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THE DESCRIPTION OF LACTATION DYNAMIC OF HOLSTEIN COWS WITH THE USE OF P.WOOD AND PRASAD-SINGH MODELS

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In the article was analyzed the efficiency of lactation models of P.Wood and Prasad-Singh to characterize the dynamics of monthly milk yield of Holstein cows. Established that equation of Prasad-Singh describes the beginning of lactation curve better than the other one and its theoretical curve closely approximates the factual indicators.

Holstein cattle, lactation curve, descriptive modeling, the model of P.Wood, Prasad-Singh model.

Problem definition. The cows' lactation dynamic is an important technological and selection indicator. It is a «biological clock» by which we can judge about the physiological condition of the animal with high probability. The graphic representation of changes in milk yield per lactation is the easiest and the most available method for assessing the character of lactation. However, the graphs define the qualitative aspects of lactation curves. Quantitatively we can assess them by means of numerous indices of the sustainability of lactation [1]. At the same time now the descriptive modeling with using specific mathematical functions is the most advanced methods of estimation of lactation. The positive side of their using is not only the determining on their basis the stability of lactation curve and calculation of intensity of increasing of monthly yields to peak and rate of decrease thereafter, setting the maximum possible of initial milk yield, and some other characteristics depending on the model that used [2].

The analysis of recent research and publications. The interest to assessing of lactation dynamics with using mathematical models began to manifest itself at the beginning of XX century. S.Brody and colleagues proposed the first function in the

20s [3]. Later their number and diversity began to grow rapidly. Presently, there are several dozen of these functions [2].

In Ukraine, the interest in assessing of the dynamics of cows' lactation with the help of mathematical models is growing [4-7]. Nevertheless with all their range of the largest distribution has become model of P.Wood [8]. However, in our previous study [9] was found that this function might generate atypical theoretical curve (continuous decreasing) in cases where peak of productivity is reached relatively early (on average in the second month of lactation).

The purpose of research. In view of the above, we specify the aim of finding the other lactation models that would not have the drawbacks of P.Wood function. We have already proved the adequacy of using the equations of Guo-Swalve [10] and J.Nelder [11], which describe the beginning of lactation dynamics better, forming a typical curve in all cases. At the same time, relatively recently S.Prasad and R.Syngh [12] proposed a lactation model that we will analyze.

Materials and methods of researches. The research was conducted on Holstein cows of breeding plant JSC «Agro-Soyuz» in Dnipropetrovsk region. In the experiment, we used monthly values of yields of 65 breeding animals for the first, second and third lactation.

To correct the empirical data we used the method of linear interpolation of monthly yields for groups of cows [13]. Characteristics of the dynamics of monthly milk yield of cows by building theoretical curves lactations conducted with using the function proposed P.Wood:

$$y_t = at^b e^{-ct} \tag{1}$$

and the equations developed by S.Prasad and R.Syngh:

$$y_t = \frac{a}{e^{bt+\frac{c}{t}}}, \tag{2}$$

where y_t – milk yield received during time unit t (month);

a, b, c – coefficients of models.

By the coefficient of determination (R^2) was evaluated approximation between the factual values and theoretical monthly yields obtained during simulation:

$$R^2 = \left(1 - \frac{\sum (y_o - y_e)^2}{\sum (y_o - y_m)^2}\right) \times 100\% \qquad (3)$$

where y_o, y_e, y_m – factual, theoretical, average monthly milk yield accordingly;

This option allows you to express the degree of similarity of the studied rows of monthly productivity in relative units (%). Parallel with the determination coefficient for the comparison of factual and theoretical lactation dynamics we used the mean absolute and root-mean-squared errors (*MAE*, *RMSE* accordingly):

$$MAE = \frac{\sum |y_o - y_e|}{n}, \quad RMSE = \frac{\sqrt{\sum (y_o - y_e)^2}}{n}. \qquad (4, 5)$$

On the basis of the used models coefficients we have calculated the additional characteristics of the lactation curves, namely the level of peak productivity (y_{peak}) and the time of its occurrence during lactation (t_{peak}) [8, 12].

All calculations are made using application package Statistica 7 and MS Excel 2010.

Results. Comparing the theoretical curves constructed by the equations of P.Wood and Prasad-Singh for first lactation (Fig. 1) was found that smaller deviations from the factual monthly yields typical for the second model (Table. 1).

