O. Dyuzhenkova. Moduli of smoothness on the set of the complex plane with piecewise smooth boundary. (2013)

In the article we extend the definition of Ditzian-Totik modules of smoothness for the functions, continuous on piecewise smooth curves of complex plane, and investigate the properties of these modules.

The purpose of research – the analysis of theory and smoothness of Z.Ditzian and V.Totik for its extension to the set of the complex plane with piecewise smooth boundary, input of analog -DT-module of smoothness $\overline{\omega}_k(f,t)$ in regions with corners and study its properties to further build constructive characteristics in terms of introduced D-module of smoothness of function f.

Let M - closed bounded set of complex plane with limit Γ which is Jordan curves and consists of a finite number of smooth arcs Γ_j, which forms outer corners $\alpha_j\pi,\ 0<\alpha_j\le 2$ in the points of junction a_j. For all $z\in\Gamma$ and $h>0$ denote $\rho_h(z):=h(|z-a_{j_*}|^{\frac{1}{\alpha_{j_*}}}+h)^{\alpha_{j_*}}$, where j_* - the index of the closest to the point of junction. Let $L(z,f):=L(z,f;z_1,...,z_k)$ - the Lagrange polynomial of degree$\le k-1$, that interpolates the function f in different points $z_i\in\mathbf{C}$, $i=\overline{1,k}$.

Definition. D-module of smoothness in order k on the curve Γ of continuous function called the function

$$\overline{\omega}_k(\tau,f,\Gamma):=\sup_{h\in[0;\tau]}\ \sup_{\tilde{z}\in\Gamma}\ \sup_{\{z_0,z_1,...,z_k\}}\left|f(z_0)-L(z_0,f;z_1,...,z_k)\right|,$$

where the inner supremum is taken over all sets of points $\{z_0,z_1,...,z_k\}$, which satisfy the inequality $|z_i-\tilde{z}|\le\rho_h(\tilde{z})\le(3k+1)|z_i-z_j|,\quad i,j=\overline{0,k},i\ne j$.

The main result is a theorem, which proved that D-module as classic module of smoothness has the property of normality.

Theorem. If the function f is continuous on curve Γ, then for all $n \in N, \tau \geq 0$ follows the inequality $\overline{\omega}_k(n\tau, f, \Gamma) \leq cn^{\overline{\alpha}k}\overline{\omega}_k(\tau, f, \Gamma)$, where $\overline{\alpha}$ - the biggest among the numbers α_j.

To prove this theorem we use the geometric lemmas and Lagrange polynomials. In particular consider the interesting lemma.

Lemma. Let M - connected set, $z \in M$ and points $z_0, z_1, ..., z_k$ are in $M \cap U[z,r]$. If $M \cap v[z,r] \neq \emptyset$, then exist is k points $z_1^{'}, ..., z_k^{'}$, for which follows $z_i^{'} \in M \cap U[z,r], \quad i = \overline{1,k}$; $\quad r \leq (3k+1)\left|z_i^{'} - z_j^{'}\right|, \quad i, j = \overline{1,k}, i \neq j$; $r \leq (3k+1)\left|z_i - z_j^{'}\right|, \quad i = \overline{0,k}, \ j = \overline{1,k}$.

Take a point $\tilde{z} \in \Gamma$, numbers $h > 0, n \in \mathbf{N}$ and consider a set of points $\{z_0, z_1, ..., z_k\}$, that satisfy the condition $|z_i - \tilde{z}| \leq \rho_h(\tilde{z}) \leq (3k+1)|z_i - z_j|$. Denote c different constants, that can depend on k and Γ. For the proof of the theorem is sufficient to prove the inequality $|f(z_0) - L(z_0, f; z_1, ..., z_k)| \leq cn^{\overline{\alpha}k}\, \overline{\omega}_k(ch, f, \Gamma)$.

In the paper also considered the others properties of introduced D-module of smoothness.