(k_2-1\right)\sin\left(\omega t\right). \tag{12}$$

To find the coefficients M and N on the basis of this equation we use a system of equations:

$$\begin{cases}\left(-2\omega^2-fk_1\omega\right)M+\left(2f\omega^2-k_1\omega\right)N=gf\left(1-k_2\right)\\ \left(-2f\omega^2+k_1\omega\right)M+\left(-2\omega^2-fk_1\omega\right)N=g\left(k_2-1\right).\end{cases} \tag{13}$$

Solving this system by Cramer's rule [9]

$$\Delta = \begin{vmatrix} -2\omega^2 - fk_1\omega & 2f\omega^2 - k_1\omega \\ -2f\omega^2 + k_1\omega & -2\omega^2 - fk_1\omega \end{vmatrix} = \omega^2\left[4\left(\omega^2 f^2 + 1\right) + k_1^2\left(f^2+1\right)\right] \quad (14)$$

$$\Delta_M = \begin{vmatrix} gf\left(1-k_2\right) & 2f\omega^2 - k_1\omega \\ g\left(k_2-1\right) & -2\omega^2 - fk_1\omega \end{vmatrix} = \omega g k_1\left(f^2+1\right)\left(k_2-1\right) \quad (15)$$

$$\Delta_N = \begin{vmatrix} -2\omega^2 - fk_1\omega & gf\left(1-k_2\right) \\ -2f\omega^2 + k_1\omega & g\left(k_2-1\right) \end{vmatrix} = 2\omega^2 g\left(f^2+1\right)\left(k_2-1\right) \quad (16)$$

get:

$$M = \frac{\Delta_M}{\Delta} = \frac{gk_1\left(f^2+1\right)\left(k_2-1\right)}{\omega\left[4\left(\omega^2 f^2+1\right)+k_1^2\left(f^2+1\right)\right]} \quad (17)$$

$$N = \frac{\Delta_N}{\Delta} = \frac{2g\left(f^2+1\right)\left(k_2-1\right)}{4\left(\omega^2 f^2+1\right)+k_1^2\left(f^2+1\right)}. \quad (18)$$

Then the solution of partial differential equations of motion of inhomogeneous particles will look like:

$$r_{ЧР} = \frac{g\left(f^2+1\right)\left(k_2-1\right)}{4\left(\omega^2 f^2+1\right)+k_1^2\left(f^2+1\right)}\left(\frac{k_1}{\omega}\cos\left(\omega t\right)+2\sin\left(\omega t\right)\right). \quad (19)$$

Full solution inhomogeneous differential equation of a particle as a sum total (8) and partial solutions (19) will be as follows:

$$r = C_1\exp\left(\lambda_1 t\right)+C_2\exp\left(\lambda_2 t\right)+ \\ +\frac{g\left(f^2+1\right)\left(k_2-1\right)}{4\left(\omega^2 f^2+1\right)+k_1^2\left(f^2+1\right)}\left(\frac{k_1}{\omega}\cos\left(\omega t\right)+2\sin\left(\omega t\right)\right). \quad (20)$$

The relative velocity of mineral particles when driving on the pan will be:

$$\upsilon_R = \frac{dr}{dt} = \lambda_1 C_1\exp\left(\lambda_1 t\right)+\lambda_2 C_2\exp\left(\lambda_2 t\right)+ \\ +\frac{g\left(f^2+1\right)\left(k_2-1\right)}{4\left(\omega^2 f^2+1\right)+k_1^2\left(f^2+1\right)}\left(2\omega\cos\left(\omega t\right)-k_1\sin\left(\omega t\right)\right). \quad (21)$$

Having that $\frac{g\left(f^2+1\right)\left(k_2-1\right)}{4\left(\omega^2 f^2+1\right)+k_1^2\left(f^2+1\right)} = k_3$ and given the initial conditions: t = 0, r = R (where R - inner radius digesters), υ = υRp = 0, for finding permanent differentiation C1 and C2 write the system of equations:

$$\begin{cases} C_1 + C_2 = R - \dfrac{k_1 k_3}{\omega} \\ \lambda_1 C_1 + \lambda_2 C_2 = -2\omega k_3. \end{cases} \tag{22}$$

