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A THREE-DIMENSIONAL MATHEMATICAL MODEL OF LOW CAPACITY TO ELECTROMAGNETIC SYSTEM OF SERIES-CONNECTED INDUCTORS, ELECTRODES AND NONFERROMAGNETIC PLATE

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Developed a three-dimensional mathematical model for calculating the current density in massive spreading nonferromagnetic elements electromagnetic system to reduce or determination of residual stresses in the prototype.

The electromagnetic field, integral equations, current spreading, electrode system.

Let capacity C, charged to voltage U_0, closes the system of series-connected inductors L and N massive conductors occupying volume $D=\bigcup_{q=1}^{N} D_q$, limited smooth surface $S=\bigcup_{q=1}^{N} S_q$ (Fig. 1).Conductivity materials carry a constant volume of each wire and flat respectively $\gamma_1, \gamma_2, \ldots, \gamma_N$.

We assume that in terms of quasi-stationary currents flow [1], ie currents at which displacement currents in the dielectric surrounding the conductors can be neglected.

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The aim – for a given system geometry, electrical characteristics of the materials making up the structural elements, electrical connection elements given voltage across the capacitor current is found $i\ t$ in a circle discharging capacitor current density distribution $\delta\ Q,t$ a massive conductors electromagnetic force $F\ t$, acting on the disk electrode etc. In general, this requires the solution of three-dimensional boundary value problems for a system of Maxwell's equations in an unbounded domain:

$$rot\vec{H}=\vec{\delta};\ rot\vec{E}=-\frac{\partial\vec{B}}{\partial t};\ div\vec{B}=0;\ div\vec{\delta}=0;\ \vec{B}=\mu\vec{H};\ \vec{\delta}=\gamma\vec{E}.\quad (1)$$

Here $\vec{E}$ – vector of the electric field, V/m; $\vec{H}$ – vector magnetic field A/m; $\vec{B}$ – magnetic induction vector, T; $\vec{\delta}$ – vector current density, A / m^3 (out massive conductors must assume that $\vec{\delta}=0$); γ – conductivity,, $\mathrm{Cm/m}$; μ – absolute permeability environment, H / m; t – time seconds.

Material and methods research. The system of equations (1), supplemented with boundary and initial conditions, formulates initial boundary value problem, the solution is reduced electromagnetic modeling process in the system. For unambiguous determination of the field to the differential equation (1) should be added to the following additional conditions [2]:

а) continuity of the normal component of the current density on the surface of the massive conductors (arising from neglect displacement currents outside and inside massive conductors);

б) continuity of the normal component of induction and tangential component of the magnetic field on the surface of the massive conductors S;

в) know the initial distribution of current density $\vec{\delta}^{\,0}\ M$ for massive conductors;

г) induction $\vec{B}\ M,t$ tends to zero $M\to\infty$ як $1/r^2$.