1. Matching Options of factual yields to theoretical by the models of P.Wood and Prasad-Singh

Lactation	The limits of deviation of theoretical and factual yields, kg	*MAE*, kg	*RMSE*, kg	R^2, %
P.Wood's model				
First	-11,4…12,5	6,43	2,45	98,99
Second	-13,1…26,2	8,99	3,64	99,58
Third	-12,1…12,6	7,38	2,66	99,76
Prasad-Singh's model				
First	-9,4…11,2	4,83	1,87	99,42
Second	-12,6…20,3	6,98	2,87	99,74
Third	-8,8…6,2	3,63	1,53	99,92

The range of deviations of the theoretical curve P.Wood's function is -11.4 ... 12.5 kg, and function of Prasad-Singh – -9.4 ... 11.2 kg.

At the same time common vectors, abnormalities were detected. In particular, for the 1st, 4th, 5th and especially 6th, 7th month both functions overestimate monthly yield, and the third, eighth and more noticeable by the second and ninth – conversely underestimated.

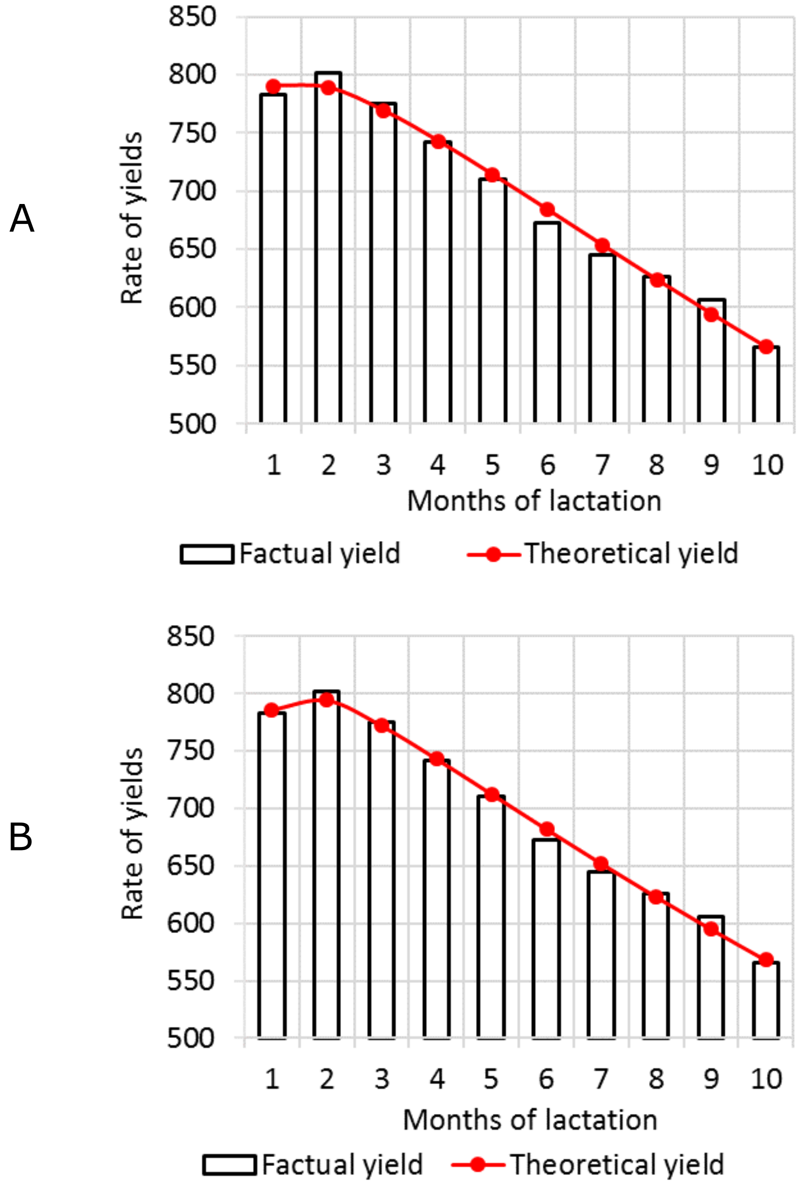


Fig. 1. **The curves of dynamics of monthly yields for the first lactation built by model of P.Wood (A) and Prasad-Singh (B)**

For the second lactation (Fig. 2) the rejection of the theoretical curves from factual data are higher when using models of P.Wood (-13.1 ... 26.2 kg) compared to the function of Prasad Singh (-12.6 ... 20.3 kg).

