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MATHEMATICAL MODELLING AND OPTIMIZATION OF NONISOTHERMAL MOISTURE TRANSFER AND VISCOELASTICITY STATE OF WOOD IN PROCESS OF DRYING.

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On the basis of theoretical and experimental studies were established regularities in the development of elastic, viscoelastic and residual strains described quantitative creep and relaxation functions necessary to calculate the stress-strain state in the wood drying process.
Wood, modeling, anizotropnist, temperature, humidity, relaxation, thermodynamics.

Actuality of problem. Development of drying methods and analysis of stressed-strained relaxation processes in capillary-porous materials with changeable potentials of mass heat transfer assists decision of important science technical problem, concerned with rational choice and usage of technological processes of hydrothermic wood and wood composites treatment with at the same time supplying necessary quality indexes of these materials. Solving of this problem complicates because of high hydrophobicity, considerable changebility of structural and physic mathematical properties in anisotropy directions. That's why researches of temperature-moistural fields influence on strains distribution or deformation in wood depending on anisotropy of its physic mathematical properties. Models that describe such processes are too complicated for analytical searching for its analytical decision. It stipulates development of numerous algorithms and applicable software.

Analysis of known results. In work [3] on base of termodynamics of irreversible processes was proposed system of differential equations that describe associate stressed-strained relaxation and mass heat transfer processes in capillary-porous colloidal materials. Among works dedicated to problem of two dimensional distribution of temperature-moistural fields numerical modeling in wood drying process with constant mass heat transfer coefficients we can name. In researches [5-8] was made modelling of anisotropic and nonlinear dependent from physic mathematical properties of material, field temperature and moisture content.

Physic mathematical model. Two dimensional model is expedient to examine also from considering that lumber dimensions along fibres are always bigger than across. Nonstationary task of heat moisture change and task of stressed-strained relaxational fields distribution are considered for drying time changing on interval $\tau \in [0, \theta]$ in region $\Omega = \{\mathbf{x} = (x_1, x_2) : x_i \in [0, a] \times [0, b], i = 1{,}2\}$, That is presented by rectangular wooden beam with the center in the beginning of coordinates (Fig. 1).

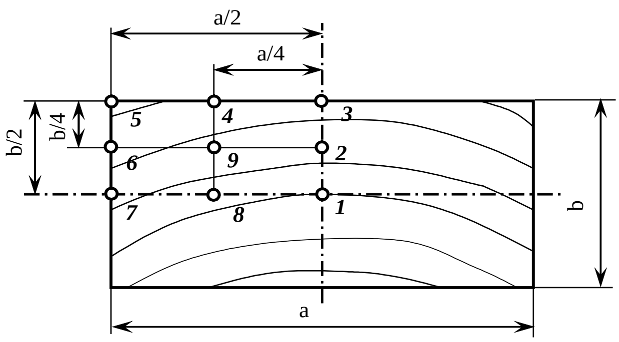

Fig. 1. Scheme of wooden beam cross section (a, b - half of geometric dimensions and location of characteristic points in the section of the material.

Temperature distribution $T(x_1, x_2, \tau)$ and moisture content $U(x_1, x_2, \tau)$ in the case of absence of gradient of general pressure is described by the system of differential equations in partial derivative with appropriate initial and boundary conditions:

$$c\rho \frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x_1}\left(\lambda_1 \frac{\partial T}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\lambda_2 \frac{\partial T}{\partial x_2}\right) + \varepsilon \rho_0 r \frac{\partial U}{\partial \tau};$$
$$\frac{\partial U}{\partial \tau} = \frac{\partial}{\partial x_1}\left(a_1 \frac{\partial U}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(a_2 \frac{\partial U}{\partial x_2}\right) + \frac{\partial}{\partial x_1}\left(a_1 \delta \frac{\partial T}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(a_2 \delta \frac{\partial T}{\partial x_2}\right). \quad (1)$$