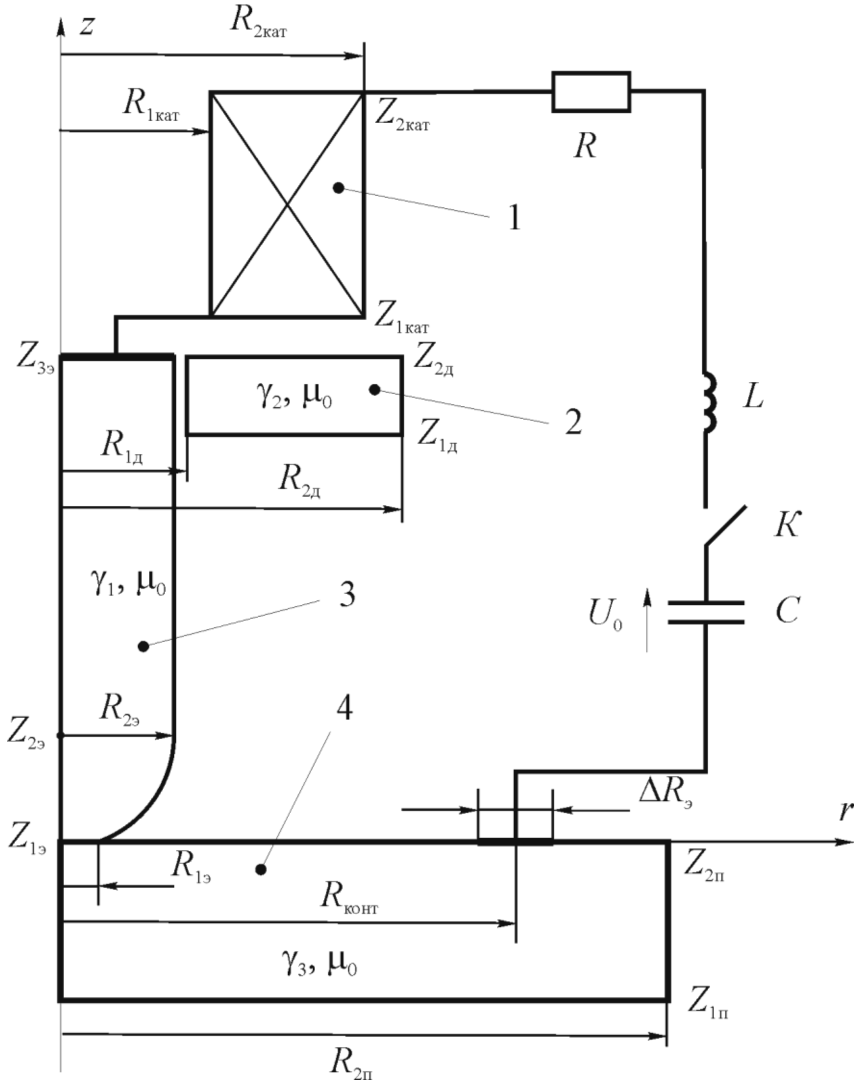

Fig.1. Meridian section of the electrode system with the image of an external electric circuit:

1 - inductor; 2 - aluminum disk; 3 - electrode; 4 - nonferromagnetic plate

Results. Insertion electromagnetic potentials $\vec{A}$ and φ simplify the original system of equations (1). Given that $div\vec{B}=0$, define the vector potential:

$$\vec{B}=rot\vec{A}; \quad (2)$$

$$div\vec{A}=0. \quad (3)$$

Substituting (2) into the second equation (1):

$$rot\vec{E}=-\frac{\partial}{\partial t}rot\vec{A} \text{ or } rot\left(\vec{E}+\frac{\partial\vec{A}}{\partial t}\right)=0.$$

The last ratio implies that $\vec{E}+\frac{\partial\vec{A}}{\partial t}$ – potential field [2] and for him there is a scalar field φ, that

$$\vec{E}+\frac{\partial\vec{A}}{\partial t}=-grad\varphi. \qquad (4)$$

From the relation (4) we obtain the following equation

$$\vec{E}=-\frac{\partial\vec{A}}{\partial t}-grad\varphi. \qquad (5)$$

From the first equation of (1) we find

$$rot\vec{H}=rot\left(\mu^{-1}\vec{B}\right)=rot\left(\mu^{-1}rot\vec{A}\right)=\vec{\delta},$$

that is,

$$rot\left(\mu^{-1}rot\vec{A}\right)=\vec{\delta}. \qquad (6)$$

Further, considering the last equation (1) we arrive at the equation

$$\vec{\delta}=-\gamma\frac{\partial\vec{A}}{\partial t}-\gamma grad\varphi. \qquad (7)$$

Equation (6), (7) form a system of equations is equivalent to the entire system of Maxwell's equations. Equivalence is understood in the following sense: if the field $\vec{A}$, φ, $\vec{\delta}$ satisfy the equation (6), (7), the magnetic induction $\vec{B}$, magnetic field $\vec{H}$ and the electric field $\vec{E}$ defined as follows:

$$\vec{B}=rot\vec{A};\ \vec{H}=\mu^{-1}\vec{B};\ \vec{E}=\gamma^{-1}\vec{\delta}$$

and in their substitution in Maxwell's equations they satisfy it identically.

