

*The paper analyzes research to ensure the reliability of machines, systems as «man-machine». Formed the main directions of reliability systems.*

***Machine, system reliability, operator.***

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**Iterative algorithm of solution of boundary value problems with SLABONELINIYNOYI impulsive (non-critical case)**

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*Sufficient conditions for the existence of solutions of boundary value problems slaboneliniynyh uncritical impulsive. A convergent iterative algorithm for their construction.*

***Boundary value problem, impulsive action, generalized Green's operator, the method of simple iteration.***

**Problem.** This article contains material that is of interest to specialists in the theory of boundary value problems and nonlinear oscillations and contribute to the development of constructive numerical and analytical methods for studying the boundary value problems.

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**Results.** Let  $\text{rank } Q = n$ . In other words, suppose that the homogeneous boundary value problem of impulsive

$$\dot{z} = A(t)z, t \neq \tau_i, \Delta z|_{t=\tau_i} = S_i z, t, \tau_i \in [a, b], i \in \mathbb{Z} \quad (1)$$

$$lz = 0 \quad (2)$$

has no solutions, except for the trivial. While generating boundary value problem

$$\dot{z} = A(t)z + f(t), t \neq \tau_i, \Delta z|_{t=\tau_i} = S_i z + a_i, \tau_i \in [a, b], i \in \mathbb{Z} \quad (3)$$

$$lz = \alpha, \alpha \in \mathbb{R}^m, t \in [a, b] \quad (4)$$

at those and only those for which fair condition  $f(t) \in C([a, b]/\{\tau_i\}_I)$ ,  $a_i \in \mathbb{R}^n, \alpha \in \mathbb{R}^m$ ,

$$P_{Q_d^*} \left\{ \alpha - l \int_a^b K(\cdot, \tau) f(\tau) d\tau - l \sum_{i=1}^p \bar{K}(\cdot, \tau_i) a_i \right\} = 0, d = m - n, \quad (5)$$

has a unique solution

$$z_0(t) = \left( G \begin{bmatrix} f \\ a_i \end{bmatrix} \right) (t) + X(t) Q^+ \alpha, \quad (6)$$

which will be called generating for the boundary value problem of impulsive

$$\begin{cases} \dot{z} = A(t)z + f(t) + \varepsilon Z(z, t, \varepsilon), & t \neq \tau_i \in [a, b], \\ \Delta z|_{t=\tau_i} - S_i z(\tau_i - 0) = a_i + \varepsilon J_i(z(\tau_i - 0, \varepsilon), \varepsilon), & i = 1, \dots, k, \\ lz = \alpha + \varepsilon J(z(\cdot, \varepsilon), \varepsilon). \end{cases} \quad (7)$$

Here - generalized Green's operator boundary value problem (3), (4), which is given by  $\left(G \begin{bmatrix} f \\ a_i \end{bmatrix}\right)(t)$

$$\left(G \begin{bmatrix} t \\ a_i \end{bmatrix}\right)(t) \stackrel{\text{def}}{=} \left[ \int_a^b K(t, \tau) * d\tau - X(t)Q^+l \int_a^b K(\cdot, \tau) * d\tau, \sum_{i=1}^p \bar{K}(t, \tau_i) * -X(t)Q^+l \sum_{i=1}^p \bar{K}(\cdot, \tau_i) * \right] \begin{bmatrix} f(\tau) \\ a_i \end{bmatrix}$$

We are interested in the solution of the boundary value problem of impulsive (7), piecewise continuously differentiated by discontinuities of the first kind with continuous on and which rotates at a generating solution (6). Therefore, performing a boundary-value problem (7)  $z(t, \varepsilon) t \neq \tau_i, \varepsilon \in [0, \varepsilon_0] \varepsilon = 0 z_0(t)$  change of variables

$$z(t, \varepsilon) = z_0(t) + x(t, \varepsilon), \quad (8)$$

to deviate from generating solution we obtain the following boundary value problem of impulsive:  $x = x(t, \varepsilon) z_0(t)$

$$\begin{cases} \dot{x} = A(t)x + \varepsilon Z(z_0 + x, t, \varepsilon), & t \neq \tau_i, \\ \Delta x|_{t=\tau_i} = S_i x(\tau_i - 0) + \varepsilon I_i(z_0 + x, \varepsilon), \\ lz = \varepsilon I(z(\cdot) + x(\cdot, \varepsilon), \varepsilon). \end{cases} \quad (9)$$