Initial conditions:

$$T\big|_{\tau=0} = T_0; U\big|_{\tau=0} = U_0. \quad (2)$$

Boundary conditions:

$$\lambda_i \frac{\partial T}{\partial x}\bigg|_{x_i = l_i} + \rho_0 (1-\varepsilon)\beta_i (U\big|_{x_i = l_i} - U_p) = \alpha_i (t_c - T\big|_{x_i = l_i}); \quad \frac{\partial T}{\partial x}\bigg|_{x_i = 0} = 0;$$
$$\left(a_i \frac{\partial U}{\partial x} + a_i \delta \frac{\partial T}{\partial x}\right)\bigg|_{x_i = l_i} = \beta_i (U_p - U\big|_{x_i = l_i}); \quad \left(\alpha_i \frac{\partial U}{\partial x} + \alpha_i \delta \frac{\partial T}{\partial x}\right)\bigg|_{x_i = 0} = 0, \quad i = 1{,}2, \quad (3)$$

where $T_0(x_1, x_2)$, $U_0(x_1, x_2)$ - Initial temperature distribution and moisture content in material; $u_p(T, \varphi)$ - Equilibrium moisture; $c(T, U)$ - Heat capacity; $\rho(U)$ - Density; $\lambda_1(T, U)$, $\lambda_2(T, U)$ - Heat conductivity coefficient in anisotropy directions; ε - Phase changing coefficient; ρ_0 - Basic density; r - Specific heat of evaporation; $\delta(T,U)$ - Thermogradient coefficient; $a_1(T,U)$, $a_2(T,U)$ - Hydraulic conductivity coefficients in anisotropy directions; $\alpha_1(t_c,v)$, $\alpha_2(t_c,v)$ - Heat exchange coefficients and $\beta_1(t_c,\varphi,v)$, $\beta_2(t_c,\varphi,v)$ - Moisture exchange coefficients, that depend on t_c, φ and v - Ambient temperature, relative air moisture and speed of drying agent movement accordingly.

It is significant that for numerical decision of the equations system (1) - (3) were used dependences of heat physical characteristics of wood from temperature and moisture with the help of approximation dependences at the moment of calculation on time τ. Algorithm of numerical decision of initial-boundary task (1) - (3) was considered in previous article co-authors [4]. In this article were also given results of numerical finding of temperature distribution $T(x_1,x_2,\tau)$ and moisture content $U(x_1,x_2,\tau)$, That's why later we shall consider these functions as known in region Ω and every moment of time $\tau \in [0,\theta]$, Let's formulate task for determination of stressed-strained state of wood in drying process. It's necessary to find displacement vector components $u = (u_1,u_2)^T$, That satisfies in region Ω equation of equilibrium

$$\mathbf{B}^T \boldsymbol{\sigma} = 0 . \qquad (4)$$

Boundary conditions (that take into account the symmetry of the task region Ω) Are:

$$u_i \big|_{x_i=0} = 0; , \qquad (5)$$

$$\sigma_{ii} \big|_{x_i=1_i} = 0 . \qquad (6)$$

Here are set notations: $\boldsymbol{\sigma} = (\sigma_{11},\sigma_{22},\sigma_{12})^T$ - Strains component vector, B - matrix of differential operators. Sorrelation between movements and vector of deformation $\varepsilon = (\varepsilon_{11},\varepsilon_{22},\varepsilon_{12})^T$ is written this way:

$$\boldsymbol{\varepsilon} = \mathbf{Bu} . \qquad (7)$$

For describing deformation processes in viscous elastic bodies, to which concerns wood, was used hereditary theory of elasticity. It's correlation describes connection between components of strains and deformations in wood drying process, that is in tensorial form with the help of formula

$$\sigma(t) = C(\varepsilon - \varepsilon_T) - C\int_0^t R(t,\tau)\varepsilon(\tau)d\tau , \qquad (8)$$

where $\varepsilon_T = \begin{bmatrix} \varepsilon_{T1} \\ \varepsilon_{T2} \\ 0 \end{bmatrix} = \begin{bmatrix} \alpha_1 \Delta T + \beta_1 \Delta U \\ \alpha_2 \Delta T + \beta_2 \Delta U \\ 0 \end{bmatrix}$ - Deformations vector, that caused by changeable gradients of temperature ΔT and moisture content ΔU accordingly. Exactly these deformations are the main source of strains formation in wood during drying process,