Us solve this system by Cramer:

$$\Delta = \begin{vmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{vmatrix} = \lambda_2 - \lambda_1; \tag{23}$$

$$\Delta_{C_1} = \begin{vmatrix} R - \dfrac{k_1 k_3}{\omega} & 1 \\ -2\omega k_3 & \lambda_2 \end{vmatrix} = \lambda_2 \left(R - \frac{k_1 k_3}{\omega} \right) + 2\omega k_3; \tag{24}$$

$$\Delta_{C2} = \begin{vmatrix} 1 & R - \dfrac{k_1 k_3}{\omega} \\ \lambda_1 & -2\omega k_3 \end{vmatrix} = -2\omega k_3 - \lambda_1 \left(R - \frac{k_1 k_3}{\omega} \right); \tag{25}$$

$$C_1 = \frac{\Delta_{C_1}}{\Delta} = \frac{\lambda_2}{\lambda_2 - \lambda_1} \left[\left(R - \frac{k_1 k_3}{\omega} \right) + \frac{2\omega k_3}{\lambda_2} \right]; \tag{26}$$

$$C_2 = \frac{\Delta_{C_2}}{\Delta} = \frac{\lambda_1}{\lambda_2 - \lambda_1} \left(\frac{k_1 k_3}{\omega} - \frac{2\omega k_3}{\lambda_1} - R \right). \tag{27}$$

Knowing the constants of integration can be found distance running mineral particles in pan and set change in velocity of moving a prescribed period of time.

Similarly, to determine the relative speed of installation and the trajectory of organic biomass particles based on Fig. 1 (b) is made differential equation of motion organic substrate particles in the form of a point on a rotating pan digesters. With regard to the current organic particle forces, equation of motion for pan will look like:

$$m \frac{d\upsilon_R}{dt} = F_{na} - F_{в} - F_{m} - F_{no} - F_{ng}. \tag{28}$$

Friction is determined by the forces of prytyskuyut to the blades:

$$F_m = f \left(F_{\tau o} + F_{\tau a} - F_{\tau g} - F_{к} \right). \tag{29}$$

Subject (28) and (29) the equations of motion organic substrate particles on the surface of the rotating blades digesters can be written as follows:

$$\frac{d^2 r}{dt^2} + (2 f\omega - k_1) \frac{dr}{dt} - \left(\omega^2 + f k_1 \omega \right) r = \\ = g \left[f (k_2 - 1) \cos(\omega t) + (1 - k_2) \sin(\omega t) \right]. \tag{30}$$

The equation is also linear second-order equations with constant coefficients and right side in a trigonometric polynomial, and therefore its solution similar to the above is the solution of the equations of motion of mineral particles.

A partial solution of equation (30) will look like:

$$r_{ЧР}=\frac{g\left(f^2+1\right)\left(k_2-1\right)}{4\left(\omega^2 f^2+1\right)+k_1^2\left(f^2+1\right)}\left(2\sin\left(\omega t\right)-\frac{k_1}{\omega}\cos\left(\omega t\right)\right). \quad (31)$$

Full solution inhomogeneous differential equation (30), the sum total and partial solutions to the following:

$$r=C_1\exp\left(\lambda_1 t\right)+C_2\exp\left(\lambda_2 t\right)+ \\ +\frac{g\left(f^2+1\right)\left(k_2-1\right)}{4\left(\omega^2 f^2+1\right)+k_1^2\left(f^2+1\right)}\left(2\sin\left(\omega t\right)-\frac{k_1}{\omega}\cos\left(\omega t\right)\right). \quad (32)$$

The relative speed of the organic particles when driving on the pan will be:

$$\upsilon_R=\frac{dr}{dt}=\lambda_1 C_1\exp\left(\lambda_1 t\right)+\lambda_2 C_2\exp\left(\lambda_2 t\right)+ \\ +\frac{g\left(f^2+1\right)\left(k_2-1\right)}{4\left(\omega^2 f^2+1\right)+k_1^2\left(f^2+1\right)}\left(2\omega\cos\left(\omega t\right)+k_1\sin\left(\omega t\right)\right). \quad (33)$$

Given the initial conditions: t = 0, r = R (where R - inner radius digesters), υ = υRp = 0, we find the constant differentiation C1 and C2:

