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TOPOLOGICALEQUIVALENCE OFFUNCTIONS

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Topologicalequivalenceoffunctionswasstudiedbythelanguageofalternatingsequences. Itisprovedthatforevery piecewise-linear functionwith n-1 localextrema, therepolynomialdegree n, whichistopologicallyisequivalenttothisfunction.

Topological equivalence, polynomial, alternatingsequence, piecewise-linear function.

Among the many problems facing mathematicians, one of the most important is to study the behavior of functions on different manifolds. It is known that many processes can be described by smooth functions [1], [4]. For example, such sciences as biology, biophysics, physics is essentially using mathematical knowledge concerning the behavior of smooth functions in a neighborhood of the critical points.

In the theory of functions important area of research is the issue of the conditions of topological equivalence of functions. This problem involved M. Morse, B. Kaplan, VV Sharko, A. Pryshlyak, A. Arnold, A. Fomenko, S. Stoilov.

The purpose of research - examining topological equivalence of functions with a finite number of extreme points, giving the topological classification of such functions in terms of further research of topological equivalence.

Materials and methods research. For each equivalence class introduces an essential function that takes in all the extreme points of the set of integers $0,1,2,\ldots,l$. Invariant function that provides an answer to these questions is alternuyuchi sequences [3], [4], [5].

Studies. Unloading Us the problem of the existence of polynomial topologically equivalent piecewise linear function.

Let X, Y - topological spaces; $f, g : X \to Y$ continuous display. Continuous map $f : X \to Y$ between topological spaces X and Y is called a homeomorphism if $f$is inverse bijections display $f^{-1} : X \to Y$ – continuous.

Definition 1.Continuous display $f, g : X \to Y$ called topologically equivalent if there are homeomorphisms $h : X \to X$ and $h' : Y \to Y$ for which the equality $g \circ h = h' \circ f$.

Topological equivalence mappings – a commutative diagram

$$\begin{array}{ccc} M & \xrightarrow{f} & N \\ h \downarrow & & \downarrow h' \\ M & \xrightarrow{g} & N \end{array}$$

where the vertical arrows are homeomorphisms.

Consider the case where N = R, that is, when asked two continuous (smooth) function $f$and$g$, and we call them topologically equivalent if the homeomorphism h' preserves the orientation of the line R.

The question when two continuous functions f and g with a finite number of extrema that are defined on the interval [a, b], are topologically equivalent and as there are different types of topologically equivalent functions in which the same number of extrema lead to the study of combinatorial objects [2]. It is known that the sequence of positive integers $0,1,2,\ldots,n,$ is (n +1)! permutations of these numbers.

Adequate invariant function that provides an answer to these questions is alternuyuchi sequence [4].

Definition 2.Alternuyuchoyu sequence $A\ (k, l)$

Type 1: <> call sequence $x_i, (0 \le i \le k)$, based on numbers $0,1,\ldots,l$, for which fair condition: $x_0 < x_1 > x_2 < x_3 > \ldots < x_{k-1} > x_k$, where $l \le k-1$;

Type 2: << call sequence $x_i, (0 \le i \le k)$ for which fair condition: $x_0 < x_1 > x_2 < x_3 > \ldots > x_{k-1} < x_k$, where $l \le k-1$;

Type 3: >> call sequence $x_i, (0 \le i \le k)$ for which fair condition: $x_0 > x_1 < x_2 > x_3 < \ldots < x_{k-1} > x_k$, where $l \le k-1$;

Type 4:><call sequence $x_i, (0 \le i \le k)$ for which fair condition: $x_0 > x_1 < x_2 > x_3 < \ldots > x_{k-1} < x_k$.

That language alternuyuchyh sequences will be answered to the questions of the topological equivalence of functions.

