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In the work given sovremennыh analysis methods uprochnenyya workers organs soil-cultivating machines rassmotrenы s Advantages and disadvantages. Shown something most эffektyvnыm by uprochnenyya workers surfaces of parts soil-cultivating machines javljaetsja tochechnoe uprochnenye - Arc welding tochechnaya poroshkovoy provolokoy.

Abrazyvnoe yznashyvanye, tochechnoe uprochnenye, Lemekh plow, cultivator paw, boronы drive.

In paper present introduce the present method hardening working tool cultivation machine them advantage and defect Demonstrate what the greatest effective method hardening force surface part cultivation machine have-point hardening point consumable-electrode are welding flux cored electrode.

Abrasive wear, pointwise reinforcement, plough share, sweep, harrow disk.

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RESEARCH AND ANALYSIS METHODS FOR OPTIMAL CONTROL OF DYNAMIC SYSTEMS

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In this paper we solve the problem of optimal control of a dynamic system described by the differential equation of second order. Shows the relationship between the known methods of optimal control: the calculus of variations, maximum principle and dynamic programming. Optimal control found in the form of feedback while respecting the constraints on the value of control.

Dynamic programming, maximum principle, variational calculus, limit management.

Problem. The operation of modern engineering systems associated with significant dynamic loads, energy costs and fleeting transient. The phenomena that accompany the transition flow

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processes in different dynamical systems are often undesirable character. For example, an undesirable heating coils are electrically driven, as it leads to aging of insulation and reduces the energy efficiency of the machine. To minimize unwanted and / or increase the desired factors used machines various optimization procedures, which are based on known mathematical methods. This study covers only the optimization of dynamic systems and control are not considered how to optimize its design parameters.

An important task that is not only theoretical but also applied nature, is to establish relationships between the known methods of optimal control. This enables you to uncover the critical relationships between these methods that result can identify ways to solve optimization problems by alternative means.

Analysis of recent research. For optimization of motion of mechanical systems use various mathematical methods. The oldest of these methods is the method of the calculus of variations [1], the origin and development of which is associated with the names of Euler, Lagrange, Poisson, Weierstrass and other scientists. One of the best known methods for solving optimal control is the maximum principle [2], developed by LS Pontryagin and his students. This method allows to take into account the limitations imposed on the dynamic control system. A powerful method for solving optimal control not only technical, but also economic, social and other natural systems are dynamic programming [3]. The author of this method is an American mathematician R. Bellman. In addition to these methods, also used the method of moments [4], the development of which is associated with the name Acad. MM Krasovskii.

Also known theorem VF Krotov sufficient conditions of optimality process control [5].

This analysis indicated no significant array of approximate optimal control methods that allow a more or less closer to solving the problem. Analysis of these methods can be found in [6, 7].

All of these methods allow to find the optimal control as a function of time and only some of them - as the optimal feedback (the problem of synthesis of optimal control). Overall analysis of these methods is quite a challenge, as each of them has its own advantages and disadvantages.

The purpose of research. The most common methods for solving optimal control problems is the calculus of variations, maximum principle and dynamic programming. The purpose of this paper to clarify the links between these methods as an example of a simple optimal control problem still massive dynamic system. To achieve this goal it is necessary to solve the following problem: 1) to provide optimal control problem formulation; 2) solve the problem of optimal control using the maximum principle; 3) show how the ratio of the maximum principle method can get the necessary condition for extremum classical calculus of variations - Euler-Poisson; 4) to the optimal control using dynamic programming method; 5) Find the correlation between dynamic programming method and the maximum principle; 6) analyze optimal results and indicate ways to further research.

Results. *1. Statement of the problem of optimal control.* For a large number of technical momentum in the first approximation can be represented as a simple differential equation:

$$m\ddot{x} = F - W, \tag{1}$$

where m - Reduced to the translational motion of the mass of the system; x - Generalized coordinate system (translational movement); F - Driving force acting on the system; W - Force static resistance of the system, in including technological nature. The dot over a symbol means differentiation with time.

Equation (4), with the notation $x = x_1, \frac{F-W}{m} = u$ can be represented in canonical form:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u. \end{cases} \tag{2}$$

The criterion optimization process, which should minimize, choose the integrated functionality that the structure is complex criteria:

$$I = \int_0^T (\delta_1 x_1^2 + \delta_2 x_2^2 + \delta_3 u^2)dt, \tag{3}$$

where *T* - The length of the system; δ_1, δ_2 and δ_3 - Weights that take into account the importance of the relevant terms in the integrand criterion (3).

We seek optimal control under such boundary conditions:

$$\begin{cases} x_1(0) = s_0,\ x_2(0) = v_0; \\ x_1(T) = x_2(T) = 0, \end{cases} \tag{4}$$

where s_0 and v_0 - Initial position and initial velocity of the dynamic system.

