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USING THE THEORY OF COMBINATIONS AT THE MODELING OF LAND PLOTS EXCHANGE IN AGRICULTURAL LAND MASSES

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Abstract. *The problem of land reallotment modeling in agricultural land masses aiming at land consolidation is scrutinized in the article. The topicality of the research is predefined by the need for the optimization of the structure of land tenure within an agricultural land mass, the implementation of measures on land consolidation. The research is aimed at the substantiation of the theory of combinations aiming at the improvement of the exchange of land plots, i.e. the choosing of alternative land plots within an agricultural land mass. The calculations are based on choosing the land plots to be exchanged to provide the optimal for agricultural activities spatial characteristics of land plots. The calculation of the variants of the formation of the consolidated land tenure by a land owner who has the right to own the prevailing share of a typical agricultural land mass has been suggested. The results of the research can be used at the realization of complex and individual land consolidation projects; at the optimization of land tenure by the sublease of land plots; as a material for the substantiation of the decision making on the land tenure optimization by territorial communities; in the following research on spatial land tenure and land ownership spatial improvement.*

Key words: *combinations theory, land consolidation, land plots exchange, land reallotment, mathematical modeling.*

Topicality

The international experience approved that land consolidation is one of the most efficient mechanisms of land plots spatial characteristics improvement, with the help of which the aims of sustainable land tenure are achieved. Among a great number of methods and mechanisms of land consolidation land plots exchange is very important [1]. Taking into consideration the structure of agricultural land and the problem of the formation of overlapped land plots, considerable attention is given to the land tenure optimization within the agricultural land mass. For example, there was an attempt to regulate this issue with the Law of Ukraine № 2498-VIII «On the Alteration of some legislative acts of Ukraine concerning the issue of the collective ownership of land, the improvement of the rules of land use in agricultural land masses, preventing raidership and stimulating the irrigation in Ukraine» [2].

But, the questions of the improvement of the mechanisms of exchange (sublease -as the exchange of the right of use) is an open question, especially, in the context of the improvement of the technical constituent of the process.

Analysis of recent research and publications

The question of the optimization of the consolidation of land is widely scrutinized in the researches of A. Hendricks, A.Vitikainen, D. Demetriou, R. Giovarelli, D. Bledsoe, M. Hartvigsen, J. Thomas and others. In the works of Ja. Janus, E. Ertunc, Ya. Inceyol, T. Cay and others the problems of the modeling of the exchange as a part of land consolidation and as a separate measure are scrutinized. But, the question of the mathematical modeling of the exchange of of land in Ukraine is not reflected enough in the scientific sources. As a rule, the existing researches draw attention on the legal and institutional aspects of the exchange and consolidation of land. The wide possibilities of the implementation of the mathematical apparatus for the automatization and optimization of the exchange of land are not used.

The aim of the research is to substantiate the use of the theory of combinations aiming at the improvement of the exchange of land plots.

Materials and methods of research

Land consolidation aims at the formation of land plots with optimal spatial

characteristics among the possible variants. The question of the exchange of land plots within the agricultural land mass predefines the search of optimal variants of reallotment [3]. At the peer land plots exchange within an agricultural land mass, it is suggested to calculate the readjustment options using the theory of combinations [4].

At the substantiation of the placement of land plots, it is considered that the consolidated land tenure should be compact and close to a rectangle with the side ration of no more than 1:4 [5].

Let us scrutinize a land mass which includes *n* land plots. The quantity of the placement options of *m* consolidated land plots provided $n>m$ is:

$$N = \frac{n!}{(n-m)!}. \tag{1}$$

Provided the mass includes $n_h \times n_l = n$ land plots, the quantity of options *N* of the formation of the consolidated land tenure $d \times p$ land plots with the possibility of the involvement of all land plots of the mass is:

$$N = (n_l - p + 1) \times C^1_{n_l - d + 1} \times P_{p \times d}, \tag{2}$$

$$N = (n_l - p + 1) \times \left(\frac{(n_h - d + 1)!}{(n_h - d + 1 - 1)!} \times (d \times p)! \right), \tag{3}$$

$$N = (n_l - p + 1) \times (n_h - d + 1) \times (d \times p)!. \tag{4}$$

To remove overlapping, let us scrutinize the placement of the consolidated land plots at the boundary of the land mass.