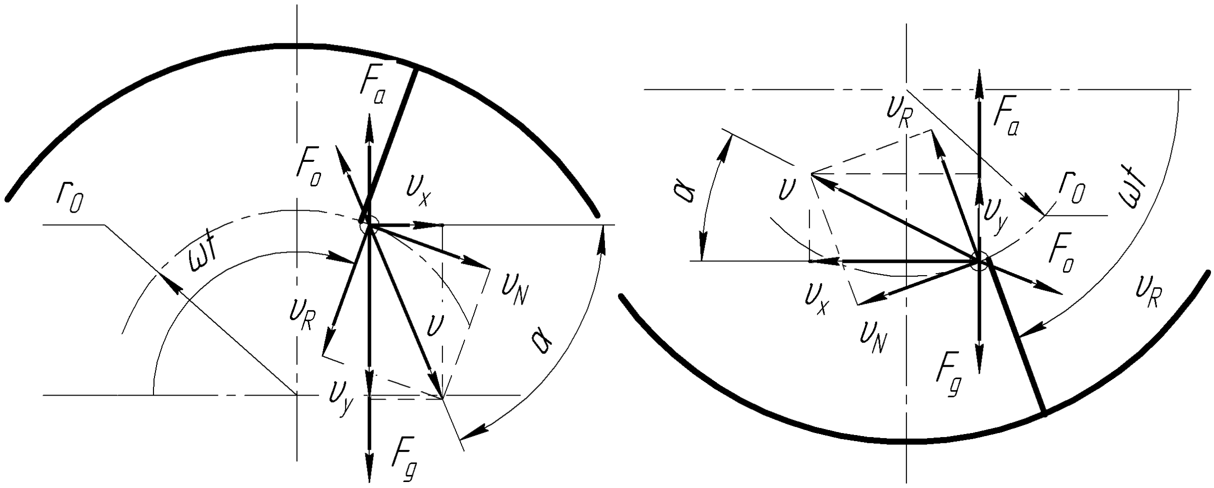
$$C_1=\frac{\lambda_2}{\lambda_2-\lambda_1}\left[\left(R+\frac{k_1 k_3}{\omega}\right)+\frac{2\omega k_3}{\lambda_2}\right]; \quad (34)$$

$$C_2=\frac{\lambda_1}{\lambda_2-\lambda_1}\left(-\frac{2\omega k_3}{\lambda_1}-R-\frac{k_1 k_3}{\omega}\right). \quad (35)$$

Knowing the permanent integration of the equation (32) we can find the distance that is organic pan particle in a prescribed period of time. But the equation (33) can set the velocity change of organic particles in pan on a set amount of time.

Diagram of the forces on the mineral (s) and organic (b) particles substrate, which are in free movement in a rotary volume digesters shown in Fig. 2. To install the trajectory of the mineral particles of biomass based on Fig. 2 (a) is made differential equation of motion of particles of mineral substrate after the accession of the blades in volume digesters. Obviously, mineral and organic substrate particles that move inside digesters after the accession of the blades, the following forces:

$$F_g=m_ч g;\; F_a=m_ч k_2 g;\; F_o=m_ч k_1\upsilon. \quad (36)$$



a) b)

Fig. 2. Scheme of forces in the mineral (s) and organic (b) particles substrate, which are in free movement in a rotary volume digesters: Fg - gravity, H; Fa - Archimedes force, N; Fo - the resistance of the substrate, N; r0 - radius at which the blade ends, m; *υ* - absolute velocity, m / s; υR - relative velocity during ascent of blades, m / s; υN - normal velocity during ascent of blades, m / s; υh - the projection of the absolute velocity of a particle at x-axis, m / s; υu - the projection of the absolute velocity of a particle on the axis in m / s; α - angle between the absolute speed and its projection on the x-axis, rad; ωt - angle digesters while climbing particles with shovels, boards.

According to equation (36), and given the fact that the mineral particles will sink, built the equation of the center of mass of this particle:

$$\begin{cases} \frac{d^2x}{dt^2} = -k_1\upsilon\cos\alpha \\ \frac{d^2y}{dt^2} = g\left(k_2 - 1\right) + k_1\upsilon\sin\alpha, \end{cases} \tag{37}$$

where: x - mineral particles moving along the horizontal axis, m; y - mineral particles move along the vertical axis, m; t - the movement of mineral particles after climbing a blade digesters, p.

