O. Dyuzhenkova. Structural characteristic of classes of functions defined by D-moduli of smoothness. (2014)

In this article introduced the analog of modules of smoothness Ditziana-Totika - D-module of smoothness for functions on continuous piecewise smooth curves of the complex plane, investigated the properties of D-module. Considered the structural characteristic of functions classes in terms of smoothness -module.

In approximation theory is widely used -modules smoothness (moduli of smoothness of Ditzian-Totik) for functions continuous on the interval. First structural characteristic of uniform approximation of functions on sets of the complex plane with piecewise smooth boundary was obtained in the work of V.Dzyadyk, G.Alibekov, Y.Volkov and others. It was seen in terms of the of smoothness modules of the function $\tilde{f}(\omega) = f(\psi(\omega))$, where $\psi(\omega)$ - conformal mapping of the exterior of the circle on the exterior of the considered set.

The purpose of research - distribution theory and smoothness of Z.Ditzian and V.Totik for the sets of the complex plane with piecewise smooth boundary, the study of the properties of analog D-module of smoothness in domains with corners and building structural characteristics in terms of smoothness introduced -module function.

The paper used methods of uniform approximation of functions, in particular interpolation functions by algebraic polynomials and Lagrange polynomials.

Let M - closed bounded set of complex plane with limit Γ which is Jordan curves and consists of a finite number of smooth arcs Γ_j, which forms outer corners $\alpha_j \pi,\ 0 < \alpha_j \le 2$ in the points of junction a_j. The class of such sets is denoted by $\mathbf{(C)}$. Denote $\overline{\omega}_k\left(t, f, \Gamma\right)$ of D-module of smoothness.

The main result is the direct and inverse theorems of uniform approximation of functions on the sets $M \subset \mathbf{(C)}$. Because of these theorems built constructive characterization of classes of functions defined by D-module of smoothness.

A continuous and nondecreasing on $[0;\infty)$ function φ is called majorant, when $\varphi(0)=0$. Class of majorants denoted by Φ. Let $\underline{\alpha}$ - the smallest, and $\overline{\alpha}$ - most of numbers $\alpha_1, \alpha_2, ..., \alpha_k$.

Theorem 1. Let $k \in N$, $\varphi \in \Phi$. If for a given function f on the set M for every natural $n \geq k-1$ follows $E_n(f) \leq \varphi\left(\frac{1}{n}\right)$, then it is evaluation

$$\overline{\omega}_k(\tau, f) \leq c\tau^{\underline{\alpha}k} \int_{\tau}^{diamM} \frac{\varphi(u)}{u^{\underline{\alpha}k+1}} du, \qquad 0 < \tau < \frac{1}{2} diamM.$$

Denote $A(M)$ the class of functions analytic in the region $M \setminus \Gamma$ and continuous on the set M.

Theorem 2. Let $k \in N$, $\varphi \in \Phi^{\overline{\alpha}k}$. If the function $f \in A(M)$ and $\overline{\omega}_k(\tau, f, \Gamma) \leq \varphi(\tau)$, then for each $n \geq k-1$ follows inequality $E_n(f) \leq c\varphi\left(\frac{1}{n}\right)$.

The consequence of direct Theorem 2 and inverse Theorem 1 of uniform approximation of functions in domains with corners is constructive characteristic of classes of functions in terms of D-module smoothness.