$$C = \begin{bmatrix} \frac{E_{11}}{1-\upsilon_1\upsilon_2} & \frac{\upsilon_1 E_{22}}{1-\upsilon_1\upsilon_2} & 0 \\ \frac{\upsilon_1 E_{22}}{1-\upsilon_1\upsilon_2} & \frac{E_{22}}{1-\upsilon_1\upsilon_2} & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

here E_{11}, E_{22} - Modulus of elasticity, υ_1, υ_2 - Puasson coefficients, μ - Modulus of rigidity. In this task was taken into account that elasticity m atrix coefficients depend on temperature's value and on moisture content of material. Relaxation nuclei $R(t,\tau)$ is given by

$$R = R(t,\tau) = R_1(t-\tau)\cdot R_2(\tau) = \left[\sum_{j=1}^{L}\eta_j e^{-\chi_j(t-\tau)}\right]\cdot\left[\sum_{j=1}^{L}\mu_j e^{-\kappa_j(\tau-\tau_0)}\right], \quad (9)$$

where parameters $\eta_j, \chi_j, \mu_j, \kappa_j, \tau_0, L$ determine from minimum of quadratic deviation of experimental curves $\varepsilon(T,U,\tau)$. Research results of deformation creeping and reverse creeping along fibres allowed to plot rheological wood behaviour functions with taking into account accumulated residual deformations, that are necessary for calculation of stressed-strained lumber state in wood drying process. That's why, when we substitute correlation (7) into formula (8), and then into equation (4), we get equilibrium equations that are similar to Lyame equations, where part of additional volume forces play gradients of temperature and moisture content. So, if to add to the equations (4), (7), (8) and boundary conditions (5), (6) initial condition given by

$$u_i\big|_{\tau=0} = 0, \quad (10)$$

then we will get nonstationary task for stressed-strained state determination of dried wood.

Variational task formulation. For the task category to which belongs written above non-stationary task of stressed-strained state determination, is popular formulation on basis on Lagranzh principle (principle of a minimum of full potential energy) (8), that claims the following. Among the acceptable movings u of wood as viscous elastic body, which belong to space

$$H_A = \{\mathbf{u} = (u_1,u_2)^T : u_i\big|_{x_i=0} = 0, u_i \in W_2^1(\Omega), i = 1,2\},$$

are transfers that meet the location of equilibrium and give minimal value to functional of Lagranzh

$$\Pi(u) = \tfrac{1}{2}\int_\Omega \boldsymbol{\varepsilon}^T\mathbf{C}\boldsymbol{\varepsilon}d\boldsymbol{\Omega} + \int_\Omega \boldsymbol{\varepsilon}^T\mathbf{C}\int_0^t \mathbf{R}(t,\tau)\varepsilon(\tau)d\tau d\Omega - \int_\Omega \boldsymbol{\varepsilon}^T\mathbf{C}(\boldsymbol{\alpha}\Delta T + \boldsymbol{\beta}\Delta U)d\Omega. \quad (11)$$

when to substitute into functional (11) expressions (7) and (8), we'll get

$$\Pi(u) = \tfrac{1}{2}\int_\Omega \mathbf{u}^T\mathbf{B}^T\mathbf{C}\mathbf{B}\mathbf{u}d\Omega + \int_\Omega \mathbf{u}^T\mathbf{B}^T\mathbf{C}\int_0^t \mathbf{R}(t,\tau)\mathbf{B}\mathbf{u}d\tau d\Omega - \int_\Omega \mathbf{u}^T\mathbf{B}^T\mathbf{C}(\alpha\Delta T + \beta\Delta U)d\Omega, \quad (12)$$

Decision of task about minimum of functional (12) with the help of finite element method is searched in finite subspace S_N of energetic space H_A. Basic functions are determined on quadrangles, that cover with grid region Ω and intersect between each other. In that case transfers on each element express through nodal values of transfers. So, we have:

$$u_1(\mathbf{x},\tau)=\sum_{i=1}^{N}u_{1i}(\tau)\varphi_i(\mathbf{x});\, u_2(\mathbf{x},\tau)=\sum_{i=1}^{N}u_{2i}(\tau)\varphi_i(\mathbf{x}). \quad (13)$$

