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# CONNECTING THE ARCS OF TWO CIRCULAR CURVES IN *THE DESIGN AND RECONSTRUCTION OF HIGHWAYS*

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***Abstract.*** *The safety and conditions of road transport traffic significantly depend on the influence of curved sections. For practical use, methods for designing horizontal curves are constantly being improved. At present, there are no simple and reliable methods for designing transition curves for connecting two one-way circular curves in scientific publications. Existing methods for finding optimal transition curves use iterative processes and specially developed software products. Therefore, improving the methodology for solving the problem of connecting circular curves is of practical importance. The paper considers the main options for forming curved sections with two circular curves - connecting by straight inserts, arcs of circles of larger radius and clothoids, as well as searching for a clothoid that is common to two circular curves and ensures the preservation of their centers. It is proved that with a known position of the centers and arcs of circular curves of given radii , the search for the optimal clothoid can be performed by the standard function "Solution Search" of the " Microsoft" menu. Excel ". Possible options for the location of the extreme points of the clothoid on existing or designed circular curves are given by the directional angles between the centers of the curves and the starting and ending points of the circular curves. Examples of calculations of direct inserts and clothoids for connecting two circular curves are given.*

***Keywords:*** *circular curve, geometric connection conditions, direct insertion, clothoid, optimization, solution search.*

**Introduction.** Design , construction and reconstruction of curved sections of highways requires further development and implementation of innovative technologies for their geodetic support. Smooth changes in the radius of curvature of the road are achieved by inserting transitional curves, which reduce the negative impact of centrifugal acceleration on vehicles and road users. In this case, a clothoid with a linear dependence of curvature on the length of its arc is mainly used. Less commonly, other spirals, cubic parabolas and polynomials of higher degrees are used as transitional curves.

***Topicality.*** A special case can be considered the presence of two horizontal curves turned in one direction, the distance between which does not allow the traditional construction of transition curves. Connecting such circular curves with short straight inserts is not recommended, and replacing them with one circular curve of a larger radius leads to a significant planned displacement of the primary curves. A transition curve can serve as the optimal connection of two adjacent circular curves. Existing methods for modeling such a transition curve usually use an iterative optimization process and have limited practical application. In this regard, the search for simple and reliable methods for designing transition curves between two curvilinear sections is of important practical importance.

***Analysis of recent research and publications.*** The current state building codes (DBN) for transport structures [1; 2; 3] regulate the issue of connecting two equally directed circular curves on highways, railways and in the subway in different ways. It should also be noted that the relevant standards for highways and subways have changed compared to the previous (no longer valid) DBN [4; 5]. For example, earlier for highways, depending on the mutual location of such circular curves, it was recommended to replace them with one circular curve of a larger radius, connect them with a transitional curve or a direct insert [4, item 2.30]. For subway tracks, it was also provided that transitional curves are inserted between circular curves of different radii [5, item 7.2]. The current standards [1, item 5.1.11] and [2, item 7.8] only note the need in such cases to design a single-slope transverse profile with a bend offset on part of the circular curve and on the adjacent straight section of the route.

The construction of a transition curve between such circular curves is considered, for example, in works [6-10]. The authors of the monograph [6] most fully and in detail investigate the modeling of various curves with variable radii and optimization of the search for unknown coefficients of the equation of curvature distribution and arc length between endpoints using the Hooke- Jeeves algorithm . The formulas for calculating the coordinates of the clothoid connecting two circular curves are given in work [7] with an unknown length of the transition curve, which must be determined by the approximation method. Variants of constructing a transition curve in the form of a quadratic spiral [8], a swarm of arc particles [9], a cubic parabola [10], etc. are also considered. It is obvious that the need to use the iteration method to find the final version of the transition curve complicates the implementation of these algorithms in engineering practice.

***Purpose and objectives of the study.*** The study was conducted to assess the possibility of directly determining the unknown parameters of a transition curve connecting two equally directed circular curves, and to develop algorithms for calculating and objectively monitoring the transition curve.

***Materials and methods.*** The justification of algorithms for connecting two circular curves by transition curves was carried out using methods of analysis, synthesis, and mathematical modeling.

