TERMS OF SOLUTIONS OF *WEAKLY PERTURBEDLINEAR* BOUNDARY PROBLEMS (IF $k=-2$)

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It is proposed and proved a theorem to obtain sufficient conditions for the existence of solutions of weakly perturbedlinear inhomogeneous boundary value problem in the case where the condition $P_{B_0}=0,\;\; P_{B_0^*}P_{Q_d^*}=0$ is not fulfilled.

Weakly perturbedlinear inhomogeneous boundary value problem, homogeneous boundary value problem with impulse action, orthogonal projector, series, psevdouniverse.

The relevance of this topic is due, above all, the importance of the practical application of the theory of boundary value problems in the theory of nonlinear oscillations, the theory of stability of motion, control theory, a number of geophysical problems. On the other hand, the article received significant new findings complement research on the theory of nonlinear oscillations for slabozburenyh boundary problems.

The aim – to find sufficient conditions for the existence of solutions of linear nonhomogeneous impulsive boundary value problems with small perturbations when generating boundary value problem of impulsive has solutions for arbitrary right–hand side.

Materials and methods research. In the study of solutions of the problem used methods of perturbation theory developed in the writings A. Lyapunov and his followers, asymptotic methods of nonlinear mechanics, developed in the writings M. Krylov, M. Bogolyubov, Y. Mytropolsky, A. Samoilenko.

Results. We introduce the following notation: $Q=lX\,\cdot\, -(m\times n)$ – dimensional matrix; $Q^*=Q^T\;\; -(n\times m)$ – dimensional matrix; $P_{Q^*}-(m\times n)$ – dimensional matrix (orthoprojector), that projects $\mathbb{R}^m$ на $N(Q^*)$, $P_{Q^*}\colon \mathbb{R}^m\to N(Q^*)$; $P_{Q_d^*}-(d\times m)$ –dimensional matrix, which string is a complete system d linearly independent rows of the matrix P_{Q^*}; Q^+ – uniqe psevdouniverse to $Q\;\, n\times m$ –

dimensional matrix; P_{B_0} – $r \times r$ – to dimensional matrix (orthoprojector), that projects R^r to null space $N\ B$ $d \times r$ – dimensional matrix $B_0, P_{B_0}: R^r \to N\ B_0$; $P_{B_0^*}$ – $(d \times d)$ – dimensional matrix (orthoprojector), that projects R^d to null space $N(B_0^*)$ $(r \times d)$– dimensional matrix $B_0^* = B_0^T$, $P_{B_0^*}: \mathbb{R}^d \to N\ B_0^*$.

If $P_{B_0} = 0$, $P_{B_0^*} P_{Q_d^*} = 0$ false, then to obtain sufficient conditions for the existence of solutions of the boundary value problem

$$\begin{aligned} z &= A\ t\ z + \varepsilon A_1\ t\ z + f\ t\ ,\ t \neq \tau_i; \\ \Delta z_{|t=\tau_i} - S_i z &= a_i + \varepsilon A_{1i} z\ \tau_i - 0\ ; \\ lz &= \propto + \varepsilon l_1 z \end{aligned} \quad (1)$$

with random inhomogeneities $f\ t\ \in C\ \ a,b \setminus \tau_i\ {}_I$, $a_i \in \mathbb{R}^n$ and $\alpha \in R^m$we have a theorem which generalizes the corresponding result for boundary value problems with impulses from [1]

Theorem. Let $rank\ Q = n_1 < n$ and

$$P_{Q_d^*} \propto - l \int_a^b K\ \cdot,\tau\ f(\tau) d\tau - l \sum_{i=1}^{p} K\ \cdot,\tau_i\ a_i\ = 0, d = m - n_1$$

false. In other words, suppose that the generating boundary value problem that results from (1) with $\varepsilon = 0$:

$$\begin{aligned} z &= A\ t\ z + f\ t\ ,\ t \neq \tau_i; \\ \Delta z_{|t=\tau_i} - S_i z &= \alpha_i; \\ lz &= \propto \end{aligned} \quad (2)$$

hasn't solutions for arbitrary inhomogeneities $f\ t\ \in C\ \ a,b \setminus \tau_i\ {}_I$, $a_i \in \mathbb{R}^n$, $\alpha \in \mathbb{R}^m$. Then just following statements are equivalent:

a) for arbitrary $f\ t\ \in C\ \ a,b \setminus \tau_i\ {}_I$, $a_i \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}^m$ boundary value problem (1) has a unique solution $z\ \ t,\varepsilon$ as convergent with $\varepsilon \in (0, \varepsilon_*]$ series

$$z\ \ t,\varepsilon\ = \sum_{i=-2}^{+\infty} \varepsilon^i z_i\ \ t\ ; \quad (3)$$