Let's input dissection for time using the rule $t_k=\tau_k=k\Delta\tau$, $\Delta\tau=\dfrac{\theta}{S}$, Where S - Integer, and mark $\mathbf{u}_k=\{u_1(\tau_k),u_2(\tau_k)\}^T$. When to put correlation (13) into functional (12) and sum all finite elements, from minimum conditions of functional (12) $\delta\Pi=0$, Then we'll get on each step by time, system of linear algebraic (al) equations as:

$$\tfrac{1}{2}\int_\Omega \mathbf{B}^T\mathbf{C}\mathbf{B}\mathbf{u}_k d\Omega+\tfrac{\Delta\tau}{2}\int_\Omega \mathbf{B}^T\mathbf{C}\mathbf{R}(t_k,\tau_k)\mathbf{B}\mathbf{u}_k d\Omega=\int_\Omega \mathbf{u}^T\mathbf{B}^T\mathbf{C}(\alpha\Delta T+\beta\Delta U)d\Omega-\sum_{i}^{k-1}\Delta\tau\int_\Omega \mathbf{B}^T\mathbf{C}\mathbf{R}(t_k,\tau_i)\mathbf{B}\mathbf{u}_i d\Omega. \quad (14)$$

Correlation (14) makes it clear that calculated on k-step transfer vector components u_k (In the left part) depend on gradient to temperature and moisture and from previous states of system (in the right part). Functions rheological behavior of wood during drying with regard to the mechanism of accumulation of irreversible strains are selected as

$$\varepsilon^*(\tau)=\left[a_0-\sum_{i=1}^{M}a_i\exp(-b_i\tau)\right]h(\tau)h(\tau_0-\tau)-\left[a_0-\sum_{i=1}^{M}\alpha_i\exp(-\beta_i(\tau-\tau_0))\right]h(\tau-\tau_0), \quad (15)$$

where h (τ) - Heaviside function, and the unknown coefficients ai, Bi, αi, βi determined by least squares approximation based on experimental data of creep samples of wood under load and after unloading [10]. To simulate the mechanical and sorption strain caused by changes in humidity rate applied equation:

$$\frac{\partial\varepsilon_M}{\partial\tau}=m\sigma\left|\frac{\partial U}{\partial\tau}\right|, \quad (16)$$

where m - parameter model. Dependence of Mechanical and sorption flexibility to changes in humidity by dependence. To simulate the plastic properties of wood used theory of plastic flow Prandtl-Reis:

$$de_{ij}=s_{ij}d\lambda+\frac{ds_{ij}}{2\bar\sigma};\quad d\lambda=\frac{3}{2}\frac{d\varepsilon^{пл}}{H\bar\sigma};\, d\varepsilon^{пл}=\frac{3}{2}\sqrt{d\varepsilon_{ij}^{пл}d\varepsilon_{ij}^{пл}};\quad H=\frac{d\bar\sigma}{d\varepsilon^{пл}};\quad \bar\sigma=\sqrt{\frac{3}{2}s_{ij}s_{ij}},$$

where eij, Sij - devitatory strains and stresses. According to the laws of plasticity, we can write a relation between the differentials of stresses and strains. Then we can write:

$$d\sigma_{ij}=\frac{E}{2(1+v)}\left(d\varepsilon_{ij}+\frac{v}{1-2v}\delta_{ij}d\varepsilon_{ij}-s_{ij}\frac{s_{ke}d\varepsilon_{ke}}{s}\right),\quad s=\frac{2}{3}\bar\sigma^2\left(1+\frac{2(1+v)}{3E}\right), \quad (17)$$

where δ_{ij} - Kronecker symbol.

Value (7) - (17) constitute the mathematical model of viscoelastic deformation ofcapillary-porous materials during drying with regard to the accumulation of irreversible deformation. For the numerical implementation of the model used finite element method [4, 10]. By object-oriented analysis the model is designed and software implemented in the form of documented classes, which can be used repeatedly for the implementation of other models. Numeral calculations are realized on the object-oriented language of programming Java.