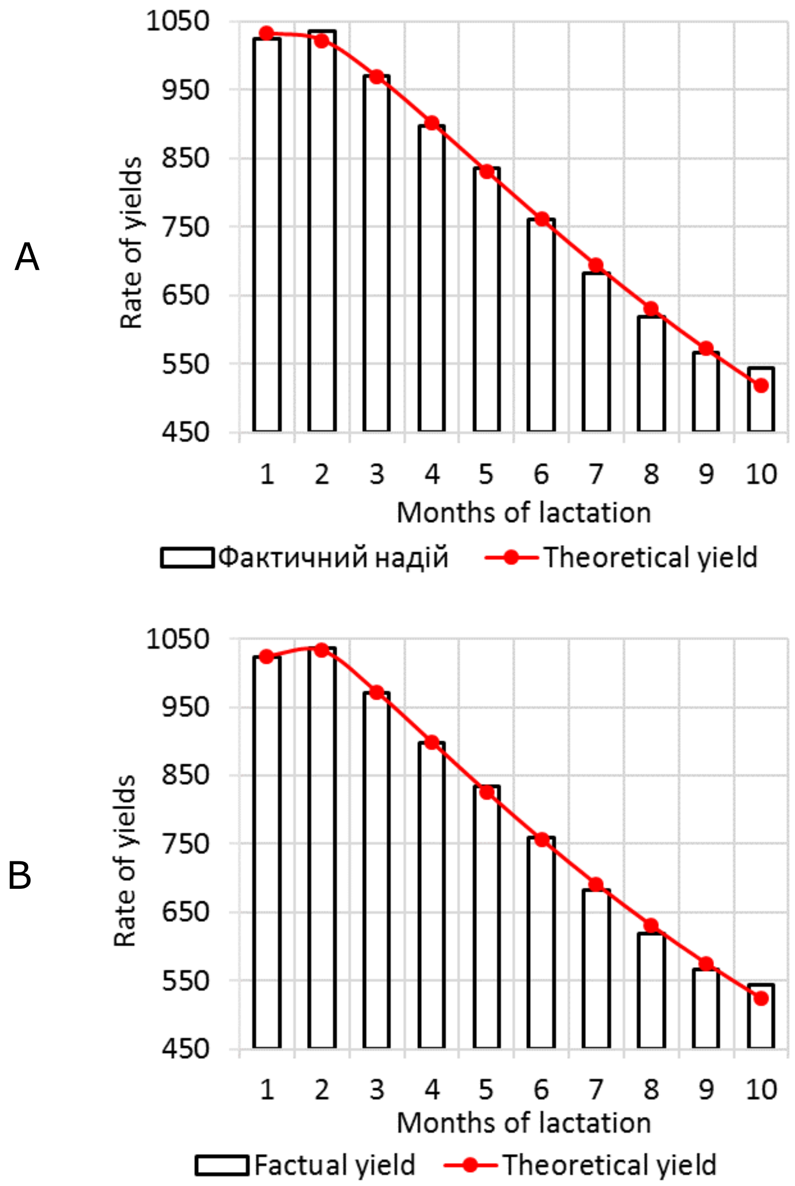
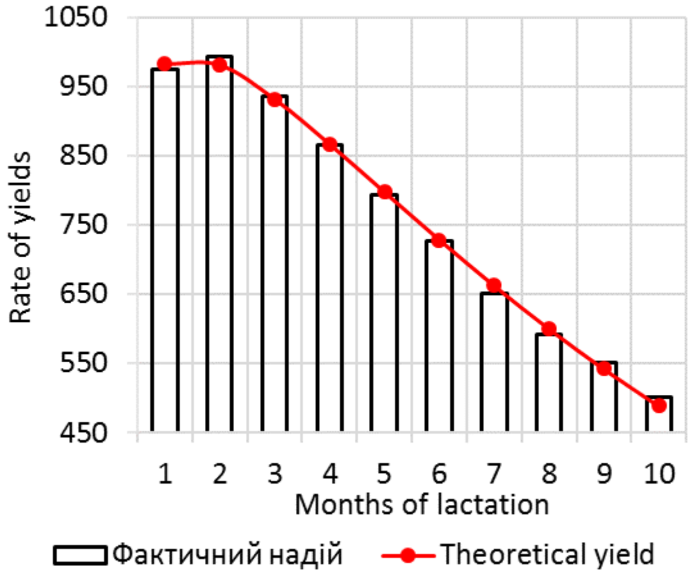


Fig. 2. **Curves of dynamics of monthly yields for the second lactation built by models of P.Wood (A) and Prasad-Singh (B)**

The comparison of theoretical values of lactation dynamics obtained by both models also revealed a number of common features. By the model of P.Wood the most notable overestimation of the theoretical monthly yields was observed for the 1st, 7th and 8th month, and by the equation of Prasad-Singh – for the 7th, 8th and 9th. A significant understatement of factual values of P.Wood's model gives on the 2nd and 10th months, and the function of Prasad-Singh – on the 5th and 10th also, to the same in both cases, the most notable in the last period (more than 20 kg).