б) an arbitrary r–dimensional constant vector $\varphi_0 \in R^r$ r–dimensional algebraic system

$$B_0 + \varepsilon B_1 + \ldots\ u_\varepsilon = \varphi_0 \quad (4)$$

has a unique solution in the form of convergent with $\varepsilon \in (0, \varepsilon_*]$series

$$u_\varepsilon = \ {}_{i=-1}^{+\infty} \varepsilon^i u_i; \tag{5}$$

в) the conditions

$$P_{B_0} \neq 0, \ \ P_{B_0} P_{B_1} = 0, \ \ P_{B_0^*} P_{B_1^*} P_{Q_d^*} = 0. \tag{6}$$

Under this condition$P_{B_0} \neq 0, P_{B_0} P_{B_1} = 0$ ensure uniqueness, and the condition $P_{B_0^*} P_{B_1^*} P_{Q_d^*} = 0$– existence of solutions.

Proof. Substituting the series (3) in the boundary problem (1) and compare the coefficients of the same powers ε.

If ε^{-2} then we have homogeneous boundary value problem:

$$\begin{aligned} & z_{-2} = A \ \ t \ \ z_{-2}, \ \ t \neq \tau_i; \\ & \Delta z_{-2 \ |t=\tau_i} = S_i z_{-2} \ \ \tau_i - 0 \ \ ; \\ & l z_{-2} = 0, \end{aligned} \tag{7}$$

which by assumption of the theorem has r–parametric system $r = n - n_1, n_1 = rankQ$ solutions of the form $z_{-2} \ \ t \ \ = X_r \ \ t \ \ c_{-1}$, where c_{-1}– random r–dimensional column vector from R^r.

If ε^{-1}, then boundary value problem:

$$\begin{aligned} & z_{-1} = A \ \ t \ \ z_{-,} + A_1 \ \ t \ \ z_{-2}, \ \ t \neq \tau_i; \\ & \Delta z_{-1 \ |t=\tau_i} = S_i z_{-1} \ \ \tau_i - 0 \ \ + A_{1i} z_{-2} \ \ \tau_i - 0 \ \ ; \\ & l z_{-1} = l_1 z_{-2}. \end{aligned} \tag{8}$$

For this boundary value problem with regard to

$B_0 = P_{Q_d^*} \ \ l_1 X_r \ \cdot \ - l \ \ {}_a^b K \ \ \cdot, \tau \ \ A_1 \ \ \tau \ \ X_r \ \ \tau \ \ d\tau - l \ \ {}_{i=1}^{p} K \ \ \cdot, \tau_i \ \ A_{1i} X_r \ \ r_i - 0$ obtain the algebraic relation $c_{-1} \in R^r$system

$$B_0 c_{-1} = 0.$$

This boundary value problem (8) has r–parametric system of solutions of the type

$$z_{-1} \ \ t \ \ = X_r \ \ t \ \ c_0 + G_1 \ \ t \ \ c_{-1},$$

where c_0– random r–dimensional column vector from R^r;

$$G_1\ t\ c_{-1} = z_{-1}\ t\ = \ G \begin{matrix} A_1\ \tau\ z_{-2}\ \tau, c_{-1} \\ A_{1i} z_{-2}\ \tau_i - 0 \end{matrix} \quad t\ + X(t)Q^+ l_1 z_{-2}\ \cdot, c_{-1}$$

– particular solution of (8);

expression $G \begin{matrix} * \\ * \end{matrix}\ t$ 7 has the form:

$$G \begin{matrix} A_1\ \tau\ z_{-2}\ \tau, c_{-1} \\ A_{1i} z_{-2}\ \tau_i - 0 \end{matrix} \quad t \stackrel{\text{def}}{=}$$

$$\int_a^b K\ t,\tau\ * d\tau - X\ t\ Q^+ l \int_a^b K\ \cdot,\tau\ * d\tau, \sum_{i=1}^{p} K\ t,\tau_i\ * - X\ t\ Q^+ l \sum_{i=1}^{p} K\ \cdot,\tau_i\ * \quad \times$$

$$\times \begin{matrix} A_1\ \tau\ z_{-2}\ \tau, c_{-1} \\ A_{1i} z_{-2}\ \tau_i - 0 \end{matrix} \ .$$

After equating the coefficients of the ε^0 we obtain the boundary value problem to determine $z_0\ t$:

$$\begin{aligned} & z_0 = A\ t\ z_0 + A_1\ t\ z_{-1}\ t\ + f\ t\ ,\ t \neq \tau_i; \\ & \Delta z_0\ _{|t=\tau_i} = S_i z_0\ \tau_i - 0\ + A_{1i} z_{-1}\ \tau_i - 0\ + a_i; \\ & l z_0 = \alpha + l_1 z_{-1}, \end{aligned} \qquad (9)$$

where $z_1\ t\ = X_r\ t\ c_0 + G_1\ t\ c_{-1}$.

