

# MATRIX SOLUTION BASIC MATHEMATICAL MODEL OF RELIABILITY DESIGN OF FUNKTSIONUVANNYASKLADNOYI "man - machine"

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*The resulting model to determine the probability of failure of the system to improve the professional conditions and psycho-physiological level operator. Analytical dependences to determine the probability of failure of the system.*

**Reliability, system model, machine, man - operator state graph.**

**Problem.** In mathematical description of the behavior of complex technical systems "man - machine" (hereinafter - JTS "LM") apply stochastic differential equations [1, 2]. Combining these equations to the appropriate system to make the transformation matrix, the solution of which is the basis for the criteria of reliability STS "LM", which are agricultural machinery, equipment and machinery for livestock.

**Analysis of recent research.** In the present article earlier [3] shows that reliability study CTC "LM" provided "aging" component "machine" and part of "man - operator", the system takes the following form:

$$\begin{cases} S\varphi_0(S) + S\varphi'_0(t) + S\varphi''_0(t) + S\varphi_1(S) + S\varphi'_1(t) = 1; \\ -\lambda'_0 P_0(S) + (S + \lambda'_1)\varphi'_0(S) = 0; \\ -h\lambda''_0\varphi_0(S) + (S + \lambda''_1)\varphi_1(S) = 0; \\ (S + \mu)\varphi_1(S) - \lambda'_1\varphi'_0(S) - \lambda''_1\varphi''_0(S) - \lambda_2\varphi'_1(S) = 0; \\ (S + \lambda_2)\varphi'_1(S) - t\lambda''_0\varphi_0(S) = 0. \end{cases} \quad (1)$$

For the resulting system (1) form the determinant, which is a factor of the unknowns in a matrix:

$$\Delta = \begin{vmatrix} S & S & S & S & S \\ -\lambda'_0 & (S + \lambda'_1) & 0 & 0 & 0 \\ -h\lambda''_0 & 0 & 0 & (S + \lambda''_1) & 0 \\ 0 & -\lambda'_1 & -\lambda''_1 & (S + \mu) & -\lambda_2 \\ -t\lambda''_0 & 0 & 0 & 0 & (S + \lambda_2) \end{vmatrix} \begin{vmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}. \quad (2)$$

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As seen from (2), is a determinant of a square matrix fifth order. To solve the matrix and of the unknown  $\varphi_i(S)$  You can use Gauss method:

$$\varphi_0(S) = \frac{\Delta_0}{\Delta}; \varphi'_0(S) = \frac{\Delta'_0}{\Delta}; \varphi''_0(S) = \frac{\Delta''_0}{\Delta}, \varphi_1(S) = \frac{\Delta_1}{\Delta}, \varphi_1(S) = \frac{\Delta'_1}{\Delta}. (3)$$

where  $\Delta$  - determinant decision (2);  $\Delta_0$  - Decision determinant for unknown  $\varphi_0(S)$ ;  $\Delta'_0$  - Decision determinant for unknown  $\varphi'_0(S)$ ;  $\Delta''_0$  - Decision determinant for unknown  $\varphi''_0(S)$ ;  $\Delta_1$  - Decision determinant for unknown  $\varphi_1(S)$ ;  $\Delta'_1$  - Decision determinant for unknown  $\varphi_1(S)$ .

**The purpose of research** - To provide the basic decision matrix model reliability of CTC "PM" in case of component "man - operator".

**Results.** Based on the above, to calculate the probability of any of the states of interest in the research necessary to solve the basic matrix  $\Delta$ . Methods of solution matrix includes several stages. Consider the first stage solution

1.1a. Define the multiplier as  $\frac{-\lambda'_0}{1} = -\lambda'_0$ .

1.1b. Multiply the first row of the matrix (1) on the resulting multiplier. After multiplying we get:

$$[-\lambda'_0]; [-\lambda'_0]; [-\lambda'_0]; [-\lambda'_0]; [-\lambda'_0]; [-\lambda'_0].$$

1.1v. We calculate the second row of the matrix (2) line that is received in p.1.1b:

$$[(-\lambda'_0 - (-\lambda'_0)); [(S + \lambda'_1 - (-\lambda'_0)); [(0 - (-\lambda'_0)); [(0 - (-\lambda'_0)); [(0 - (-\lambda'_0)); [(0 - (-\lambda'_0)).$$

When done, we can write:

$$[0]; [(S + \lambda'_1 + \lambda'_0)]; [\lambda'_0]; [\lambda'_0]; [\lambda'_0]; [\lambda'_0].$$

The resulting string sets the second equation of the new transformed system of equations as follows:

$$0 + (S + \lambda'_1 + \lambda'_0)P'_0 + \lambda'_0 P''_0 + \lambda'_0 P_1 + \lambda'_0 P''_1 = \lambda'_0. (4)$$

The second step changes the original matrix  $\Delta$  (2) hold for the first and third line 1.1a - 1.1v.