Therefore, you must change the dynamic system from some initial position, which is characterized by non-zero position s_0 and speed v_0 in zero position while minimizing criterion by the expression (3). For mechanical systems such mode of movement means zahalmovuvannya.

2. Solving the optimization problem using the maximum principle. In order to solve the problem using the maximum principle must be written Hamiltonian function:

$$Í = \psi_1 \tilde{o}_2 + \psi_2 u - \delta_1 x_1^2 - \delta_2 x_2^2 - \delta_3 u^2, \tag{5}$$

where ψ_1 and ψ_2 - Conjugate variables. According to the maximum principle must therefore manage the process to Hamiltonian (5) was maximized. For open field control ($u \in (-\infty; \infty)$) Is a management function of the conditions of stationary Hamilton:

$$\frac{\partial Í}{\partial u} = \psi_2 - 2\delta_3 u = 0. \tag{6}$$

From equation (6) we get:

$$u = \frac{\psi_2}{2\delta_3}. \tag{7}$$

In order to ensure that the resulting control (7) delivers maximum Hamiltonian (5), it is necessary to analyze the sign of the second derivative of the Hamiltonian in management:

$$\frac{\partial^2 Í}{\partial u^2} = -2\delta_3 < 0, \quad (\delta_3 > 0). \tag{8}$$

Expression (7) is valid for open field control. However, generally, the control imposed restrictions as nestrohyh inequalities:

$$u_{\max} \geq u \geq u_{\min}, \tag{9}$$

where $u_{\max}$ and $u_{\min}$ - Respectively the maximum and minimum control. Then the region of admissible controls will be limited boundaries $u_{\max}$ and $u_{\min}$. Given the constraints (9) optimal control can be represented in the following form:

$$u=\begin{cases} u_{\max}, & \text{якщо } \dfrac{\psi_2}{2\delta_3}\ge u_{\max};\\ \dfrac{\psi_2}{2\delta_3}, & \text{якщо } u_{\max}\ge\dfrac{\psi_2}{2\delta_3}\ge u_{\min};\\ u_{\min}, & \text{якщо } \dfrac{\psi_2}{2\delta_3}\le u_{\min}.\end{cases} \qquad (10)$$

3. Finding the connection between maximum principle and classical variations. Thus, the structure of the optimal control (10). To find the unknown variable conjugate ψ_2 need to find solutions of the conjugated system of differential equations:

$$\begin{cases} \dot{\psi}_1=-\dfrac{\partial\acute{I}}{\partial\tilde{o}_1}=2\tilde{o}_1\delta_1;\\ \dot{\psi}_2=-\dfrac{\partial\acute{I}}{\partial\tilde{o}_2}=-\psi_1+2\tilde{o}_2\delta_2.\end{cases} \qquad (11)$$

Prodyferentsiyuyemo last equation of (11) in time and given the first equation (11) we get:

$$\ddot{\psi}_2=-2\tilde{o}_1\delta_1+2u\delta_2. \qquad (12)$$

Further double prodyferentsiyuyemo expression (7) time and substitute the resulting expression in (12). As a result, we have:

$$\ddot{u}=\frac{\ddot{\psi}_2}{2\delta_3}=\frac{-2\tilde{o}_1\delta_1+2u\delta_2}{2\delta_3}. \qquad (13)$$

Given that $\ddot{u}=\overset{IV}{\tilde{o}}$ and $u=\ddot{x}$ write the equation (13) in the following way:

$$\overset{IV}{\tilde{o}}-\ddot{x}\frac{\delta_2}{\delta_3}+x\frac{\delta_1}{\delta_3}=0. \qquad (14)$$

Equation (14) is the Euler-Poisson for functional (3).

4. Synthesis of optimal control using dynamic programming method. To find the optimal control as a function of the phase variable dynamic systems use dynamic programming method. Main functional Bellman equation takes the following form:

$$\min_u\left\{\frac{\partial S}{\partial x_1}x_2+\frac{\partial S}{\partial x_2}u+\delta_1x_1^2+\delta_2x_2^2+\delta_3u^2\right\}=0, \qquad (15)$$

where S - Bellman function.