Alternuyuchi sequences are built according to the sequence of the function under extreme points, which is defined on the interval. In all types of sequences may be cases when not all the numbers $1,2,\ldots,l$ present. If alternuyuchiy sequence contains all numbers $1,2,\ldots,l$, then we say that the type alternuyucha sequence is complete.

From the definition of topological equivalence for continuous functions with a finite number of extrema follows. that each equivalence class there is an essential function that takes in all the extreme points of the integer values from the set $1,2,\ldots,l$ [3], [4].

Put into compliance function *f* on the interval [*a*, *b*] alternuyuchu complete sequence of the appropriate type $A(k,l)$. For this we consider the interval [*a*, *b*] the sequence $y_i, (0 \le i \le k)$ of local extrema of functions $f\,(y_0 = a, y_k = b), y_i < y_{i+1}$.

Denote $x_i = f(y_i)$. Suppose that $f(a) < x_i; f(b) < x_{k-1}$. Then, it is clear that the sequence $x_0 < x_1 > x_2 < x_3 > \ldots < x_{k-1} > x_k$ forms a complete alternuyuchu type sequence $A(k,l)$ <>. Construction of other typesalternuyuchyh sequences are similar.

If we consider the interval [*a*, *b*] an arbitrary continuous function $y = f(x)$ with a finite number of local extrema, then using homeomorphism real line $s: R^1 \to R^1$, preserving the orientation function $y = f(x)$ can be replaced by a function $z = s \circ f(x)$ that will be a marked feature. Obviously, if the point y_i– local maximum (minimum) forfunction *f*, then it will be a local maximum (minimum) for the function $s \circ f$ and vice versa [1].

Since the homeomorphism s saves the points located on the real axis, then the adjacent extreme points y_i and y_{i+1} function *f* inequality $f(y_i) < f(y_{i+1})$ $(f(y_i) > f(y_{i-1}))$ holds for the function $s \circ f$, $s \circ f(y_i) < s \circ f(y_{i+1})$ $(s \circ f(y_i) > s \circ f(y_{i-1}))$. Thus, continuous functions with a finite number of local extrema clearly can be associated alternuyuchu complete sequence $A_f(k,l)$ of a certain type depending on the number of extrema of the function *f* and the values that the function *f* takes the extreme points.

Thus, for any equivalence class of continuous functions $y = f(x)$ with a finite number of local extrema, which is given on the interval [*a, b*], is uniquely constructed alternuyucha sequence.

Have given full alternuyuchasequence $A(k,l)$. Then, the sequence $A(k,l)$ can be associated piecewise linear curve $L(A(k,l))$. It is clear that the piecewise linear curve $L(A(k,l))$ can be interpreted as the graph of a continuous function $L(A(k,l))$ of *k +1*local extremeand *l* values in them, which is defined on the interval [*a, b*]. It is clear that the extreme interior point chosen is ambiguous, but their arrangement on the interval [*a, b*] is always the same.

Definition. For full alternuyuchoyisequence $A(k,l)$ piecewise linear function $L(A(k,l))$ will be called *PL*– implementation sequence.

Lemma 1.Let the interval [*a, b*] given continuous function *f* with a finite number of extreme points. Put it in its full compliance alternuyuchusequence $A_f(k,l)$. Then the function *f*and $L(A_f(k,l))$ to be topologically equivalent.

Theorem 1. Two continuous functions $y = f(x)$and $y = g(x)$ on the interval [*a, b*] with a finite number of local extrema are topologically equivalent if and only if they are full of the same sequence and alternuyuchi $A_f(k,l)$ and $A_g(k,l)$.

For any full alternuyuchoyi sequence $A(k,l)$ can construct of continuous on the interval [*a, b*] function from a finite number of local extrema that its full alternuyucha sequence $A_f(k,l)$ is $A(k,l)$ [4].