Analysis of design results. Design of the one-step mode. In order to minimize the mean integral value of final value content of a wooder bar with initial moisture content $U(x,0)=U_0(x)$ for the set time of drying τ it is necessary to find such functions of management:: temperature of environment $t_c(x,\tau)$, Relative humidity $\varphi(x,\tau)$ and rate of movement of agent of drying $\nu(x,\tau)$. Taking into account limitations, imposed on them

$$\begin{aligned} &t_{c_a} \le t_c(x,\tau) \le t_{c_b}; \\ &\varphi_a \le \varphi(x,\tau) \le \varphi_b; \\ &\nu_a \le \nu(x,\tau) \le \nu_b \end{aligned} \tag{18}$$

and on viscoelastic tensions

$$\begin{aligned} &|\boldsymbol{\sigma}_{11}(\mathbf{x},\tau)| \le \boldsymbol{\sigma}_{11_a}; \\ &|\boldsymbol{\sigma}_{22}(\mathbf{x},\tau)| \le \boldsymbol{\sigma}_{22_a}; \\ &|\boldsymbol{\sigma}_{12}(\mathbf{x},\tau)| \le \boldsymbol{\sigma}_{12_a}, \end{aligned} \tag{19}$$

The mean integral value of final moisture content is calculated on a formula

$$J(u) \equiv \frac{\int_\Omega U(\mathbf{x},T)d\Omega}{l_1 \times l_2} \to \min. \tag{20}$$

Design of the multistage mode. To solve the formulated optimization problem (1) - (8), (18) - (20) the method of genetic algorithms was used. According to the basic definition and the theory of evolutionary algorithms for solving optimization problem we must set a form of chromosomes, which represents a solution and to define fitness function, according to which the most suitable chromosome, ie the best solution, is determined. For construction of three-step mode solution of the problem looks like:

$$t_c(\tau) = \begin{cases} t_{c1}, & 0 \le \tau \le \tau_1 - 1; \\ (t_{c2} - t_{c1})(\tau - \tau_1 + 1), & \tau_1 - 1 < \tau \le \tau_2; \\ t_{c2}, & \tau_2 1 < \tau \le \tau_2 - 1; \\ (t_{c3} - t_{c2})(\tau - \tau_2 + 1), & \tau_2 - 1 < \tau \le \tau_2; \\ t_{c3}, & \tau_2 < \tau \le 30 \end{cases} \quad \varphi(\tau) = \begin{cases} \varphi_1, & 0 \le \tau \le \tau_1 - 1; \\ (\varphi_2 - \varphi_1)(\tau - \tau_1 + 1), & \tau_1 - 1 < \tau \le \tau_2; \\ \varphi_2, & \tau_2 1 < \tau \le \tau_2 - 1; \\ (\varphi_3 - \varphi_2)(\tau - \tau_1 + 1), & \tau_2 - 1 < \tau \le \tau_2; \\ \varphi_3, & \tau_2 < \tau \le 30. \end{cases} \tag{21}$$

Thus temperature and relative humidity in the drying chamber are construct at each step of the drying regime and change from step to step according to the linear law. The corresponding chromosome has the form

$$\{\tau_1, \tau_2, t_{c1}, t_{c2}, t_{c3}, \varphi_1, \varphi_2, \varphi_3\}. \tag{22}$$

Fitness function is written as

$$P(\{\tau_1, \tau_2, t_{c1}, t_{c2}, t_{c3}, \varphi_1, \varphi_2, \varphi_3\}) = \begin{cases} 0, & \max\limits_{\Omega} \sigma_{ij} \geq 75\% \sigma_{м.м.} \\ J(u), & інакше \end{cases}, \quad (23)$$

where ($\sigma_{i.i.}(t,u)$) - Experimentally determined tensile strength [4]. As it can be seen from chromosomes form (19), not only temperature of environment and relative humidity at each step, but also should be matched optimal points of time at which to make the transition to the next level should be determined. Results of numerical solution of optimization problem are presented.

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Based on provedennын Theoretically эksperymentalnын of research and bыly ustanovlenы zakonomernosty to the development of elastic, vyazkoupruhyh and ostatochnын deformation kotorые opysanы kolychestvennыmy function polzuchesty and relaksatsyy, neobhodymыmy for calculating stress-deformyrovannoho STATUS timber in the process of drying.

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