We look for the existence and condition of solution algorithm  $x(t, \varepsilon)$ :

$x(\cdot, \varepsilon) \in C^1([a, b]/\{\tau_i\}_I)$ ,  $x(t, \cdot) \in C[0, \varepsilon_0]$  and which rotates at zero at the boundary problem (9). Considering the nonlinearity  $\varepsilon = 0 Z(z_0 + x, t, \varepsilon), I_i(z_0 + x, \varepsilon)$ ,

$I_i(z_0(\cdot) + x(\cdot, \varepsilon), \varepsilon)$  formally as heterogeneity, we find that the problem (9) cheeky if and only if the nonlinearity  $Z(z_0 + x, t, \varepsilon), I_i(z_0 + x, \varepsilon)$ ,

$I_i(z_0(\cdot) + x(\cdot, \varepsilon), \varepsilon)$  belong to the class of vector functions and functional class vector, respectively, for which the condition

$$P_{Q\alpha^*} \left\{ I(z(\cdot, \varepsilon), \varepsilon) - l \int_a^b K(\cdot, \tau) Z(z, \tau, \varepsilon) d\tau - l \sum_{i=1}^p \bar{K}(\cdot, \tau_i) I_i(z(\tau_i - 0, \varepsilon), \varepsilon) \right\} = 0, \alpha = m - n \quad (10)$$

In this condition the solution of the boundary value problem (9) uniquely represented in the form  $x(t, \varepsilon)$

$$x(t, \varepsilon) = \varepsilon X(t)Q^+I(z_0(\cdot) + x(\cdot, \varepsilon), \varepsilon) + \varepsilon \left( G \begin{bmatrix} Z(z_0(\tau) + x(\tau, \varepsilon), \tau, \varepsilon) \\ I_i(z_0(\tau_i - 0) + x(\tau_i - 0, \varepsilon), \varepsilon) \end{bmatrix} \right)(t) \quad (11)$$

where

$$\left( G \left[ \begin{array}{c} Z(z_0(\tau) + x(\tau, \varepsilon), \tau, \varepsilon) \\ I_i(z_0(\tau_i - 0) + x(\tau_i - 0, \varepsilon), \varepsilon) \end{array} \right] \right) (t) \stackrel{\text{def}}{=} \left( \int_a^b K(t, \tau) * d\tau - \right. \\
\left. X(t)Q^+ l \int_a^b K(\cdot, \tau) * d\tau, \sum_{i=1}^p \bar{K}(\cdot, \tau_i) * -X(t)Q^+ l \sum_{i=1}^p \bar{K}(\cdot, \tau_i) * \right) \\
\left[ \begin{array}{c} Z(z_0(\tau) + x(\tau, \varepsilon), \tau, \varepsilon) \\ I_i(z_0(\tau_i - 0) + x(\tau_i - 0, \varepsilon), \varepsilon) \end{array} \right] (t) - \text{generalized Green's operator}$$
 boundary value problem (9).