To set the trajectory of organic biomass particles based on Fig. 2 (b) and given the fact that organic particles will emerge, built the equation of the center of mass of this particle:

$$\begin{cases} \frac{d^2x}{dt^2} = k_1\upsilon\cos\alpha \\ \frac{d^2y}{dt^2} = g\left(k_2 - 1\right) - k_1\upsilon\sin\alpha. \end{cases} \tag{38}$$

To determine the trajectories of mineral and organic particles after climbing blades of digesters used method of sequential differentiation [9, 10], which gives approximate solutions of equations (38) and (39) as a power series Taylor.

Given the initial conditions corresponding to the absolute speed at the time of the biomass particles climbing blade υ = υ0, between the absolute angular velocity and its projection on the axis at this point α = α0, and taking x0 = 0, y0 = 0, we write:

- For mineral particles:

$$\begin{cases} x = \dfrac{\upsilon_0 \cos\alpha_0}{k_1}\left[1 - \exp\left(-k_1 t\right)\right] \\ y = \dfrac{\upsilon_0 \sin\alpha_0}{k_1}\left[\exp\left(-k_1 t\right) - 1\right] + \dfrac{g\left(1-k_2\right)}{k_1^2}\left[1 - k_1 t - \exp\left(-k_1 t\right)\right]; \end{cases} \quad (39)$$

- For organic particles:

$$\begin{cases} x = \dfrac{\upsilon_0 \cos\alpha_0}{k_1}\left[\exp\left(-k_1 t\right) - 1\right] \\ y = \dfrac{\upsilon_0 \sin\alpha_0}{k_1}\left[1 - \exp\left(-k_1 t\right)\right] - \dfrac{g\left(1-k_2\right)}{k_1^2}\left[1 - k_1 t - \exp\left(-k_1 t\right)\right]. \end{cases} \quad (40)$$

According to the obtained systems of equations can construct the trajectory of mineral and organic particles after the accession of the blades. Based on the calculations found that the average density of the substrate ρs = 1025-1050 kg / m3 density of mineral substrate ρm = 1150-1250 kg / m3 and the density of the organic substrate ρo = 800-900 kg / m3, rational values of angular velocity digesters are ω = 0,035-0,08 s-1.

Based on the equations of motion found mineral and organic particles on the substrate components digesters and blades after the blades of their ascent trajectory of particles found inside the drum (Fig. 3).

For values of angular velocity and inner radius digesters adopted in the calculation, and the length of the blade digesters l = (0,7750,825) R, mineral particles will climb to the top of digesters, after which the particles are separated from the blade and move down, and organic substrate particles will dive to the bottom of the bioreactor and then be separated from the blade and moving up, because this will be provided uniform mixing and interpenetration of the components of the substrate.