**Presentation of the main material.**

***Problem statement.*** The combination of two circular curves of different radii into a continuous path can be done by straight inserts (Fig. 1, a) or by transitional curves, which at the connection points must be tangent to the circular curves. In addition, the circular curves and the straight insert can be replaced by one circular curve (Fig. 1, b ), which, if necessary, can be connected to the straight sections by transitional curves (Fig. 2, a) or replaced by a biclothoid [11] (Fig. 2, b ) In the considered cases of using circular and transitional curves, the existing or designed route is displaced (see Fig. 1-2), which may be impossible or inappropriate due to local conditions (presence of obstacles, urban planning or land management restrictions, etc.).

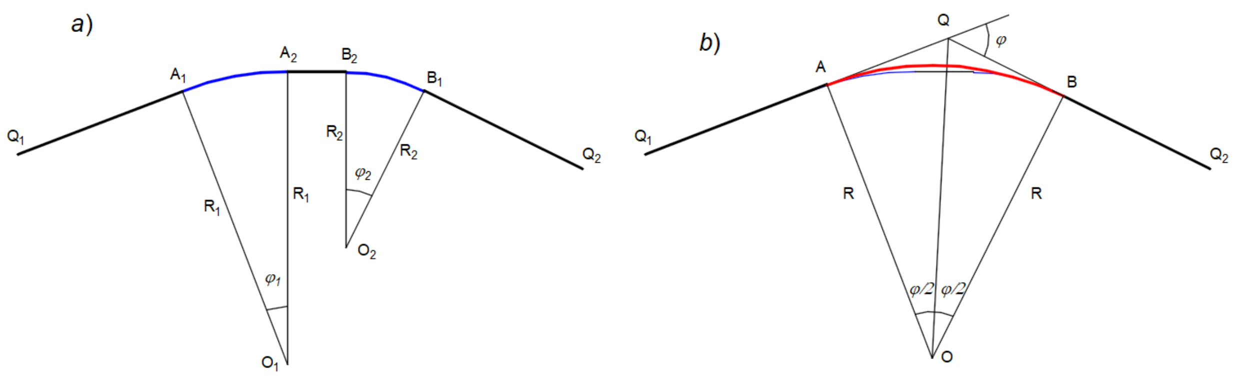

**Figure. 1 - Route diagram: a) two circular curves ($A_1A_2$, $B_1B_2$) and a straight insert ($A_2B_2$); b) replacement of curves with one circular curve ($AB$)**

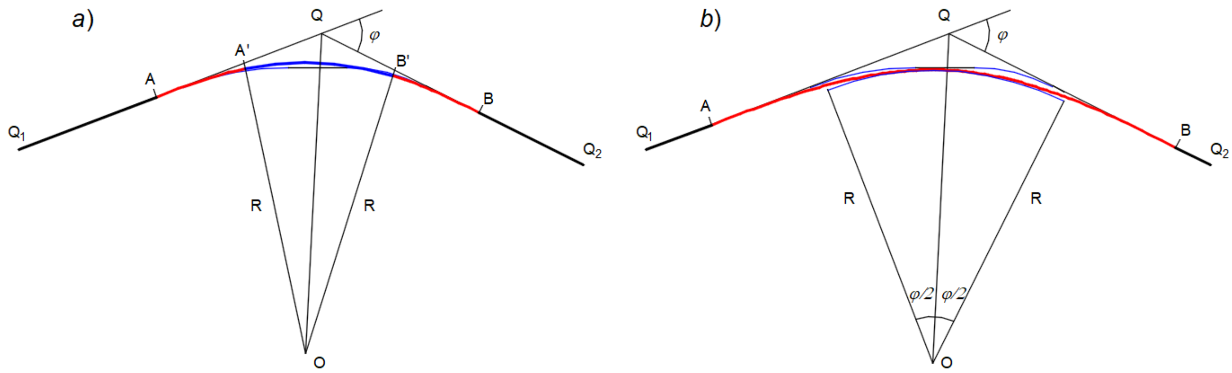


**Figure. 2 - Route diagram: a) two clothoids (AA', BB') and a circular curve (A'B'); b ) replacing curves with a biclothoid (AB)**

In such cases, it is better to connect two circular curves with a transition curve, the spatial location of which will be close to the section $A_1B_1$ of the existing route.