In the third lactation period (Fig. 3) theoretical curve formed by the equation of P.Wood deviates from the factual monthly yields in the range of -12.1 ... 12.6 kg, and a curve based at the function of Prasad-Singh has again lower limit deviation – -8.8 ... 6.2 kg.

A



B

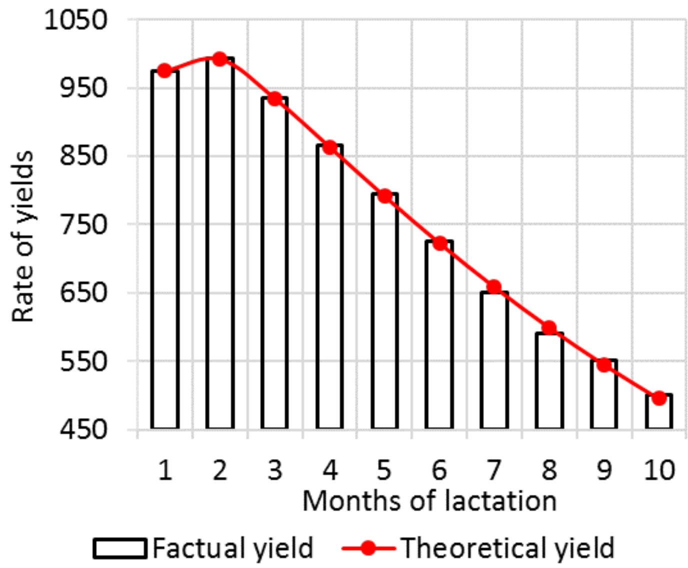


Fig. 3. **Curves of dynamics of monthly yields for the third lactation built by models of P.Wood (A) and Prasad-Singh (B)**

Comparing the distribution of values of the theoretical curves used models it was found that they significantly dominated tandem factual values of the 6th and 7th months, and when using P.Wood's function – even for the 1st one. The opposite tendency noted at the 9th and 10th months, by the equation of P.Wood also for the second.

Comparing the coefficients of determination, which characterize the degree of similarity between the factual and theoretical lactation curves (Table 1), it was found that the curves constructed by Prasad-Singh model greater extent correspond to the empirical parameters, because the parameter R^2 for all the studied lactations is higher. At the same time, along with the coefficient of determination for the comparison of factual and theoretical lactation dynamics we calculated the average absolute and square error (*MAE*, *RMSE*, respectively). Smaller values of these errors are typical for the lactation curves that formed by the function of Prasad-Singh, which indicates about higher compliance with factual values.

Also, we calculated the additional parameters that characterize the theoretical curves of the studied models, such as the level of peak productivity (y_{peak}) and the time of its occurrence (t_{peak}). It was found (Table 2) that the maximum of yield for all lactations for theoretical curves of P.Wood model is less than the same indicator calculated by the coefficients of the equation of Prasad-Singh.

2. Coefficients and parameters of the models of P.Wood and Prasad-Singh

Lactation	The coefficients of models			y_{peak}	t_{peak}
	a	*b*	*c*		
P.Wood's model					
First	837,5	0,080	0,058	793,7	1,39
Second	1158,4	0,153	0,116	1037,1	1,32
Third	1109,5	0,174	0,122	991,7	1,43
Prasad-Singh's model					
First	925,9	0,048	0,116	797,8	1,56
Second	1385,3	0,095	0,206	1047,2	1,47
Third	1357,7	0,099	0,232	1002,7	1,53

At the same time, the beginning of lactation to its maximum level for the curves constructed by the first function for all lactation is less than the second analog equation. In our opinion this is because the model P.Wood to the peak productivity too rapidly increasing theoretical lactation curve, which leads to a decrease t_{peak} and y_{peak}, as well as reducing the approximation between the theoretical and factual values of lactation dynamics.

The conclusions and recommendations for further researches. Thus, the analysis of application of lactation models of P.Wood and Prasad-Singh to describe the lactation dynamics of Holstein cows indicates about their high efficiency. The comparison of these models revealed a higher level of approximation of the factual monthly yields using equation Prasad-Singh, that confirmed by the corresponding parameters. Furthermore, the theoretical curve by the model of P.Wood at the initial stage relatively rapidly growing that causes displacement of the peak of the theoretical curve closer to the first month and underestimation of peak productivity. In the context of foregoing the model of Prasad-Singh builds the lactation curve much closer to the factual data, and accordingly capable adequately characterize the dynamics of monthly milk yield of Holstein cows.

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