Minimum right-hand side of equation (9) will look for parameter control u what prodyferentsiyuyemo her u and compare the resulting zero:

$$2\delta_3u+\frac{\partial S}{\partial x_2}=0. \qquad (16)$$

We find from equation (16) control u:

$$u = -\frac{1}{2\delta_3}\frac{\partial S}{\partial x_2} \tag{17}$$

and substitute obtained in equation (15) with the result that we have:

$$\frac{\partial S}{\partial x_1}x_2 - \frac{1}{4\delta_3}(\frac{\partial S}{\partial x_2})^2 + \delta_1 x_1^2 + \delta_2 x_2^2 + = 0. \tag{18}$$

Equation (18) is a nonlinear differential equation in partial derivatives. We seek its solution in the form of a quadratic form:

$$S = A_1 x_1^2 + A_2 x_1 x_2 + A_3 x_2^2. \tag{19}$$

Take the partial derivative of expression (19) for the parameters x_1 and x_2:

$$\frac{\partial S}{\partial x_1} = 2A_1 x_1 + A_2 x_1, \tag{20}$$

$$\frac{\partial S}{\partial x_2} = A_2 x_1 + 2A_3 x_2. \tag{21}$$

Substituting expressions (20) and (21) in equation (18) and obtain:

$$x_1^2(\delta_1 - \frac{A_2^2}{4\delta_3}) + x_2^2(\delta_2 + A_2 - \frac{A_3^2}{\delta_3}) + x_1 x_2(2A_1 - \frac{A_2 A_3}{\delta_3}) = 0. \tag{22}$$

Equation (22) is valid in the case when the expressions in brackets are equal to zero because $x_1 \neq 0, \; x_2 \neq 0$. Therefore, equation (22) can be replaced by a system of nonlinear algebraic equations:

$$\begin{cases} \delta_1 - \frac{A_2^2}{4\delta_3} = 0, \\ \delta_2 + A_2 - \frac{A_3^2}{\delta_3} = 0, \\ 2A_1 - \frac{A_2 A_3}{\delta_3} = 0. \end{cases} \tag{23}$$

Solution of the system of equations (23) will have two real and two complex roots. Choose one valid, which does not lead to loss of stability of the system.

Substituting the obtained roots in expression (17) we obtain the optimal control function in the form of synthesis:

$$u = -\frac{\sqrt{\delta_1}\sqrt{\delta_3}}{\delta_3}x_1 - \frac{\sqrt{(\delta_2 + 2\sqrt{\delta_1}\sqrt{\delta_3})\delta_3}}{\delta_3}x_2. \tag{24}$$

Consider the Bellman equation (15). Convert this equation:

$$\min_{u}(-1)\left\{\left(-\frac{\partial S}{\partial x_1}\right)x_2+\left(-\frac{\partial S}{\partial x_2}\right)u-\delta_1 x_1^2-\delta_2 x_2^2-\delta_3 u^2\right\}=0 \qquad (25)$$

or

$$\max_{u}\left\{\left(-\frac{\partial S}{\partial x_1}\right)x_2+\left(-\frac{\partial S}{\partial x_2}\right)u-\delta_1 x_1^2-\delta_2 x_2^2-\delta_3 u^2\right\}=0. \qquad (26)$$

The expression in braces is a Hamiltonian function, if we accept $-\frac{\partial S}{\partial x_1}=\psi_1$ and $-\frac{\partial S}{\partial x_2}=\psi_2$. Note that the optimal control using Bellman equation performed for open field controls that limit (9) is not taken into account. The maximum principle allow for the limitation and therefore optimal control in the form of feedback with constraints (9) can be written as follows:

$$u=\begin{cases} u_{\max}, & якщо\ -\frac{\sqrt{\delta_1\delta_3}}{\delta_3}x_1-\frac{\sqrt{(\delta_2+2\sqrt{\delta_1\delta_3})\delta_3}}{\delta_3}x_2\ge u_{\max};\\ -\frac{\sqrt{\delta_1\delta_3}}{\delta_3}x_1-\frac{\sqrt{(\delta_2+2\sqrt{\delta_1\delta_3})\delta_3}}{\delta_3}x_2, & якщо\ u_{\max}\ge-\frac{\sqrt{\delta_1\delta_3}}{\delta_3}x_1-\frac{\sqrt{(\delta_2+2\sqrt{\delta_1\delta_3})\delta_3}}{\delta_3}x_2\ge u_{\min};\\ u_{\min}, & якщо\ -\frac{\sqrt{\delta_1\delta_3}}{\delta_3}x_1-\frac{\sqrt{(\delta_2+2\sqrt{\delta_1\delta_3})\delta_3}}{\delta_3}x_2\le u_{\min}. \end{cases} \qquad (27)$$

Construct graphs optimal process (Fig. 1, Fig. 2) for the following system parameters $\delta_1=0{,}2$, $\delta_1=0{,}3$, $\delta_1=0{,}5$, $u_{\max}=1$, $u_{\min}=-1$, $\tilde{o}_0=0$ m $v_0=1$ m / s.