Derive equations to be met potentials $\vec{A}$ and φ. Take into account that the medium is linear, isotropic, without hysteresis. Because of the assumptions made, from equation (6) we find:

$$rotrot\vec{A}=\mu\vec{\delta}. \qquad (8)$$

According to the ratio of vector analysis

$$rotrot\vec{a}=grad\left(div\vec{a}\right)-\Delta\vec{a}, \qquad (9)$$

record

$$-\Delta\vec{A}+grad\ \ div\vec{A}=\mu\vec{\delta}.$$

Given (3), we obtain the following equation to determine the vector potential

$$\Delta\vec{A}=-\mu\vec{\delta}. \qquad (10)$$

To determine the scalar potential with (5) we find

$$div\vec{E}=-\frac{\partial}{\partial t}div\vec{A}-div\ \ grad\varphi\ . \qquad (11)$$

Taking into account that

$$div\vec{E}=div\left(\frac{\vec{\delta}}{\gamma}\right)=\frac{1}{\gamma}div\vec{\delta}=0\ \text{i}\ div\vec{A}=0,$$

get

$$div\ \ grad\varphi=0.$$

So for the scalar electric potential we obtain the following equation:

$$\Delta\varphi=0. \qquad (12)$$

Formulate the boundary value problem for the calculation of the magnetic field in the electromagnetic system (Fig. 2):

$$\Delta\vec{A}=-\mu_0\vec{\delta}_W,\ Q\in D_W, \qquad (13)$$

$$\Delta\vec{A}=-\mu_0\vec{\delta}_q,\ Q\in D_q,\ q=1,2,...,N, \qquad (14)$$

$$\Delta\vec{A}=0,\ Q\in D_0, \qquad (15)$$

where $\vec{\delta}_W$ – current density in the winding reel D_W, A/m^3; $\vec{\delta}_q$ – current density in the solid conductor D_q, A/m ^{3}; $q=1,2,...,N$ (the problem in $N=3$); D_0 – all bodies external to the electromagnetic system space.

Boundary conditions for the vector potential on the boundary S_{qm}, $q=1,2,...,N-1$, massive bodies are as follows:

$$\left[\vec{n}_Q,\vec{A}^-\right]=\left[\vec{n}_Q,\vec{A}^+\right],\left[\vec{n}_Q,rot\vec{A}^-\right]=\left[\vec{n}_Q,rot\vec{A}^+\right],\ Q\in S\cup S_W, \qquad (16)$$

where $\vec{A}^+$, $\vec{A}^-$ – thresholds vector potential at the point Q when approaching it in accordance with internal and external parties massive conductor; $\vec{n}_Q$ – outdoor normal to the boundary q-th massive conductor; S – border massive conductors, $S = S_{20} \cup S_{12} \cup S_{10} \cup S_{13} \cup S_{30}$ (Fig. 2).

We write the boundary conditions for scalar electric potential:

$$\varphi^{+} = \varphi^{-} \text{ на } S, \tag{17}$$

where φ^{+}, φ^{-} – value of the scalar electric potential at point $Q \in S$, when approaching it in accordance with internal and external parties massive conductor D_q, $q = 1, 2, ...N$.

Then take into account the continuity of the normal component of the current density $\vec{\delta}$ on the border of massive bodies:

$$\vec{\delta}^{+}, \vec{n}_Q = \vec{\delta}^{-}, \vec{n}_Q \quad \text{на } S,$$

That is,

$$-\gamma^{+} \frac{\partial A^{+}_{n_Q}}{\partial t} - \gamma^{+} \frac{\partial \varphi^{+}}{\partial n_Q} = -\gamma^{-} \frac{\partial A^{-}_{n_Q}}{\partial t} - \gamma^{-} \frac{\partial \varphi^{-}}{\partial n_Q}, Q \in S, \tag{18}$$

where $A^{+}_{n_Q}$ ($A^{-}_{n_Q}$) – instantaneous projection vector potential on the outer normal $\vec{n}_Q$ at the point $Q \in S$, when approaching it from the inner (outer) side of a massive conductor D_q, $q = 1, 2, ..., N-1$; $\partial \varphi^{+} / \partial n_Q$ $\partial \varphi^{-} / \partial n_Q$ – normal derivative of the scalar electric potential when approaching a point $Q \in S$ from the inner (outer) side of a massive conductor D_q, $q = 1, 2, ..., N$; $\vec{n}_Q$ – outer normal to the boundary massive conductor D_q, $q = 1, 2, ..., N$; γ^{+}, γ^{-} – conductivity material internal and external border areas of concern S, Cm/m.