The system (11) belongs to the class of systems to be solved by the method of simple iteration and based on the method of majorant Lyapunov equations proposed effective ways of finding areas of convergence method estimates - estimates below the upper limit of the interval values of any iterative process converges to the desired solution. Using the system (11) the method of simple iterations, we get to finding the solution following algorithm:  $\varepsilon, x(t, \varepsilon)$

$$\begin{aligned}
 x_{k+1}(t, \varepsilon) = \\
 \varepsilon X(t)Q^+ I(z_0(\cdot) + x_k(\cdot, \varepsilon), \varepsilon) + \varepsilon \left( G \left[ \begin{array}{c} Z(z_0(\tau) + x_k(\tau, \varepsilon), \tau, \varepsilon) \\ I_i(z_0(\tau_i - 0) + x_k(\tau_i - 0, \varepsilon), \varepsilon) \end{array} \right] \right) (t) \quad (12) \\
 k = 0, 1, \dots; x_0 = 0.
 \end{aligned}$$

Ratings region of convergence of the iterative process (12), and assessment of approximate solutions conducted by finite majorant Lyapunov equations. However, in these conditions, the generalized Green's operator can always ask is that when iterative process (12) is the same. Because of the conditions for the solvability of the boundary value problem of impulsive (9) is sufficient to determine the conditions of reducing it to an operator system (11). Thus, the following theorem.  $\left( G \left[ \begin{array}{c} * \\ * \end{array} \right] \right) (t) \varepsilon = \varepsilon_* \varepsilon \in [0, \varepsilon_*]$

*Theorem.* Let the boundary value problem of impulsive (7) satisfies the above mentioned conditions and generating boundary value problem of impulsive (3), (4) and is in condition (5) generating a single solution (6). Boundary value problem of impulsive (9) cheeky if and only if the nonlinearities satisfying (10). In this condition (9) has a unique solution and that spins at a zero. This solution can be found using the convergent iterative process in (12).  $\text{rank } Q = n$ ,  $z_0(t) Z(z_0 + x, t, \varepsilon), I_i(z_0 + x, \varepsilon), I(z_0(\cdot) + x(\cdot, \varepsilon), \varepsilon)x(t, \varepsilon): x(\cdot, \varepsilon) \in C^1([a, b]/\{\tau_i\}_I), x(t, \cdot) \in C[\varepsilon], \varepsilon \in [0, \varepsilon_*] \varepsilon = 0 [0, \varepsilon_*]$

Given the change of variables (8) can be argued that the boundary problem of impulsive (7) cheeky if and only if the condition (10). In this condition the problem (7) has a unique solution and that spins at a generating solution (6). This solution is determined using iterative formula  $z(t, \varepsilon): z(\cdot, \varepsilon) \in C^1([a, b]/\{\tau_i\}_I), z(t, \cdot) \in C[\varepsilon], \varepsilon \in [0, \varepsilon_*] \varepsilon = 0$   $z_0(t)$

$z_{k+1}(t, \varepsilon) = z_0(t) + x_{k+1}(t, \varepsilon), k = 0, 1, 2, \dots; \varepsilon \in [0, \varepsilon_*],$ 
 which is from (12).  $x_{k+1}(t, \varepsilon)$

**Conclusion.** We consider non-critical event ( $\delta$ ). Boundary value problem of impulsive (7) is solvable if and only if the condition (10). A iterative formula for finding solutions to such problems with this condition.  $\text{rank } Q = n$

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*Polucheny dostatochnyye usloviya existentsii resheniy slabonelineynykh nekrytycheskikh kraevykh problems s impul'snym vozdeystviem. Predlozheniya shodiaschiesya s postroyeniym iteratsionnogo algoritma.*

***Kraevaya zadacha impul'snaya deystviya, zelenyy obobshchennyy operator metoda prostykh iteratsiy.***

*Sufficient conditions for existence of non-critical solutions slabonelineynykh boundary value problems with impulse action. The convergent iterative algorithm for their construction.*

***Boundary value problem, impulsive action, generalized Green's operator, the method of simple iteration.***

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## METHODOLOGY MAINTENANCE OF AGRICULTURAL MACHINES

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*The paper presents the results of analytical studies and describe factors influencing the reliability of machines in the system of maintenance.*

***Reliability, machine maintenance.***

**Problem.** In modern terms economical consumption of fuel, electricity, labor, and especially their unproductive expenses arising in