***General provisions for the design of a transition curve between two circular curves.*** For two circular curves $A_1A_2$ and $B_1B_2$ (see Fig. 1, a) it is possible to construct an infinite number of various transition curves, the extreme points of which will be within the arcs of the corresponding curves. The length of such transition curves will depend on their type and the distance of the beginning and end of the transition curve from the end points of the circular curves $A_2$ and $B_2$ , therefore, in practice, it is advisable to limit ourselves to the interval from the middle to the end of the circular curves.

When designing the transition curve *AB,* you can arbitrarily choose its starting point ($A$) in the given interval of the arc $SK_1$-$A_2$ of the first curve and determine its coordinates $(x_A, y_A)$ using the well-known formulas:

$$\left.\begin{aligned} x_A &= x_{O_1} + R_1 cos\alpha_{O_1A} \\ y_A &= y_{O_1} + R_1 sin\alpha_{O_1A} \end{aligned}\right\}. \tag{1}$$

where $x_{O_i}$, $y_{O_i}$ and $R_1$ – coordinates of the center and radius of the first circular curve; $\alpha_{O_1A}$ – the directional angle of the line from the center of the circle to point $A$.

Similarly, the end point of the transition curve ($B$) in a given interval is determined arc $B_2$-$SK_2$ of the second curve and its coordinates ($x_B, y_B$) are determined.

The procedure for finding a transition curve between two existing circular curves (Fig. 3) consists in finding such a tangent curve for which the geometric parameters of the transition and circular curves coincide at the connection points ($i$ = 1, 2):

$$\varphi_i = \gamma_i; \quad (2)$$

$$\rho_i = R_i, \quad (3)$$

and the calculated coordinates of the centers of the circles satisfy the equation:

$$\sqrt{\left(x_{O_2} - x_{O_1}\right)^2 + \left(y_{O_2} - y_{O_1}\right)^2} = b_{12}, \quad (4)$$

where $\varphi_i$ and $\gamma_i$ – turning angles transition and circular curves at their connection points;

$\rho_i$ and $R_i$ – radii transition and circular curves at their connection points;

$x_{O_i}$, $y_{O_i}$ - coordinates of the centers of circular curves relative to the beginning of the transition curve;

$b_{12}$ – the distance between the centers of the given circular curves.

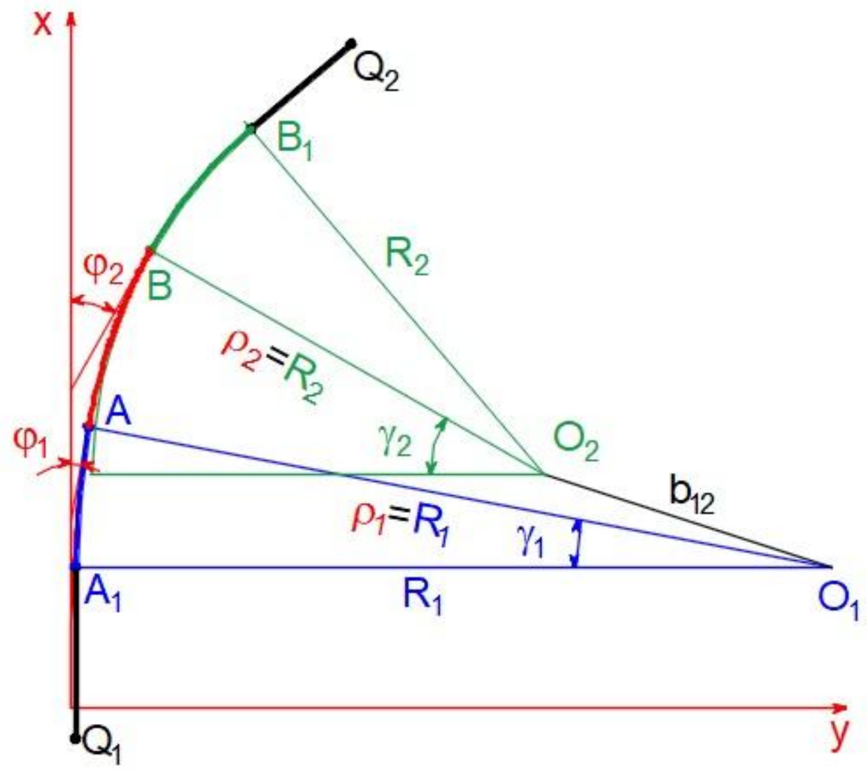


**Figure. 3. Scheme of connecting two circular curves by a clothoid**

Regardless of the type of transition curve, the coordinates of the centers of circular curves can be determined by the formulas:

$$\left.\begin{aligned} x_{O_i} &= x_i - R_i sin\varphi_i; \\ y_{O_i} &= y_i + R_i cos\varphi_i, \end{aligned}\right\} \qquad (5)$$

where $x_j$, $y_j$ ($j$ = $A$, $B$) – conditional coordinates of the connection points of the transition and circular curves.