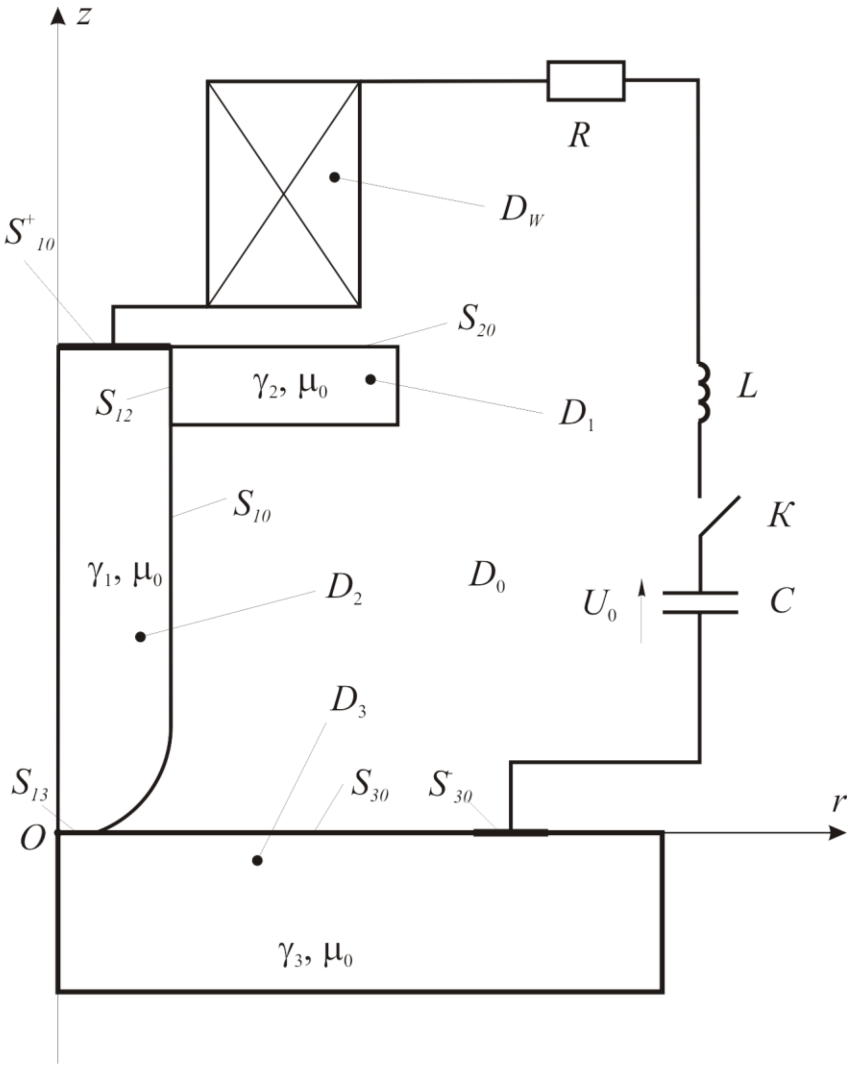

Fig.2. electrode system

Thus, we can formulate the boundary value problem for determining the magnetic vector potential and scalar electric potential:

$$\Delta\vec{A}=-\mu_0\vec{\delta}_W\,,Q\in D_W\,; \tag{19}$$

$$\Delta\vec{A}=-\mu_0\vec{\delta}_q\,,Q\in D_q\,,q=1,2,...,N\,; \tag{20}$$

$$\Delta\vec{A}=0\,,Q\in D_0\,; \tag{21}$$

$$\left[\vec{n}_Q,rot\vec{A}^-\right]=\left[\vec{n}_Q,rot\vec{A}^+\right],Q\in S\,; \tag{22}$$

$$\left[\vec{n}_Q,\vec{A}^-\right]=\left[\vec{n}_Q,\vec{A}^+\right],Q\in S\,; \tag{23}$$

$$\Delta\varphi_q=0\,,Q\in D_q\,,q=1,2,3,...,N\,; \tag{24}$$

$$\varphi^+=\varphi^-\ \text{на}\,S\,; \tag{25}$$

$$-\gamma_1 \frac{\partial A_{n_Q}^{+}}{\partial t} - \gamma_1 \frac{\partial \varphi^{+}}{\partial n_Q} = \delta_{n_Q}^{+} \text{ на } S_{10}^{+}; \tag{26}$$

$$-\gamma_N \frac{\partial A_{n_Q}^{+}}{\partial t} - \gamma_N \frac{\partial \varphi^{+}}{\partial n_Q} = \delta_{n_Q}^{-} \text{ на } S_{N0}^{-}; \tag{27}$$