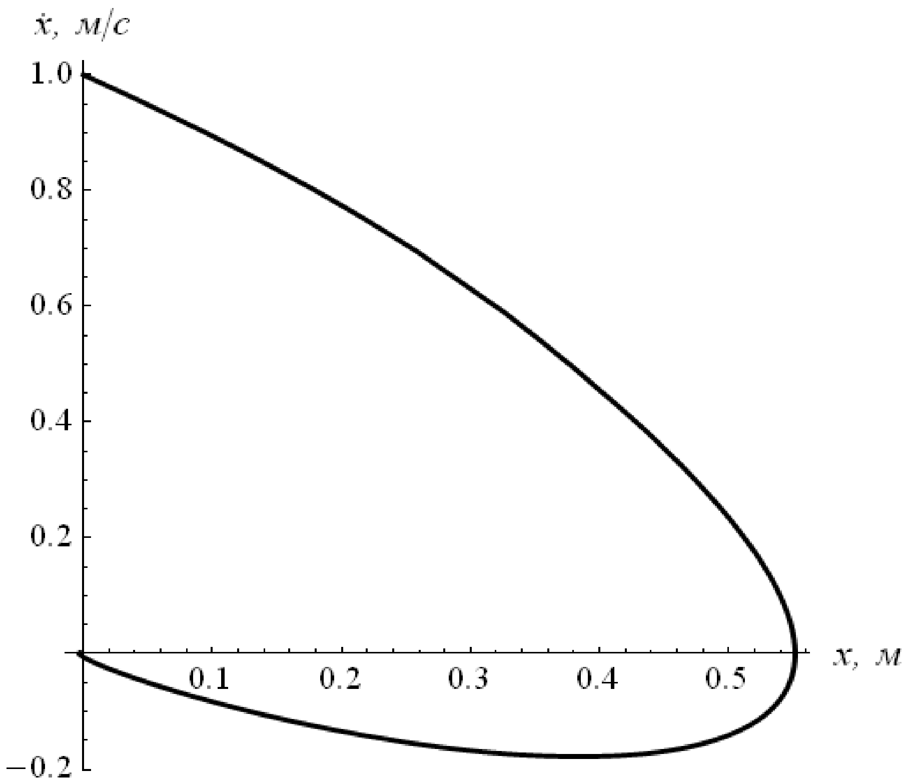

Fig. 1. Phase portrait of motion of a dynamical system.

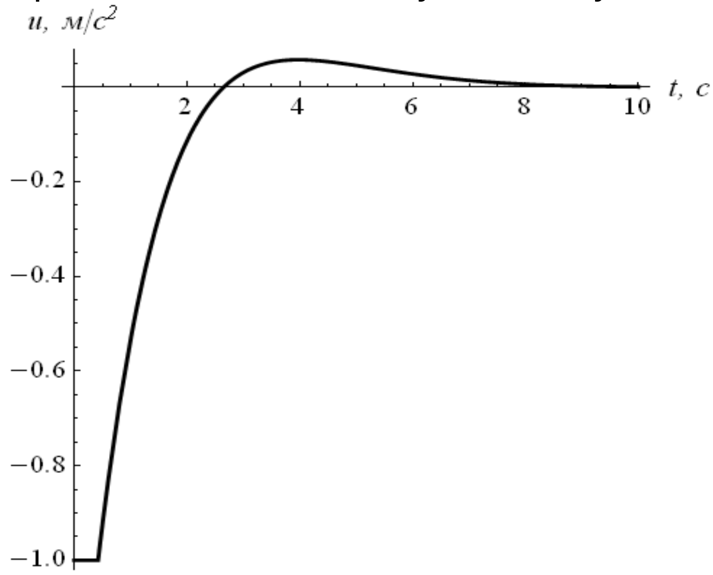

Fig. 2. The schedule features a dynamic optimal control system.

Conclusion. Use of optimal control (dynamic programming and maximum principle) allows to find the optimal control of dynamic system in the form of feedback. Use only one of these methods gives the desired result: maximum principle constitutes a "quality" picture optimal control and dynamic programming - "quantitative" and only a combination of these methods gives the desired result. In this paper established between methods of dynamic programming, the maximum principle and the calculus of variations. These links allow you to combine different stages of optimization problem solving approaches of a method.

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In this article the problem of optimal control decisions dynamycheskoy systemoy, kotoraja opysыvaetsya dyfferentsyalnым equation echoed order. Shown Communications Between yzvestnыmy optimal control methods: varyatsyonnыm yschyslenyem, the maximum principle and Dynamic Programming. Optymalnoe Management Found in video communication in Accounting obratnoy restrictions on the value of control.

Dynamycheskoe programming, the maximum principle, varyatsyonnoe yschyslenye, Peak to management.

The optimal control problem by dynamical systems has been solved in paper. Dynamical systems is describing by differential equation of second order. Connection with known methods of optimal control (variational calculus, maximum principle, dynamical programming) has been showed. Optimal control has been calculated in feedback form with accounting control limitation.

Dynamical programming, maximum principle, variational calculus, control limitation.

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ANALYSIS TECHNOLOGY biodiesel production

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