Full compliance with the geometric conditions for connecting two circular curves (2)-(4) can be achieved by using a clothoid (Fig. 3), but it does not always provide such a connection. For example, if the radii of the circular curves are the same, then it is impossible to construct a clothoid between them. The practice of connecting circular curves with straight sections shows that changing the radii at the points of their connection has less effect on traffic safety than the discrepancy in the angles of rotation. Therefore, the connection of two circular curves can be done by straight inserts and transitional curves, which are tangent to the circular curves at the connection points.

***Calculation of the straight insert.*** The boundary data for designing the transition curve should be the positions of the main points of the circular curves. To construct a straight line segment that is tangent to two circles, we find the coordinates of points $A_2$ and $B_2$ (see Fig. 1), at which the straight insert connects to the circular curves:

$$\left.\begin{aligned} x_{A_2} &= x_{O_1} + R_1 cos\alpha_{1A_2} \\ y_{A_2} &= y_{O_1} + R_1 sin\alpha_{1A_2} \end{aligned}\right\}; \qquad \left.\begin{aligned} x_{B_2} &= x_{O_2} + R_2 cos\alpha_{2B_2} \\ y_{B_2} &= y_{O_2} + R_2 sin\alpha_{2B_2} \end{aligned}\right\}, \qquad (6)$$

where $\alpha_{1A_2}$, $\alpha_{2B_2}$ – unknown directional angles of lines from the centers of the circles and points $A_2$, $B_2$.

Having solved the inverse geodesic problem by the coordinates of the centers of circular curves

$$tg\alpha_{O_1O_2} = \frac{y_{O_2} - y_{O_1}}{x_{O_2} - x_{O_1}}; \qquad (7)$$

$$b_{12} = \frac{y_{O_2} - y_{O_1}}{sin\alpha_{O_1O_2}} = \frac{x_{O_2} - x_{O_1}}{cos\alpha_{O_1O_2}} \qquad (8)$$

and determining the horizontal angle ($\omega$) between the lines $A_2B_2$ and $O_1O_2$

$$\cos\omega = \frac{R_1 - R_2}{b_{12}}, \tag{9}$$

can calculate unknown direction angles $\alpha_{O_1A_2}$, $\alpha_{O_2B_2}$ by the formula:

$$\alpha_{O_1A_2} = \alpha_{O_2B_2} = \alpha_{O_1O_2} - \omega. \tag{10}$$

An example of calculating the coordinates of the connection points of a straight insert with circular curves is given in Table 1.

**Table 1. Calculation of direct insertion**

| | | | |
|---|---|---|---|
| $x_{O_1}$ = | 1250 m | $x_{O_2}$= | 1200 m |
| $y_{O_1}$ = | 2500 m | $y_{O_2}$= | 2550 m |
| $R_1$ = | 250 m | $R2$ = | 150 m |
| $b_{12}$ = | 111,803 m | $\alpha_{O_1O_2}$= | 26.565051° |
| $\omega$= | 26.565051° | $\alpha_{O_1A_2}$= | 0.0° |
| $x_{A_2}$ = | 1 50 0.000 m | $x_{B_2}$ = | 1 5 00,000 m |
| $y_{A_2}$ = | 25 00,000 m | $y_{B_2}$ = | 255 0.000 m |
| $d_{pr.vst.}$ = | 50,000 m | $\alpha_{pr.vst.}$ = | 90.0° |

***Construction of a circular arc between given points of two circular curves.*** In the case of designing the connection of two circular curves with the same radii by a transition curve, the tangential arc of the circle *AB can be used* (Fig. 4). It can be constructed using the property of the angle bisector between the tangent and the chord. With the known position of two points (*A* – the beginning of the curve, *B* – the end of the curve) and the center (*O*) of the circular curve, it is easy to find the direction angles ($\alpha_{AO}$, $\alpha_{BO}$) of the corresponding directions to the center of the circle and construct an auxiliary point *C'* (the vertex of the angle of rotation).