$$-\gamma^{+} \frac{\partial A_{n_Q}}{\partial t} - \gamma^{+} \frac{\partial \varphi^{+}}{\partial n_Q} = -\gamma^{-} \frac{\partial A_{n_Q}}{\partial t} - \gamma^{-} \frac{\partial \varphi^{-}}{\partial n_Q}, Q \in S; \tag{28}$$

$$A(\infty) = 0; \tag{29}$$

where S_{10}^{+}, S_{N0}^{-} – of the boundary S_{10}, S_{N0}, which is set to the normal component of the current density $\delta_{n_Q}^{+}$ й $\delta_{n_Q}^{-}$ (contact in the electromagnetic system, which joins the chain consisting of tanks, reels and active resistance). If you know the current $i_W(t)$ in the winding reel D_W, then plug the current density in the contact S_{10}^{+}, S_{N0}^{-} is as follows:

$$\delta_{n_Q}^{+}(t) = \frac{i_W(t)}{S_{10}^{+}}, \delta_{n_Q}^{-}(t) = \frac{i_W(t)}{S_{N0}^{-}}. \tag{30}$$

Looking vector potential in the form:

$$\vec{A}(Q,t) = \frac{\mu_0}{4\pi} \int_{D_W} \frac{\vec{\delta}_W(M,t)}{r_{QM}} dV_M + \frac{\mu_0}{4\pi} \int_{D} \frac{\vec{\delta}_m(M,t)}{r_{QM}} dV_M, \tag{31}$$

satisfying equation (19) - (21) and the boundary conditions (22), (23). Here $\vec{\delta}_W(M,t)$ – instantaneous current density in the coil D_W, A/m^3; $\vec{\delta}_m(M,t)$ – instantaneous current density in a massive conductor D_m, $m = 1,2,...,N$, A/m^3; r_{QM} – distance from the point Q to the point M, м; $\mu_0 = 4\pi \cdot 10^{-7}$, H/m.

Substituting equation (7) expression vector magnetic potential (31), we obtain the integral-differential equation (integral for spatial variables, differential time) for the density of eddy currents in massive conductors:

$$\frac{\vec{\delta}_q(Q,t)}{\gamma_q \lambda} + \frac{\partial}{\partial t} \int_{D} \frac{\vec{\delta}(M,t)}{r_{QM}} dV_M + \frac{1}{\lambda} grad\varphi(Q,t) = -\frac{\partial}{\partial t} \int_{D_W} \frac{\vec{\delta}_W(M,t)}{r_{QM}} dV_M;$$

$$Q \in D_q, q = 1,2,...,N, \tag{32}$$

where $\lambda=\mu_0/4\pi$.

Magnetic induction vector is given by [2]:

$$\vec{B}(Q,t)=\frac{\mu_0}{4\pi}\int_{D_W}\frac{\left[\vec{r}_{QM},\vec{\delta}_W(M,t)\right]}{r_{QM}^3}dV_M+\frac{\mu_0}{4\pi}\int_{D}\frac{\left[\vec{r}_{QM},\vec{\delta}(M,t)\right]}{r_{QM}^3}dV_M\,. \tag{33}$$

To determine the $grad\varphi_q$ put internal problem (24) – (28). Solution of equation (24) are looking at a potential of a simple layer of electrical charges [3]:

$$\varphi(Q,t)=\frac{1}{4\pi\varepsilon_0}\int_S\frac{\sigma(M,t)}{r_{QM}}dS_M\,, \tag{34}$$

where $\varphi(Q,t)$ – instantaneous scalar electric potential at point Q; $\sigma(M,t)$ – instantaneous density simple layer of electrical charges at the point M border S.

