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UNDIFFERENTIATED PROBLEMS OF OPTIMIZATION THE SENSITIVITY OF DYNAMICAL SYSTEMS

L.A.PANTALIYENKO, Candidate of Physical and Mathematical Sciences

Problems of designing and managing various complex systems often solved by solving optimization problems using methods parameterization [1]. In fact, this approach is to build a parametric model that accurately reflects the properties of the real system. This original problem is transformed into a finite control problem of nonlinear programming regarding parameters that characterize transitional process, function and object management.

In practice, because of various physical and technological reasons, the value of real parameters always different from the settlement (the best), which changes the characteristics and quality of the system. With significant deviations characteristics of the object is generally optimized system unworkable. In this connection the problem of designing Relevance become insensitive to changes in system parameters. Unlike productions tasks with limited sensitivity and guaranteed, the original problem is formalized optimization problem to look sensitivity functions. With this approach, we come to the problem undifferentiated optimization, whose solution is based on methods control theory.

The aim of research – development of numerical methods for solving optimization problems of sensitivity based on the principle of maximum Pontryagin undifferentiated optimization algorithms.

Materials and methods research. The paper used mathematical methods of control theory, sensitivity and undifferentiated optimization.

Consider a nonlinear system of differential equations:

$$\frac{dx}{dt} = f(x,t,\alpha), \quad f(0,t,0) \equiv 0, \quad t \in [t_0, T] \tag{1}$$

with initial conditions of a given compact set M_0.

Solution trajectory optimization problem, for example

$$\min_{\alpha \in G_\alpha} \max_{x_0 \in M_0} \max_{t \in [t_0, T]} \Phi(x(t, x_0, \alpha))$$

ensures optimality system (1) only if the equality $\alpha = \alpha^{(0)}$. Therefore, to account for deviations of parameters from their design values for real appropriate to minimize the maximum sensitivity of system to parameter changes:

$$\min_{\alpha \in G_\alpha} \max_{t; k; x_0 \in M_0} \Phi\left(u^{(k)}(t, \alpha)\right), \tag{2}$$

where $\Phi(.)$ − convex function.

Explore undifferentiated sensitivity optimization problem (2) on the trajectories of differential equations sensitivity [2,3]

$$\frac{du^{(i)}(t, \alpha)}{dt} = \frac{\partial f(\bar{x}, t, \bar{\alpha})}{\partial x} u^{(i)}(t, \alpha) + + \frac{\partial f(\bar{x}, t, \bar{\alpha})}{\partial \alpha_i}, \tag{3}$$

Here $\bar{\alpha}$, $\bar{x} = x(t, \bar{\alpha})$ − the calculated parameters vectors and trajectories of system (1) respectively; $u^{(i)}(t, \alpha) = \dfrac{\partial x(t, \alpha)}{\partial \alpha_i}$, $i = 1, 2, ..., m$ − n- dimensional vectors sensitivity function of the first order.

If $\alpha^{(0)}$ − the solution to the problem (2) on the trajectories of the equation system sensitivity, then

$$\max_{\alpha \in G_\alpha} \min_{k \in M_k} \min_{x_0 \in \overline{M}_{0,k}} \min_{\tau \in M_{x_0,k}} \int_{t_0}^{t} \psi^*(t, \tau) \left[\frac{\partial \tilde{f}\left(x, t, \alpha^{(0)}, u^{(k)}\right)}{\partial \alpha} + \frac{\partial \tilde{f}\left(x, t, \alpha^{(0)}, u^{(k)}\right)}{\partial x} \cdot \frac{\partial x}{\partial \alpha} \right] \Big|_{x = x\left(t, \alpha^{(0)}\right)} \left(\alpha - \alpha^{(0)}\right) dt \le 0 .$$

Here M_k, $\overline{M}_{0,k}$, $M_{x_0,k}$ − a set of points, which according to maximums are reached, in the ratio (2); $\tilde{f}(.)$ − the right part of the system of differential equations sensitivity; $\psi(t, \tau)$ − the conjugated system.

For numerical solution of optimization problems of type (1), (2) applies iterative method:

$$\alpha^{(i+1)} = P_{G_{\alpha^{(i)}}} \left\{ \alpha^{(i)} - \rho_k G\left(\alpha^{(i)}\right) \right\}, \ i = 0, 1, 2, ..., \tag{4}$$

Then considered a simplified version of undifferentiated optimization problem in the case of fixed initial conditions $x(t_0) = x_0$, ie,

$$I\left(\alpha^{(0)}\right) = \min_{\alpha \in G_\alpha} \max_{t; k} \Phi\left(u^{(k)}(t, \alpha)\right). \tag{5}$$

In addition to the performances of tasks, choice of optimal parameters to ensure normal operation of the system in real conditions, you can make and the criterion of using a compatible type

$$\min_{\alpha \in G_\alpha} \max_{t;k;x_0 \in M_0} \{ \lambda_1 \Phi_1 (x(t, x_0, \alpha)) + \lambda_2 \Phi_2 (u^{(k)}(t, \alpha)) \}.$$

Conclusions

Based necessary optimality conditions and methods nedyferentsiyovnoyi optimization algorithms to minimize the sensitivity of dynamical systems. Separately considered the procedure of calculating the optimal parameters for fixed initial conditions and a compact set.