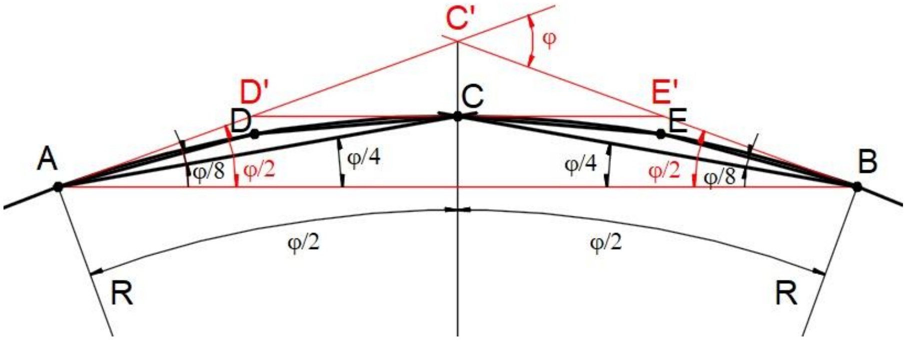


**Figure. 4. Graphical construction of arc *AB* of a circular curve**

The graphic construction of intermediate points (*C*, *D*, *E*, …) consists in the sequential division of the known arc into two equal parts. For example, the position of point *C* is found at the intersection of the bisectors *AC* and *BC* of the angles between the tangents *AC'*, *BC'* and the chord *AB* (see Fig. 4). Similarly, relative to the chord *AC* we determine the position of point *D*, and from the chord *CB* – point *E*. Repeating such actions with new chords *AD*, *DC*, *CE*, *EB*, etc., we find the positions of a sufficient number of intermediate points of the given arc *AB*.

Gaussian formulas for an azimuthal serif. But the accuracy of the serif will be significantly reduced for small chords. It is more reliable to use the traditional method of calculating conditional rectangular coordinates and other geometric parameters of circular curves. To do this, from the coordinates of points *A*, *B* ( $x_A,\ y_A;\ x_B,\ y_B$) and the directional angles of lines $AO_1$ , $BO_2$ ($\alpha_{AO_1};\ \alpha_{BO_2}$), find the angle of rotation ($\varphi$) and the radius of the connecting arc (*R*) using the formulas:

$$\varphi = \alpha_{BO_2} - \alpha_{AO_1}; \tag{11}$$

$$R = \frac{(x_B - x_A)sin\alpha_{AO_1} - (y_B - y_A)cos\alpha_{AO_1}}{sin(\alpha_{AO_1} - \alpha_{BO_2})} \tag{12}$$

or

$$R = \frac{(x_B - x_A)sin\alpha_{BO_2} - (y_B - y_A)cos\alpha_{BO_2}}{sin(\alpha_{AO_1} - \alpha_{BO_2})}. \tag{13}$$

In the case of joining two circular curves with different radii, the transition curve can be similarly projected as a tangent arc of the circle *AB* .

***Calculation transition curve in the form of a clothoid.*** The general principle of constructing a clothoid requires that both connecting circular curves be tangent to the clothoid at the initial (*A*) and final (*B*) points of the transition curve *AB* .

The basic equations of the clothoid relate the radius ($\rho_i$) and rotation angle ($\varphi_i$) of the current point to its distance ($l_i$) from the origin and the clothoid parameter (*RL*):

$$\rho_i = \frac{RL}{l_i}; \qquad \varphi_i = \frac{l_i^{\,2}}{2RL}. \tag{14}$$

Let us denote unknown parameter *RL* clothoids through *C*, then for the transition curve *AB* after transformations we have (*i* =1, 2):

$$l_i = \frac{C}{R_i}; \qquad \varphi_i = \frac{C}{2R_i^{\,2}}, \tag{15}$$

where $l_i$, $\varphi_i$, $R_i$ – elements of the clothoid at its initial *A* (*i*=1) and final *B* (*i*=2) points.