Select homeomorphism $h$and $h'$is not unambiguous. So, if fixed topological spaces *X* and *Y*, then of course there are two problems: 1) to describe the set of classes of continuous mappings from *X* and *Y* relative typed equivalence relation, and 2) provide necessary and sufficient conditions when two continuous mappings *f* and *g* of *X* in *Y* are topologically equivalent. These two problems are very complicated and the answer is not known, for example, when $X = Y = R^1$. Therefore, we will not consider the general case, and restrict narrower classes of continuous mappings, including polynomial [5].

Homeomorphism $h' : R^1 \to R^1$ preserves orientation, given a strictly monotonically increasing function. If the function $f: X \to R^1$ and $g: X \to R^1$ – topologically equivalent to X and a point $x_0 \in X$ is a point of local maximum (or minimum) of the function *f*, point *h(x)* is a point of local maximum (or minimum) of the function *g*. If the - strictly monotonic continuous function, then it is topologically equivalent to a linear function *f*, defined on [*a*, *b*] with multiple values equal to [*c*, *d*]. Here [*a*, *b*] and [*c*, *d*] – the real line spacing that can be endless.

Lemma 2. Let $f: R^1 \to R^1$ – be a continuous function with a finite number of local extrema, then f is topologically equivalent to a piecewise linear function.

Consequence. Let $f: R^1 \to R^1$ – polynomial, then it is topologically equivalent to a piecewise linear function.

Theorem 2.Let $\xi_1 \le \xi_2 \le ... \le \xi_{n-1}$, $\eta_1, ..., \eta_{n-1}$ – be real numbers. Then there exists a polynomial $f: R^1 \to R^1$ of degree *n*such that $f'(\xi_i) = 0;\ f(\xi_i) = \eta_i, i = \overline{1, n-1}$ if and only if the number $(-1)^i(\eta_i - \eta_{i-1}), i = \overline{2, n-1}$ one sign.

Theorem 3. For each piecewise-linear function $l: R^1 \to R^1$ with *n-1* local extrema, a polynomial $f: R^1 \to R^1$ of degree *n*, which is topologically equivalent to the function l.

Proof. Let l – piecewise linear and the extreme $\xi_1 < \xi_2 < ... < \xi_{n-1}$. Because $\xi_i, i = \overline{1, n-1}$ – extreme, then $(-1)^i(l(\xi_i) - l(\xi_{i-1})), i = \overline{2, n-1}$ the same sign. Theorem 2.it follows that there exists a polynomial $f: R^1 \to R^1$ of degree *n* such that $f'(\xi_i) = 0;\ f(\xi_i) = l(\xi_i), i = \overline{1, n-1}$. Obviously*f*monotone on each interval$(-\infty, \xi_1)...$ $(\xi_i, \xi_{i+1})...(\xi_{n-1}, +\infty), i = \overline{1, n-1}$(since *f* has degree *n* ⟶ f' has degree *n-1*).

Proves that *f* is topologically equivalent to *l*. Need to find a homeomorphism such that $f \circ h = h' \circ l$. Suppose that $h'(x) = x$, for any x is R^1,

$$h(x) = \begin{cases} f_0^{-1} \circ l(x), x \in (-\infty, \xi_1] \\ f_1^{-1} \circ l(x), x \in [\xi_1, \xi_2] \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ f_{n-1}^{-1} \circ l(x), x \in [\xi_{n-1}, +\infty) \end{cases},$$ where $f_k^{-1} : f([\xi_k, \xi_{k+1}]) \to [\xi_k, \xi_{k+1}] - \kappa$ and branch display f^{-1}.

Then for any x $f \circ h(x) = f(f_k^{-1}(l(x))) = l(x)$.

Therefore $f \circ h = h' \circ l = l$.

Theorem is proved.

Conclusions

The paper was established terms of topological equivalence of functions. Considered invariant continuous functions with a finite number of extreme points defined on the interval. These invariants are alternuyuchi sequence. Investigated the fact that for every piecewise-linear function with n-1 local extrema, a polynomial of degree n, which is topologically equivalent to this function.

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