Boundary conditions (25) are performed automatically. To satisfy the expression (34) boundary condition (28), use the theorem jump normal derivative of the simple layer potential [3]:

$$\frac{\partial\varphi^-}{\partial n_Q}=-\frac{\sigma(Q,t)}{2\varepsilon_0}+\frac{1}{4\pi\varepsilon_0}\int_S\sigma(M,t)\frac{(\vec{r}_{QM},\vec{n}_Q)}{r_{QM}^3}dS_M\,, \tag{35}$$

$$\frac{\partial\varphi^+}{\partial n_Q}=\frac{\sigma(Q,t)}{2\varepsilon_0}+\frac{1}{4\pi\varepsilon_0}\int_S\sigma(M,t)\frac{(\vec{r}_{QM},\vec{n}_Q)}{r_{QM}^3}dS_M\,. \tag{36}$$

Thus, we obtain a system of integral equations

$$\sigma(Q,t)+\frac{\gamma^+-\gamma^-}{\gamma^++\gamma^-}\frac{1}{2\pi}\int_S\sigma(M,t)\frac{(\vec{r}_{QM},\vec{n}_Q)}{r_{QM}^3}dS_M=-2\varepsilon_0\frac{\gamma^+-\gamma^-}{\gamma^++\gamma^-}\frac{\partial A_{n_Q}}{\partial t},\ Q\in S\,. \tag{37}$$

Substituting equation (37) the expression for the vector potential (31), we finally obtain

$$\frac{\chi(Q)\mu_0\varepsilon_0}{2\pi}\frac{\partial}{\partial t}\int_D\frac{(\vec{\delta}(M,t),\vec{n}_Q)}{r_{QM}}dV_M+\sigma(Q,t)+\frac{\chi(Q)}{2\pi}\int_S\sigma(M,t)\frac{(\vec{r}_{QM},\vec{n}_Q)}{r_{QM}^3}dS_M=$$

$$=-\frac{\chi(Q)\mu_0\varepsilon_0}{2\pi}\frac{\partial}{\partial t}\int_{D_W}\frac{(\vec{\delta}_W(M,t),\vec{n}_Q)}{r_{QM}}dV_M\,,Q\in S\,, \tag{38}$$

де $\chi(Q) = \left(\gamma^{+}(Q) - \gamma^{-}(Q)\right) / \left(\gamma^{+}(Q) + \gamma^{-}(Q)\right)$, $Q \in S$; $\gamma^{+}(Q)$, $\gamma^{-}(Q)$ – value of conductivity of the material from the inside and the outside of the border S at the point $Q \in S$ (normal $\vec{n}_Q$ directed from the inner to the outer region).

Satisfying expression (34) boundary conditions (26), (27), complementary equation (38) the following equations:

$$\frac{\mu_0\varepsilon_0}{2\pi}\frac{\partial}{\partial t}\int_D \frac{\left(\vec{\delta}(M,t),\vec{n}_Q\right)}{r_{QM}}dV_M + \sigma(Q,t) + \frac{1}{2\pi}\int_S \sigma(M,t)\frac{\left(\vec{r}_{QM},\vec{n}_Q\right)}{r_{QM}^3}dS_M =$$

$$= -\frac{\mu_0\varepsilon_0}{2\pi}\frac{\partial}{\partial t}\int_{D_W} \frac{\left(\vec{\delta}_W(M,t),\vec{n}_Q\right)}{r_{QM}}dV_M - \frac{2\varepsilon_0}{\gamma_1}\delta_{n_Q}^{+}(t),\ Q \in S_{10}^{+}; \tag{39}$$

$$\frac{\mu_0\varepsilon_0}{2\pi}\frac{\partial}{\partial t}\int_D \frac{\left(\vec{\delta}(M,t),\vec{n}_Q\right)}{r_{QM}}dV_M + \sigma(Q,t) + \frac{1}{2\pi}\int_S \sigma(M,t)\frac{\left(\vec{r}_{QM},\vec{n}_Q\right)}{r_{QM}^3}dS_M =$$

$$= -\frac{\mu_0\varepsilon_0}{2\pi}\frac{\partial}{\partial t}\int_{D_W} \frac{\left(\vec{\delta}_W(M,t),\vec{n}_Q\right)}{r_{QM}}dV_M - \frac{2\varepsilon_0}{\gamma_N}\delta_{n_Q}^{+}(t),\ Q \in S_{N0}^{-}. \tag{40}$$