Considering that the difference in the angles of rotation $\varphi_2$ and $\varphi_1$ is related to the directional angles of the lines $\alpha_{AO_1}$ and $\alpha_{BO_2}$ lines $AO_1$ and $VO_2$ ( Fig. 5)

$$\Delta\varphi = \frac{C}{2}\left(\frac{1}{R_1^{\,2}} - \frac{1}{R_2^{\,2}}\right), \tag{16}$$

we find the unknown parameter of the clothoid by the formula:

$$C = 2 \cdot \Delta\varphi \cdot R^2, \tag{17}$$

where

$$R^2 = \frac{R_1^{\,2} \cdot R_2^{\,2}}{R_1^{\,2} - R_2^{\,2}}. \tag{18}$$

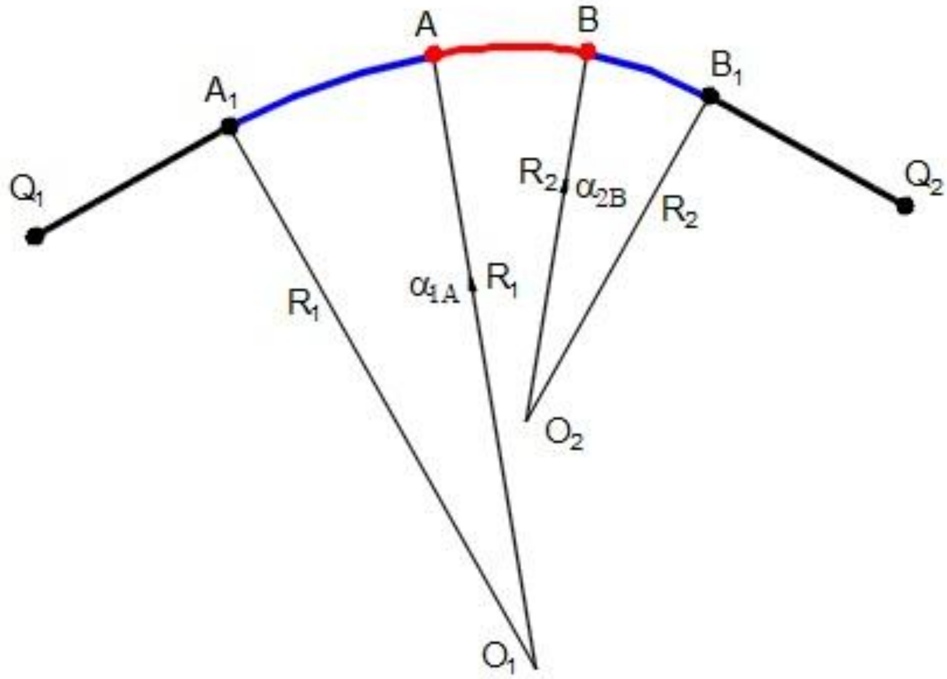


**Figure. 5. The connecting of two circular curves by a clothoid**

Let us consider an example of determining the clothoid parameter (see Fig. 4) using the initial data given in Table 2.

**Table 2. Initial data for calculating clothoid**

| | | | | | |
|---|---|---|---|---|---|
| $x_{O_1}$ = | 600,000 m | $x_{O_2}$= | 699,485 m | $\alpha_{O_1O_2}$= | 357,620° |
| $y_{O_1}$ = | 500,000 m | $y_{O_2}$= | 495,866 m | $b_{12}$= | 99,571 m |
| $R_1$= | 250,000 m | $R_2$= | 150,000 m | $R^2$= | 35156,250 m$^2$ |
| $\alpha_{O_1A}$= | 350,5° | $\alpha_{O_2B}$= | 9,5° | $\Delta\varphi$= | 19,0° |
| $x_A$ = | 846,571 m | $x_B$ = | 847,428 m | $\alpha_{AB}$= | 89,207° |
| $y_A$ = | 458,738 m | $y_B$ = | 520,623 m | $d_{AB}$= | 61,891 m |

The unknown parameter of the clothoid is calculated using formula (17):

$$C = 2 \cdot 0{,}33161256 \text{ рад.} \cdot 35156{,}250 \text{ м}^2 = 23316{,}508 \text{ м}^2.$$