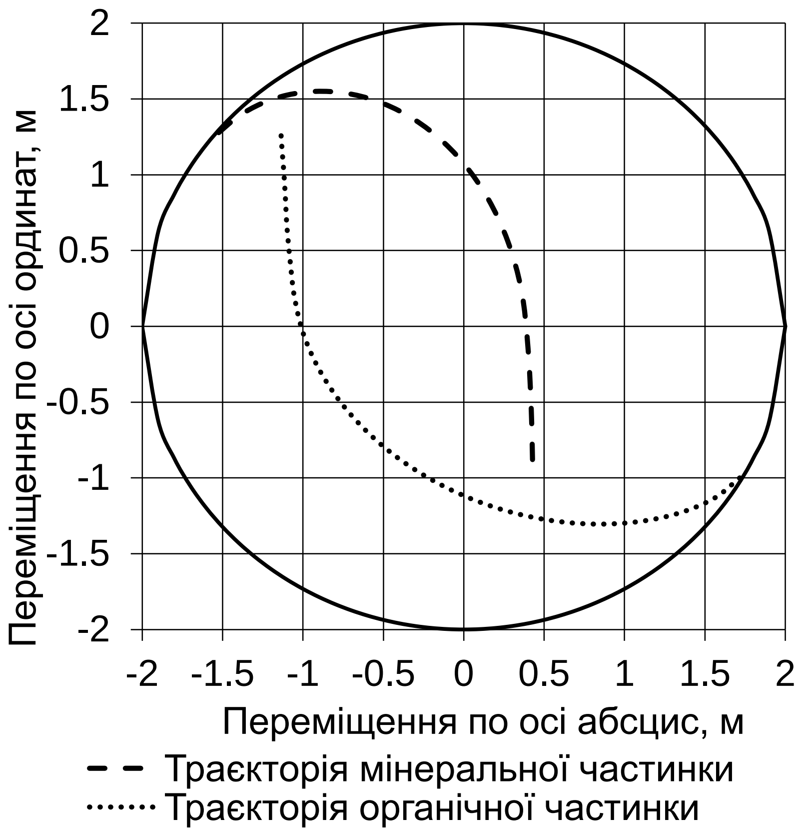


Fig. 3. Example of calculating the trajectory of particles substrate (digesters angular velocity ω = 0,08 rad / s, digesters inner radius R = 2 m).

Conclusion. The mathematical model allows you to setrational values of angular velocity rotary digesters and its design parameters by determining motion parameters of biomass particles on the blades and the volume digesters. Also established that pivnomirne mixing and interpenetration Organic and mineral components the substrate is provided for rational values of angular velocity digesters from 0.035 to 0.08 s-1 and the length of the blades digesters from 77.5 to 82.5% of the inner radius.

List of references

1. *Guidance* on biogas. That to obtaining Using / vozobnovlyaemыh agency, special resources (FNR). 5th edition. - Hyultsov: Germany, 2012. - 213 p.

2. *Golub, GA* Technical support biogas / GA Holub, OV Dubrovin, BA Ruban, VA Voitenko // Scientific works of Vinnytsia National Agrarian University. Series: Engineering. - Ball, 2012. - Vol. 10. - P. 17-19.

3. *G. Dove* Modern trends in biogas plants / G. Holub, V. Voitenko, B. Ruban, V. Ermolenko // Engineering and Technology APC. - 2012. - № 2 (29). - P. 18-21.

4. *Golub, GA* Justification of immersion fill factor and biomass digesters rotating / GA Holub, OV Dubrovin // Scientific Bulletin of National University of Life and Environmental Sciences of Ukraine. Series: Power equipment and agribusiness. - 2012. - Vol. 170, ch. 2. - P. 55-61.

5. *SM Kuharets* Rationale energy expenditure to drive the rotary reactor biogas plant / SM Kuharets, GA Dove // Technical and technological aspects of the development and testing of new techniques and technologies for agriculture Ukraine / DNU UkrNDIPVT them. L. Pogorelogo. - 2014 - Vol. 18 (32), Vol. 2. - P. 356-365.
6. *G. Dove* Design features a modular biogas plant with a rotating reactor / H. Golub, S. Kuharets, B. Ruban // Engineering and Technology APC. - 2014. - № 9 (60). - P. 10-14.
7. *Technology* processing biological waste in biogas plants with rotating reactor / [GA Holub, OV Sydorchuk SM Kuharets etc .; Ed. HA. Blue]. - K .: NUBiP Ukraine, 2014. - 106 p.
8. *SM Kuharets* Raw materials and efficiency of biogas / SM Kuharets, GA Dove // Scientific Bulletin of National University of Life and Environmental Sciences of Ukraine. Series: Power equipment and agribusiness. - 2012. - Vol. 212, p. 1. - P. 11-21.
9. *Bronstein I. N.* Handbook on mathematics for engineers and uchaschyhsya polytechnics / I. N. Bronstein, KA Semendyayev. - 13th ed., Corr. - M .: Nauka, 1986. - 544 p.
10. *Golub, GA* Agro-industrial production of edible mushrooms. Mechanical and technological bases / GA Dove. - K .: Agricultural Science, 2007. - 332 p.

Matematycheskaya model is designed for motion parameters for determining particles byomassы t vraschayuschemsya digesters, kotoraja allows us to establish ratsyonalnыe value uhlovoy Speed digesters and ego konstruktyvnыe parameters.

Byomassa, digesters, biogas, peremeshyvanyya, particle motion.

The mathematical model to determine the parameters of particle motion in a rotating biomass digesters, which allows establishing rational values of angular velocity digesters and its design parameters is developed.

Biomass, digesters, biogas, mixing, motion of particle.