The quantity of variants of placement of a separate land plot at the boundary of the land mass is:

$$N = 2C^1_{n_h} + 2C^1_{n_l - 2}, \tag{5}$$

$$N = 2\left(\frac{n_h!}{(n_h - 1)!} \right) + 2\left(\frac{(n_l - 2)!}{(n_l - 3)!} \right), \tag{6}$$

then

$$N = 2(n_h + n_l - 2). \tag{7}$$

Let us scrutinize the placement of m land plots at the boundary of the land mass consisting of n land plots with $n_h \times n_l = n$ the area of land plots.

The initial precondition for the calculation of the reallotment options:

$$\begin{cases} 1 < m \le n_h, \\ 1 < m \le n_l . \end{cases} \tag{8}$$

The quantity of variants of the placement of *m* consolidated land plots at the boundary of the land mass is:

$$N = 2(C^1_{n_h - m+1} \times P_m) + 2(C^1_{n_l - m+1} \times P_m). \tag{9}$$

i.e.,

$$N = 2m! \left(\frac{(n_h - m+1)!}{(n_h - m)!} + \frac{(n_l - m+1)!}{(n_l - m)!} \right), \tag{10}$$

$$N = 2m!(n_h + n_l - 2m + 2). \tag{11}$$

Predefined by the formula (11), number of derangements within a consolidated land mass predefines the possibility of the optimal selection of peer land plots at the exchange.

Let us scrutinize the placement of the consolidated land plots at the land mass boundary in the form of a rectangle consisting of $d \times p$ land plots:

$$d \times p = m, \tag{12}$$

where m - is the general number of the reallocated land plots.

The case when the following condition is met, is scrutinized:

$$\begin{cases} n_h \ge d, \\ n_l \ge p , \\ d \ne p. \end{cases} \tag{13}$$

The total number of land plots reallocation options according to the specified conditions:

$$N = 2(C^1_{n_h - d+1} \times P_m) + 2(C^1_{n_l - d-1} \times P_m). \tag{14}$$

Then:

$$N = 2\left(\frac{(n_h - d+1)!}{(n_h - d+1-1)!} \times (d \times p)! \right) + 2\left(\frac{(n_l - d+1)!}{(n_l - d+1-1)!} \times (d \times p)! \right), \tag{15}$$

$$N = 2(n_h - d+1)(d \times p)! + 2(n_l - d+1)(d \times p)!, \tag{16}$$

$$N = 2(\mathrm{d}\times\mathrm{p})!\left(n_h - d + 1 + n_l - d + 1\right), \tag{17}$$

$$N = 2(\mathrm{d}\times\mathrm{p})!\left(n_h + n_l - 2d + 2\right). \tag{18}$$

If p=1, then $d = m$, so, formula (18) is converted into the formula (11).

Let us scrutinize the case the mass has a form of n-sided polygon, on the side i of which n land plots are placed.

Then, provided:

$$1 < m \le n_i \tag{19}$$

the quantity of variants of the placement of m land plots at the boundary of the land mass is:

$$N = \sum_{i=3}^{n} \left(C^1_{n_i - m + 1} \times P_m\right). \tag{20}$$

i.e.:

$$N = m! \sum_{i=3}^{n} \frac{(n_i - m + 1)!}{(n_i - m)!}, \tag{21}$$

$$N = m! \sum_{i=3}^{n} n_i - m + 1. \tag{22}$$

The total quantity of the exchange options provided the following conditions are met:

$$\begin{cases} n_h \ge d, \\ n_l \ge p \end{cases} \tag{23}$$

.

is:

$$N = 2\left(C^1_{n_h - d + 1} \times P_m\right) + 2\left(C^1_{n_l - p - 1} \times P_m\right), \tag{24}$$

$$N = 2\left(\frac{(n_h - d + 1)!}{(n_h - d + 1 - 1)!} \times (d \times p)!\right) + 2\left(\frac{(n_l - p - 1)!}{(n_l - p - 1 - 1)!} \times (d \times p)!\right), \tag{25}$$

$$N = 2\left(n_h - d + 1\right)(d \times p)! + 2(n_l - p - 1)(d \times p)!, \tag{26}$$

$$N = 2(\mathrm{d}\times\mathrm{p})!\left(n_h - d + 1 + n_l - p - 1\right), \tag{27}$$

$$N = 2(\mathrm{d}\times\mathrm{p})!\left(n_h + n_l - d - p\right). \tag{28}$$

Formula (28) can be used when d=p. If d=p=1, then from formula (28) we get

the formula (7).