Now the coordinates of the initial $A$ and final $B$ points of the transition curve and the centers of the circular curves $O_1$ and $O_2$ can be calculated using the well-known formulas:

$$\left.\begin{aligned} x_i &= \frac{C}{R_i} - \frac{C^3}{40 \cdot R_i{}^5} + \frac{C^5}{3456 \cdot R_i{}^9} - \cdots; \\ y_i &= \frac{C^2}{6 \cdot R_i{}^3} - \frac{C^4}{336 \cdot R_i{}^7} + \frac{C^6}{42240 \cdot R_i{}^{11}} - \cdots \end{aligned}\right\} \quad (19)$$

and

$$\left.\begin{aligned} x_{O_i} &= \frac{C}{2 \cdot R_i} - \frac{C^3}{240 \cdot R_i{}^5} - \frac{C^5}{34560 \cdot R_i{}^9} + \cdots; \\ y_{O_i} &= R_i + \frac{C^2}{24 \cdot R_i{}^3} - \frac{C^4}{2688 \cdot R_i{}^7} + \frac{C^6}{506880 \cdot R_i{}^{11}} - \cdots \end{aligned}\right\}. \quad (20)$$

To check the correspondence of the found clothoid to the original data, it is necessary to compare the original lengths of the lines $O_1 O_2$ and *AB* with their calculated values (Table 3). In the case of satisfying the geometric conditions of the connection of two circular curves (2)-(4), all the necessary data for the transition curve are determined using the traditional method. To convert the coordinates of the clothoid points to the original system, the calculated and original coordinates of the centers of the circular curves are used.

**Table 3. Calculated data of clothoid**

| $C$= | 23316,508 m$^2$ | $\Delta\varphi$= | 19,00° | $d_{AB}$= | 61,892 m | $b_{12}$= | 99,571 m |
|---|---|---|---|---|---|---|---|
| Точки | $l$, m | $\rho$, m | $\varphi$° | $x_i$, m | $y_i$, m | $x_{O_i}$, m | $y_{O_i}$, m |
| *A* | 93,266 | 250,000 | 10,69 | 92,942 | 5,785 | 46,579 | 251,448 |
| *B* | 155,443 | 150,000 | 29,69 | 151,322 | 26,337 | 77,031 | 156,648 |

If the calculated lengths of lines *AB* ($d_{AB}$) and $O_1O_2$ ($d_{12}$) do not correspond to their original values, then the given points *A* and *B* do not belong to the same clothoid.

***Research results and their discussion.*** To determine the possibility of connecting the given circular curves with a clothoid, it is necessary to analyze the entire combination of the mutual location of points in the intervals between the beginning and end of each curve. For example, the points of the previously considered circular curves (see Table 2) can be located in the ranges of the directional angles of the lines $340° \leq \alpha_{AO_1} \leq 360°$and $0° \leq \alpha_{BO_2} \leq 20°$. Determining the optimal location of the clothoid is possible using the standard function "Solution Search" («Solver») of the menu "Microsoft Excel" (Table 4 – optimization task, Table 5 – optimization result). In this case, optimization occurs according to the objective function

$$F = b_{12} - b'_{12} = 0, \quad (21)$$

where $b_{12}$ – the initial value of the distance between the centers of circular curves (4); $b'_{12}$ – calculated value of the distance between the centers of circular curves.

**Table 4. Output data for "Solution Search"**

| Marking | *A* | *B* | Marking | *A* | *B* |
|---|---|---|---|---|---|
| $R_i$= | 250,000 m | 150,000 m | $x_i$= | 193,343 m | 290,455 m |
| $\varphi_i$= | **340,00°** | **20,00°** | $y_i$= | 25,420 m | 109,261 m |
| $\Delta\varphi$= | 40,00° | 0,69813 rad. | $d_{AB}$= | 61,892 m | 128,296 m |
| $R^2$= | 35156,250 $m^2$ | | $x_{Oi}$= | 97,672 m | 157,349 m |
| $C$= | 49087,385 $m^2$ | | $y_{Oi}$= | 256,390 m | 178,516 m |
| $l_i$= | 196,350 m | 327,249 m | $d_{12}$= | 99,571 m | 98,111 m |
| | | | ***F*** **=** | **-1,460 m** | |