1.2a. Define multiplier  $\frac{-h\lambda''_0}{1} = -h\lambda''_0$ .

1.2b. Multiply the first line of the matrix (2) on the resulting multiplier:

$$[-h\lambda''_0]; [-h\lambda''_0]; [-h\lambda''_0]; [-h\lambda''_0]; [-h\lambda''_0]; [-h\lambda''_0].$$

1.2v. We calculate the third row of the matrix (2) series that obtained in paragraph 1.2b. Therefore, we can write:

$$[(-h\lambda''_0) - (-h\lambda''_0)]; [0 - (-h\lambda''_0)]; [0 - (-h\lambda''_0)]; [(S + \lambda''_1) - h\lambda''_0]; [0 - (-h\lambda''_0)]; [0 - (-h\lambda''_0)].$$

After simplification we write:

$$[0]; [h\lambda_0'']; [h\lambda_0'']; [(S + \lambda_1'' + h\lambda_0'')]; [h\lambda_0'']; [h\lambda_0'']$$

Hence, we can write the third equation of the new system transformed as follows:

$$0 + h\lambda_0''P_0' + h\lambda_0''P_0'' + (S + \lambda_1'' + h\lambda_0'')P_1 + h\lambda_0''P_1'' = h\lambda_0''. \quad (5)$$

The third step is the initial transformation matrix  $\Delta$  (2) is a repetition of previous actions, but for the first and fourth lines.

1.3a. Define multiplier:  $\frac{0}{1} = 0$ .

1.3b. Multiply the first row of the matrix (2) to factor found:

$$[0]; [0]; [0]; [0]; [0]; [0]$$

1.3v. We calculate from the fourth row of the matrix (2) line that is received in paragraph 1.3b:

$$[0]; [-\lambda_1' - 0]; [-\lambda_1'' - 0]; [(S + \mu) - 0]; [-\lambda_2]; [0]$$

After simplifying the obtained values, we can write:

$$[0]; [-\lambda_1']; [-\lambda_1'']; [(S + \mu)]; [-\lambda_2]; [0]$$

Write a line ratios, which sets a new fourth equation of the transformed system:

$$0 - \lambda_1'P_0' - \lambda_1''P_0'' + (S + \mu)P_1 - \lambda_2P_1'' = 0. \quad (6)$$

Fourth, the initial transformation matrix  $\Delta$  is also a repeat of the early action, but relative to the first and fifth lines.

1.4a. Define the multiplier represented lines:  $\frac{-t\lambda_0''}{1} = -t\lambda_0''$ .

1.4b. Also, we multiply the first line of the matrix (2) on the resulting multiplier:

$$[-t\lambda_0'']; [-t\lambda_0'']; [-t\lambda_0'']; [-t\lambda_0'']; [-t\lambda_0'']; [-t\lambda_0'']$$

1.4v. We calculate the fifth row of the matrix (2) line that will get in paragraph 1.4b:

$$[(-t\lambda_0'') - (-t\lambda_0'')]; [0 - (-t\lambda_0'')]; [0 - (-t\lambda_0'')]; [0 - (-t\lambda_0'')]; [(S + \lambda_2) - (-t\lambda_0'')]; [0 - (-t\lambda_0'')]$$

Using the resulting film coefficients, we write the fifth equation for the new converted system:

$$0 + t\lambda_0''P_0' + t\lambda_0''P_0'' + t\lambda_0''P_1 + (S + \lambda_2 + t\lambda_0'')P_1'' = t\lambda_0''. \quad (7)$$

Given that the first equation in the new system is stored as a matrix (2), as defined by the following equations obtained relationship (4) - (7), we can write a new transformed system. The resulting new system is equivalent to address:

$$\begin{cases} P_0 + P'_0 + P''_0 + P_1 + P''_1 = 1; \\ 0 + (S + \lambda'_1 + \lambda'_0)P'_0 + \lambda'_0 P''_0 + \lambda'_0 P_1 + \lambda'_0 P''_1 = \lambda'_0; \\ 0 + h\lambda''_0 P'_0 + h\lambda''_0 P''_0 + (S + \lambda''_1 + h\lambda''_0)P_1 + h\lambda''_0 P''_1 = h\lambda''; \\ 0 - \lambda'_1 P'_0 - \lambda'_1 P''_0 + (S + \mu)P_1 - \lambda_2 P''_1 = 0; \\ 0 + t\lambda''_0 P'_0 + t\lambda''_0 P''_0 + t\lambda''_0 P_1 + (S + \lambda_2 + t\lambda''_0)P''_1 = t\lambda_0. \end{cases} \quad (8)$$