From condition (necessary and sufficient) solvability

$$P_{Q_d^*}\ \propto - l \int_a^b K\ \cdot,\tau\ f(\tau) d\tau - l \sum_{i=1}^{p} K\ \cdot,\tau_i\ a_i\ = 0, d = m - n_1$$

for the boundary value problem (9) with respect $c_0, c_1 \in R^r$ are algebraic system

$$B_0 c_0 + B_1 c_{-1} = \varphi_0,$$

where φ_0 – r –dimensional continuous random vector from R^r because of the arbitrariness $f\ t\ \in C\ \ a,b\ \backslash\{\tau_i\}_I\ , a_i \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}^m$;

$$\varphi_0 = -P_{Q_d^*}\left(\propto - l\int_a^b K(\cdot,\tau)f(\tau)d\tau - l\sum_{i=1}^{p} K(\cdot,\tau_i)a_i\right),$$

$d \times r$–dimensional matrix B_1 has the form:

$$B_1 = P_{Q_d^*}\left(l_1 G_1(\cdot) - l\int_a^b K(\cdot,\tau)A_1(\tau)G_1(\tau)d\tau - l\sum_{i=1}^{p} K(\cdot,\tau_i)A_{1i}G_1(r_i - 0)\right). \quad (10)$$

Boundary value problem (9) has r–parametric system solutions:

$$z_0(t) = X_r(t)c_1 + G_1(t)c_0 + G_2(t)c_{-1} + f_1(t),$$

where c – arbitrary r–dimensional column vector from R^r;

$$G_2(t)c_{-1} = G\left[\begin{matrix} A_1(\tau)G_1(\tau)c_{-1} \\ A_{1i}G_1(\tau_i - 0)c_{-1}\end{matrix}\right](t) + X(t)Q^+ l_1 G_1(\cdot)c_{-1};$$

$$f_1(t) = G\left[\begin{matrix} f(\tau) \\ a_i\end{matrix}\right](t) + X(t)Q^+\alpha.$$

Continuing this procedure, we note that to the series (3) has a solution of the boundary value problem (1) with random inhomogeneities $f(t) \in C([a,b] \setminus \{\tau_i\}_I)$, $a_i \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}^m$ necessary and sufficient that it should be relatively soluble $c_i \in R^r$ $(i = -1,0,1,\ldots)$ the following system of algebraic equations for arbitrary $\varphi_0 \in R^r$:

$$\begin{aligned} B_0 c_{-1} &= 0; \\ B_0 c_0 + B_1 c_{-1} &= \varphi_0 \end{aligned} \quad (11)$$

and further

$$\begin{aligned} B_0 c_1 + B_1 c_0 + B_2 c_{-1} &= \varphi_1; \\ B_0 c_2 + B_1 c_1 + B_2 c_0 + B_3 c_{-1} &= \varphi_2. \end{aligned} \quad (12)$$

On the other hand, substituting the series (5) into (4) we find that to the series (5) was the solution of system (4) is necessary and sufficient that the coefficients $u_i \in R^r$ satisfy a system similar to (11), (12) but with $\varphi_i = 0, i = 1,2,\ldots$:

$$\begin{aligned} B_0 u_{-1} &= 0; \\ B_0 u_0 + B_1 u_{-1} &= \varphi_0 \end{aligned} \quad (13)$$

and further

$$B_0 u_1 + B_1 u_0 + B_2 u_{-1} = 0;$$
$$B_0 u_2 + B_1 u_1 + B_2 u_0 + B_3 u_{-1} = 0. \qquad (14)$$

To solve the system of algebraic equations (13) is necessary and sufficient to fulfill the condition (6). Then, to complete the proof of the theorem will confirm that in the case of solvability of equations (13), the system of equations (14) is also solved, and the last of the conditions of solvability of equation (13) we find the ratio u_{-1}. From condition of solvability (which coincides with the precondition) of the first equation (14) we find u_0, ect. The proof of this theorem repeats the proof of the corresponding theorem in [1], so it will not repeat.

Summary

If $P_{B_0} = 0,\ \ P_{B_0^*} P_{Q_d^*} = 0$ false, is to obtain sufficient conditions for the existence of solutions of boundary value problem (1) with random inhomogeneities $f \ \ t \ \ \in$ $C \ \ a,b \setminus \tau_i \ \ I$, $a_i \in \mathbb{R}^n, \alpha \in \mathbb{R}^m$ should be involved $d \times r$ – measurable matrix B_1 (10). Solution $z \ \ t, \varepsilon$ boundary value problem (1) is sought in this case in the form of convergent with $\varepsilon \in (0, \varepsilon_*]$ series from $k \geq 2$.

References

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