When forming the optimization task (Table 4), the initial data are the radii of the circular curves ($R_1$ , $R_2$), one of the possible options for the angles of rotation ($\varphi_1 = \alpha_{O_1A}$, $\varphi_2 = \alpha_{O_2B}$) and the distance between the centers of the given circular curves ($b_{12}$). In the control example (see Table 3), in addition, the chord length between points *A* and *B* ($d_{AB}$ ) is known. All other values in Table 4 are calculated by formulas (15), (17)-(21) and (4) from the radii of the circular curves ($R_1$ , $R_2$) and the difference in the initial angles of rotation ($\Delta\varphi$). Since the objective function (F=-1.460 m) does not satisfy condition (21), it is necessary to find the optimal solution by changing the possible values of the angles of rotation ($\varphi_1$) and ($\varphi_2$) in the given ranges.

After accessing the standard "Solution Search" function, the calculated parameters take values corresponding to the optimal solution (Table 5) and coincide with the calculated data of the adopted clothoid model to an accuracy of 1 mm (see Table 3).

**Table 5. Result of optimization of the objective function**

| Marking | *A* | *B* | Marking | *A* | *B* |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| $R_i$= | 250,000 m | 150,000 m | $x_i$= | 92,943 m | 151,323 m |
| $\varphi_i$= | **350,498°** | **9,498°** | $y_i$= | 5,785 m | 26,337 m |
| $\Delta\varphi$= | 19,00° | 0,33161 rad. | $d_{AB}$= | 61,892 m | 61,892 m |
| $R^2$= | 35156,250 $m^2$ | | $x_{Oi}$= | 46,579 m | 77,032 m |
| $C$= | 23316,679 $m^2$ | | $y_{Oi}$= | 251,448 m | 156,648 m |
| $l_i$= | 93,267 m | 155,445 m | $b_{12}$= | 99,571 m | 99,571 m |
| | | | ***F*=** | **4,1E-10 m** | |

In cases where there is no optimal solution, instead of a clothoid, we can recommend, for example, an arc of a circle of a larger radius (see Fig. 4) to connect two given circular curves.

***Conclusions.*** The most complete fulfillment of the geometric conditions for the connection of two circular curves is achieved using a clothoid. In the work-It has been proven that there is not always a clothoid that provides a correct connection of two circular curves. In such cases, for example, an arc of a circle of larger radius between selected points of the connected circular curves can be used.

The formation of possible options for the location of the initial and final points of the clothoid is achieved by specifying the initial coordinates of the centers of the circular curves and setting the range of directional angles of the lines between the points of the circular curves and their centers. To find a clothoid that is common to two circular curves and ensures the preservation of their centers, It is suggested to use the standard "Find a solution" function of the menu "Microsoft Excel".

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**З'ЄДНАННЯ ДУГ ДВОХ КОЛОВИХ КРИВИХ ПРИ ПРОЄКТУВАННІ ТА РЕКОНСТРУКЦІЇ АВТОМОБІЛЬНИХ ДОРІГ**

***Анотація.*** *Безпека та умови руху автомобільного транспорту суттєво залежать від впливу криволінійних ділянок. Для практичного використання постійно удосконалюються методи проєктування горизонтальних кривих. На цей час в наукових виданнях відсутні прості та надійні методи проєктування перехідних кривих для з'єднання двох однобічно направлених колових кривих. Наявні методи пошуку оптимальних перехідних кривих використовують ітераційні процеси та спеціально розроблені програмні продукти. Тому вдосконалення методики розв'язання задачі з'єднання колових кривих набуває практичного значення. В роботі розглянуті основні варіанти формування криволінійних ділянок з двома коловими кривими - з'єднання прямими вставками, дугами кіл більшого радіусу та клотоїдами, а також пошук клотоїди, яка є спільною для двох колових кривих та забезпечує збереження їх центрів. Доведено, що при відомому положенні центрів та дуг колових кривих заданих радіусів, пошук оптимальної клотоїди може виконуватися стандартною функцією «Пошук рішення» меню «Microsoft Excel». Можливі варіанти розташування крайніх точок клотоїди на існуючих або запроєктованих колових кривих задається дирекційними кутами між центрами кривих та початковими і кінцевими точками колових кривих. Наведені приклади розрахунків прямих вставок та клотоїд для з'єднання між собою двох колових кривих.*

***Ключові слова:*** *колова крива, геометричні умови з'єднання, пряма вставка, клотоїда, оптимізація, пошук рішення.*