To solve the resulting reduced system (8) can also use the method of Gauss. For the second stage of the calculation of the system (8) we obtain an extended matrix  $\Delta'$ :

$$\begin{array}{ccccc|c} a'_{11} = 1 & a'_{12} = 1 & a'_{13} = 1 & a'_{14} = 1 & a'_{15} = 1 & b'_1 = 1, \\ a'_{21} = 0 & a'_{22} = (S + \lambda'_1 + \lambda'_0) & a'_{23} = \lambda'_0 & a'_{24} = \lambda'_0 & a'_{25} = \lambda'_0 & b'_2 = \lambda'_0, \\ a'_{31} = 0 & a'_{32} = h\lambda''_0 & a'_{33} = h\lambda''_0 & a'_{34} = (S + \lambda''_1 + h\lambda''_0) & a'_{35} = h\lambda''_0 & b'_3 = h\lambda''_0, \\ a'_{41} = 0 & a'_{42} = -\lambda'_1 & a'_{43} = -\lambda'_1 & a'_{44} = (S + \mu) & a'_{45} = -\lambda_2 & b'_4 = 0, \\ a'_{51} = 0 & a'_{52} = t\lambda''_0 & a'_{53} = t\lambda''_0 & a'_{54} = t\lambda''_0 & a'_{55} = (S + \lambda_2 + t\lambda''_0) & b'_5 = t\lambda_0 \end{array} \quad (9)$$

By analogy, the conversion will hold the second and third stages of the calculation of the matrix (2). After transformations we obtain the probability of system stay in one of the states:

$$P_1 = \frac{b''_4 a'''_{55}}{a'''_{44} a'''_{55} - a'''_{45} a'''_{54}}. \quad (10)$$

$$P''_1 = -\frac{b''_4 a'''_{54}}{a'''_{44} a'''_{55} - a'''_{45} a'''_{54}}. \quad (11)$$

$$P''_0 = \frac{h\lambda_0(S + \lambda'_1) - a'''_{34} \left( \frac{b''_4 a'''_{55}}{a'''_{44} a'''_{55} - a'''_{45} a'''_{54}} \right) + a'''_{35} \left( \frac{b''_4 a'''_{54}}{a'''_{44} a'''_{55} - a'''_{45} a'''_{54}} \right)}{h\lambda''_0(S + \lambda'_1)}. \quad (12)$$

$$\begin{aligned} P'_0 &= \lambda'_0 - \\ &- \lambda'_0 \frac{h\lambda_0(S + \lambda'_1) - a'''_{34} \left( \frac{b''_4 a'''_{55}}{a'''_{44} a'''_{55} - a'''_{45} a'''_{54}} \right) + a'''_{35} \left( \frac{b''_4 a'''_{54}}{a'''_{44} a'''_{55} - a'''_{45} a'''_{54}} \right)}{h\lambda''_0(S + \lambda'_1)} + \\ &+ \lambda'_0 \frac{b''_4 a'''_{55}}{a'''_{44} a'''_{55} - a'''_{45} a'''_{54}} - \lambda'_0 \frac{b''_4 a'''_{54}}{a'''_{44} a'''_{55} - a'''_{45} a'''_{54}} \Big/ (S + \lambda'_1 + \lambda'_0). \end{aligned} \quad (13)$$

Based on the analytical relations (10) - (13) and setting probabilities host system in the states  $P'_0, P''_0, P_1, P''_1$  Probability  $P_0$  will be:

$$P_0 = 1 - (P'_0 + P''_0 + P_1 + P''_1). \quad (14)$$

**Conclusion.** To simplify the analysis of the analytical relations (10) - (13) and the possibility of practical use to assess the reliability CCC "LM" can impose the condition that  $\mu \gg \lambda$ . Based on the conditions

imposed in Formulas (10) - (13) resulting analytical studies to ensure reliability CCC "LM" while reducing their technical level ("aging" of the machine) and increased professional and technological level of the human operator can not loss of credibility theoretical results negligible quantities failure rate  $\lambda_i$  second and higher degrees. Further conversion and installation determinant  $\Delta$  core matrix can be made, going back to determine the coefficients  $a_i$  and  $b_i$ .

### References

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*Will provide a model for definitions probability bezotkaznoy work system to Increase PROFESSIONAL terms and psyhofyzyolohycheskoho urovnja operator. Polucheny analytycheskye dependence for definitions probability bezotkaznoy work system.*

***Nadezhnost system, model car, man - operator graph states.***

*The model for determining the probability of failure-free operation of system for provision of professional development and psycho-physiological levels of operator. The analytical dependence for determination of probability of failure of system.*

***Reliability, system, model, machine, man - operator, graph states.***