Taking into account that

$$grad\varphi = -\frac{1}{4\pi\varepsilon_0}\int_S \sigma(M,t)\frac{\vec{r}_{QM}}{r_{QM}^3}dS_M,$$

write the equation (32)

as: $$\frac{\vec{\delta}_q(Q,t)}{\gamma_q\lambda} + \frac{\partial}{\partial t}\int_D \frac{\vec{\delta}(M,t)}{r_{QM}}dV_M - \frac{1}{\mu_0\varepsilon_0}\int_S \sigma(M,t)\frac{\vec{r}_{QM}}{r_{QM}^3}dS_M = -\frac{\partial}{\partial t}\int_{D_W}\frac{\vec{\delta}_W(M,t)}{r_{QM}}dV_M;$$

$$Q \in D_q,\ q = 1,2,\ldots,N. \tag{41}$$

Thus, the system of integro-differential equations for the density of eddy current density and simple layer of electric charges (in the considered electrode system (Fig. 2) $N = 3$).

$$\frac{\vec{\delta}_q(Q,t)}{\gamma_q\lambda} + \frac{\partial}{\partial t}\int_D \frac{\vec{\delta}(M,t)}{r_{QM}}dV_M - \frac{1}{\varepsilon_0\mu_0}\int_S \sigma(M,t)\frac{\vec{r}_{QM}}{r_{QM}^3}dS_M = -\frac{\partial}{\partial t}\int_{D_W}\frac{\vec{\delta}_W(M,t)}{r_{QM}}dV_M,$$

$$Q \in D_q,\ q = 1,2,\ldots,N; \tag{42}$$

$$\frac{\chi(Q)\,\mu_0\varepsilon_0}{2\pi}\frac{\partial}{\partial t}\int_D \frac{\vec{\delta}(M,t),\vec{n}_Q}{r_{QM}}dV_M+\sigma(Q,t)+\frac{\chi(Q)}{2\pi}\int_S \sigma(M,t)\frac{(\vec{r}_{QM},\vec{n}_Q)}{r_{QM}^3}dS_M=$$

$$=-\frac{\chi(Q)\,\mu_0\varepsilon_0}{2\pi}\frac{\partial}{\partial t}\int_{D_W} \frac{\vec{\delta}_W(M,t),\vec{n}_Q}{r_{QM}}dV_M-F(Q,t),\ Q\in S\cup S_{10}^{+}\cup S_{30}^{-}, \qquad (43)$$

where

$$\chi(Q)=\begin{cases}\dfrac{\gamma^{+}(Q)-\gamma^{-}(Q)}{\gamma^{+}(Q)+\gamma^{-}(Q)}, & \text{если} \quad Q\in S;\\ 1, & \text{если} \quad Q\in S_1^{+}\cup S_3^{-};\end{cases} \qquad (44)$$

$$F(Q,t)=\begin{cases}0, & \text{если} \quad Q\in S;\\ \dfrac{2\varepsilon_0}{\gamma_1}\delta_{n_Q}^{+}(t), & \text{если} \quad Q\in S_{10}^{+};\\ \dfrac{2\varepsilon_0}{\gamma_3}\delta_{n_Q}^{-}(t), & \text{если} \quad Q\in S_{30}^{-}.\end{cases} \qquad (45)$$

Solve the system of equations (42) - (45) we find the current density in each massive conductor through which it is possible to calculate the density of heat sources in massive conductors electromagnetic force acting on the disk electrode, and so on.

Conclusions

Developed a three-dimensional mathematical model to calculate the current spreading in nonferromagnetic plate (prototype) while passing through her current pulse allows to study the effect of electromagnetic parameters of the system to heat, distribution electrodynamic forces in a massive plate close contact with the electrode.

References

1. Tozoni O.V. calculation of transients in bus / O.V. Tozoni, E.I. Petrushenka // Theoretical Electrical Engineering. – Lions: Lions. University Press, 1966. – Vol. 2. – P. 144 – 147.

2. Beans K. Analysis and calculation of electric and magnetic fields / K. Binns, P. Laurenson // – M .: Energy, 1970. – 376 p.

3. Tozoni O.V. Calculation of three-dimensional electromagnetic fields / O.V. Tozoni, I.D. Maergoiz // – K .: Tehnika, 1974. – 352 p.

Разработано трехмерную математическую модель для расчета плотности токов растекания в массивных неферромагнитных элементах электромагнитной системы для уменьшения или определения остаточных напряжений в опытном образце.

Электромагнитное поле, интегральные уравнения, токи растекания, электродная система.

Developed a three-dimensional mathematical model for the calculation of the current densities in the massive spreading of non-ferromagnetic elements of the electromagnetic system to reduce or determination of residual stresses in the prototype.

The electromagnetic field, integral equations, spreading currents, the electrode system.