According to the formula (28), the placement of the consolidated land plots $d \times p$ has the same number of options, irrespective of the placement along a certain side of the land mass.

Research results and discussion

Let us scrutinize the search of the alternative land plots exchange options within land masses depicted in the Fig.1.

Fig.1. Agricultural land masses, within of which the substantiation of the alternative land plots exchange options is performed

For the land mass in the Fig. 1 «а», which includes 162 land plots, the quantity of the placement options for five consolidated land plots at the boundary of the land mass according to the formula (9) is:

$$N = 5!\ 50 = 6000.$$

For the land mass in the Fig. 1 «б», which includes 197 land plots, the quantity of the placement options for five consolidated land plots at the boundary of the land mass according to the formula is 5!45=5400.

Let us scrutinize the formation within the land mass in the Fig.1 «б» of a consolidated land tenure by a person who has a right to use the major share of the agricultural land mass (further - consolidated land tenure). For the given mass, according to the Land Code of Ukraine, such a consolidated land tenure includes at least 122 land plots.

The minimal quantity of variants of the formation of the consolidated land tenure is calculated using formulas (9) and is:

$$N = P_{40} \times C_{27}^{1}$$

$$N = 40!\ 27 = 8{,}16*10^{47}$$

The precondition, that the overlapped land plots are at the land mass boundary, is taken into consideration. In this case the consolidated land tenure is formed so, that every land plot included in it (but for those at the angles) borders on at least two land plots which are also a part of such land tenure. If such a precondition is not set, the quantity of variants of the formation of the consolidated land tenure is:

$$N = P_{40} \times C_{66}^{40} = 1{,}35*10^{66}$$

The quantity of variants of the formation of the consolidated land tenure by the land owner who has the ownership right for the major share of land mass points out at the practicability of the automatization and modeling of the exchange of land plots aiming at the consolidation within an agricultural land mass.

The modeling of the exchange of land plots based on the theory of combinations was included into land consolidation and reallotment models presented in the works [6-9], which are based on the heuristic [6] and optimizational methods [7, 8] and their combination [9].

Conclusions and perspectives

Taking into consideration the structure of agricultural land, it is reasonable to define the possible variants of the exchange of land plots using the theory of combinations. The formation of a land tenure with the most preferable for the agricultural activity configuration and placement is in the gist of the substantiation. It has been substantiated, how the search for the variants of exchange for a separate

overlapped land plot as well as for the formation of the consolidated land tenure by a land owner who has a right to own the major share of the land mass.

Defining the variants of exchange of land plots was used at the modeling using the heuristic and optimizational method, particularly, at the developing of the optimizational model. The suggested approaches can be used for the evaluation of the reallotment by the optimizational method.

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ВИКОРИСТАННЯ КОМБІНАТОРИКИ ПРИ МОДЕЛЮВАННІ ОБМІНУ ЗЕМЕЛЬНИХ ДІЛЯНОК В МАСИВАХ ЗЕМЕЛЬ СІЛЬСЬКОГОСПОДАРСЬКОГО ПРИЗНАЧЕННЯ

***Анотація**. У статті розглядається питання моделювання перерозподілу земель в масивах земель сільськогосподарського призначення з метою консолідації земель. Актуальність дослідження обумовлена необхідністю оптимізації структури землекористування в межах масивів земель сільськогосподарського призначення, впровадження заходів щодо консолідації земель. Дослідження має на меті обґрунтування застосування комбінаторики з метою вдосконалення обміну земельних ділянок, а саме підбору альтернативних варіантів земельних ділянок в межах масивів земель сільськогосподарського призначення. Розрахунки базуються на виборі обмінюваних земельних ділянок таким чином, щоб забезпечити оптимальні для сільськогосподарської діяльності просторові характеристики земельних ділянок. Запропоновано розрахунок варіантів формування консолідованого землекористування землевласником, який має право володіння переважною частиною типового масиву земель сільськогосподарського призначення. Отримані результати можуть бути використані в при реалізації проектів комплексної та індивідуальної консолідації земель; при оптимізації землекористувань шляхом суборенди земельних ділянок; як матеріали для підтримки прийняття рішень територіальними громадами щодо оптимізації землекористування; в наступних наукових дослідженнях, які стосуються просторового впорядкування землеволодінь і землекористувань.*

Ключові слова: *комбінаторика, консолідація земель, обмін земельних ділянок, перерозподіл земель, математичне моделювання.*